# Modified numerals and polarity sensitivity: Between O(nly)$_{\text{DA}}$ and E(ven)$_{\text{SA}}$[1]

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**Abstract.** Bare numerals (BNs), comparative-modified numerals (CMNs), and superlative-modified numerals (SMNs) exhibit interesting differences with respect to scalar implicatures, ignorance, sensitivity to grammatical polarity, and sensitivity to pragmatic polarity. Each of these differences has been used to conclude fundamental difference—between BNs and MNs, within MNs, and within SMNs. Revisiting the manifestation of the effects within numerals and across numerals and disjunction, indefinites, and minimizers, as well as connections between the effects, in this paper I argue for fundamental similarity and propose a unified account. The account crucially explores interactions between subdomain alternatives and classic, Horn-style scalar alternatives as well as between O(nly) and E(ven), arguing in particular that sensitivity to grammatical/pragmatic polarity comes from $\text{O}_{\text{DA}}/\text{E}_{\text{SA}}$, and that both O and E are ultimately driven by the same pressure: To justify the choice of the assertion in light of its true alternatives.

**Keywords:** modified numerals, subdomain alternatives, scalar alternatives, O(nly), E(ven)

## 1. Introduction

Naively speaking, comparative-modified numerals (CMNs; e.g., *more/less than 3*) and superlative-modified numerals (SMNs; *at least/most 3*) should simply denote a range on a scale, and, given reference to this scale, they should be pairwise truth-conditionally equivalent. However, they often exhibit effects much richer than that. Four interesting effects are as follows:

First, Horn (1972) showed that, in a plain upward-entailing (UE) environment, a bare numeral (BN; e.g., *3*) utterance such as *Jo called 3 people* is compatible with situations where she called 3 or 4 or ... people but strongly suggests that she did not in fact call 4 or ... people, so, that she called exactly 3 people. Horn proposes that this comes about as follows: BNs entail one bound—*3* entails 'at least 3'—but they belong to a natural scale—$\langle 0, 1, 2, 3, 4, \dots \rangle$—which naturally activates scalar alternatives—the scalemates—which, when factored into meaning, give rise to scalar implicatures—'not at least 4', 'not at least 5', etc.—such that BNs acquire another bound—*3* implicates 'not at least 4'—which is how their 'exactly' meaning. However, Krifka (1999) showed that on this view not just BNs but also CMNs and SMNs are predicted to give rise to exact meanings in that context also and that, while for BNs this is correct, for CMNs and SMNs this is incorrect. This is what we will call the Scalar Implicatures (SI) puzzle.

(1) a. Jo called **3 / more than 2 / at least 3** people.
$\rightsquigarrow$ not ✓4 / # more than 3 / # at least 4 (predicted meaning: exactly 3)

Second, Geurts and Nouwen (2007) showed that in the same UE environment CMNs are compatible with exact speaker knowledge but SMNs are not. Since this has been tied to an effect of ignorance, this is what we will call the Ignorance (IG) puzzle.

(2) a. Jo called **4** people. Therefore, she called ✓**more than 2 / # at least 3** people.

[1] I would like to thank Gennaro Chierchia, Kathryn Davidson, Anamaria Fălăuş, Roger Schwarzschild, the members of the Laboratoire Linguistique de Nantes, and the audience at *Sinn und Bedeutung 25* for useful suggestions and discussion. Any errors are my own.

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Third, Geurts and Nouwen (2007) showed that in a downward-entailing (DE) environment such as the scope of negation CMNs are fine but SMNs are degraded (i.e., they cannot take scope below negation), though in a DE environment such as the first argument of a conditional or a universal they are both fine. This is what we will call the Polarity Sensitivity 1 (POL1) puzzle.

(3) a. Jo did**n't** call **✓more than 2 / # at least 3** people.
b. **If** Jo called **✓more than 2 / ✓at least 3** people, she won.
c. **Every**one who called **✓more than 2 / ✓at least 3** people won.

Fourth, and last, Cohen and Krifka (2014) (and refs. therein) argued that, even in those DE environments where SMNs are reported to be fine, this is in fact not always so: In a positive antecedent of a conditional or restriction of a universal, SMNs are actually degraded if the continuation is pragmatically negative (lose), and in a negative antecedent of a conditionals or restriction of a universal they are degraded if the continuation is pragmatically positive (win). This is what we will call the Polarity Sensitivity 2 (POL2) puzzle.

(4) a. **If** Jo called **✓more than 2 / # at least 3** people, she lost.
b. **If** Jo didn't call **✓more than 2 / # at least 3** people, she won.

What is the reason for these patterns? How can we make sense of them all?

The existing literature has proposed answers of fundamental difference: SI has been used to argue that CMNs and SMNs are fundamentally different from BNs (Krifka, 1999 and most of the literature since); IG and POL1 has been used to argue that SMNs are fundamentally different from CMNs (for IG: Geurts and Nouwen, 2007 and most of the literature since, in various ways; for POL1: Geurts and Nouwen, 2007; Cohen and Krifka, 2014; Spector, 2015, in various ways); and POL2 has been used to argue that SMNs are fundamentally different from SMNs—that is, that SMNs have two different lexical entries (Cohen and Krifka, 2014).

In this paper I propose an answer of fundamental similarity: I revisit SI, IG, POL1, and POL2; reject the conclusion of a fundamental difference for all; and provide a unified account for all.

The plan is as follows: Expanding on my work in Mihoc (2020), and considering things in broader perspective (disjunction, indefinites), in §2 I reject the claim for IG and show that this helps us reject the claim for POL1 also. I propose a unified solution in terms of silent exhaustification relative to subdomain alternatives via contradiction-based O(nly). Building on this, and in broader perspective (disjunction, indefinites, minimizers), in §3 I reject the claim for SI and show that this helps us reject the claim for POL2 also. I propose a unified solution in terms of silent exhaustification relative to classic, Horn-style scalar alternatives via O(nly) and E(ven). In §4 I discuss empirical and theoretical predictions arising from all of this, especially concerning the relation between the subdomain alternatives and the classic, Horn-style scalar alternatives; O and E; O and *only* and E and *even*; and O/*only* and E/*even*. In §5 I conclude.

## 2. IG and POL1: A matter of subdomain alternatives and O(nly)

In this section I review and expand on the discussion and solution for IG and POL1 that I developed in Mihoc (2020). In the existing literature IG has been used to conclude fundamental difference between CMNs and SMNs, and POL1 has been used to support this conclusion. I reject the conclusion for IG and show that it helps us reject the conclusion for POL1 also.

2.1. IG: A matter of obligatory $O_{ExhDA}$ $\pm$ ability to use just SgDA or just NonSgDA

2.1.1. IG in numerals is richer and also includes considerable similarity

As mentioned, Geurts and Nouwen (2007) showed that, in a plain UE environment, CMNs are compatible with knowing that *n*, for some *n* in the range (what we will call a ONE-WINNER scenario, reflecting the fact that one element of the domain, *n*, is true in all the worlds compatible with what the speaker knows), but SMNs are not, (5a). This contrast, supported experimentally, has been used to argue that SMNs give rise to IG but CMNs do not (Geurts and Nouwen, 2007 and most of the literature since). Consistently, CMNs are also compatible with knowing that *not n* (what we will call a ONE-LOSER scenario), whereas SMNs are not, (5b). However, something not acknowledged enough: Both CMNs and SMNs are compatible with IG (Nouwen, 2015: 244), (5c), and both CMNs and SMNs can give rise to IG (Mayr and Meyer, 2014), (5d)—two similarities, also supported experimentally, which are especially striking when we consider that, in the same contexts, BNs can do neither. (Here and throughout, experimental data/sources are indicated on the right-hand margin of the relevant example line.)

(5) a. Jo called **4** people. So, she called **✓more than 2 / # at least 3** people. (100%/51%)[2]
b. Jo called **✓more than 2 / # at least 3** people, but not, 5.
c. I am not sure how many people Jo called, but it was **✓more than 2 / ✓at least 3**.[3]
d. Jo called **more than 2 / at least 3** people. $\rightsquigarrow$ / $\rightsquigarrow$ speaker not sure how many[3]

2.1.2. IG in numerals is similar to IG in disjunction and indefinites

Most of the existing literature on IG in SMNs assumes that it is similar to IG in disjunction (Büring, 2008 and most literature since) or, more rarely, indefinites (Nouwen, 2015). However, this comparison is rarely qualified. Still, qualifying it is worthwhile because, as it turns out, sometimes we find profiles not just like SMNs but also like CMNs. More concretely, in disjunction we find the following: English *or* is incompatible with either ONE-WINNER or ONE-LOSER; and French *ou* is the same.[4]

(6) a. (i) Jo called Alice. Therefore, she called # **Alice, Bob, or Cindy**.
(ii) Jo called # **Alice, Bob, or Cindy**, but not Alice.
b. (i) John a triché. # Donc, John ou Bill est un tricheur.
John cheated therefore John *ou* Bill is a cheater
(ii) John est dans la cuisine ou la salle de bain, # mais pas dans la cuisine.
John is in the kitchen or the bathroom but not in the kitchen

However, in modal indefinites we find the following: Romanian *un NP oarecare*—and seemingly also Italian *un NP qualsiasi/qualunque*, *un qualsiasi/qualunque NP*, a.o.—is incompatible with either ONE-WINNER and ONE-LOSER; German *irgendein*—or Italian *un qualche NP*, or

[2]Cf. Geurts and Nouwen (2007: 546). For similar findings, see also Geurts et al. (2010), Cummins and Katsos (2010), or Cremers et al. (2017), and for an overview see Nouwen et al. (2019). The UE modifiers are usually accepted at higher rates, though, for the same level of monotonicity, the contrast between CMNs and SMNs is always significant.

[3]For experimental evidence see Westera and Brasoveanu (2014) or Cremers and Chemla (2017).

[4]For observations about ONE-WINNER in *or* see Strawson (1952), Grice (1989), Rips (1994: 48), Chierchia (2013: 252), Lauer (2014), Nouwen (2015: 250), and for experimental evidence see Rips (1994: 156) who, testing the natural language counterparts of various valid logical inference rules, found disjunction introduction to be the absolute worst, or Marty, Picat, and Mascarenhas (work in progress, p.c.), who find acceptance to be intermediate (60%). Sources for the French examples: Aurore Gonzalez (p.c.) and Laurence B. Violette (p.c.).

Spanish *algún*, a.o.—are incompatible with ONE-WINNER but compatible with ONE-LOSER; and English *some* is compatible with both ONE-WINNER and ONE-LOSER.[5]

(7) a. (i) A sunat Victor. Deci, a sunat # **un** student **oarecare**.
has called Victor so has called UN student OARECARE
(ii) A sunat # **un** student **oarecare**, dar nu Victor.
has called UN student OARECARE but not Victor
b. (i) John hat geschummelt. Deshalb ist # **irgendein** Student ein Betrüger.
John has cheated therefore is IRGENDEIN student a cheater
(ii) John ist in ✓ **irgendeinem** Zimmer im Haus aber nicht in der Küche.
John is in IRGENDEIN room in.the house but not in the kitchen
c. (i) Arnold, a physicist, is cooking. Therefore, ✓**some** physicist is cooking.
(ii) Jo called ✓**some** student, but not Alice.

2.1.3. The solution for IG in disjunction and indefinites

Chierchia (2013) (and refs. therein, esp. Kratzer and Shimoyama, 2002, Alonso-Ovalle and Menéndez-Benito, 2010) identifies the source of IG in indefinites with the fact that indefinites make reference in their truth conditions not just to a scalar element but also to a domain. This naturally gives rise not just to scalar alternatives (SA) but also to subdomain alternatives (DA). The alternatives crucial to IG are the DA (although their SA also play a role; see §3.1). These are factored into meaning via the silent contradiction-based exhaustivity operator O(nly), which asserts the prejacent and negates all the non-entailed alternatives; they are used in pre-exhaustified form (as if prefixed by O); and they are used obligatorily. Without an intervening operator, this leads to contradiction (due to their SA; see §3.1) but with an intervening possibility or necessity modal, this yields a Free Choice effect. IG is a silent Free Choice effect that arises when $O_{ExhDA}$ happens across a last resort matrix-level, speaker-oriented, silent epistemic necessity modal, $\Box_S$. In the default case, this Free Choice effect is total, but if an item allows $O_{ExhDA}$ relative to just singleton DA (SgDA) it is also compatible with ONE-LOSER. This proposal does not address ONE-WINNER and also aligns imperfectly with the existing approaches to disjunction, which often resort to disjunct-based alternatives derived using the structural theory of alternatives and to contradiction-free exhaustification. In Mihoc (2020) I address these as follows: I show that, if an item allows $O_{ExhDA}$ relative to just the non-singleton DA (NonSgDA), it becomes compatible with ONE-WINNER. And I argue that, once we recognize that disjunction too involves a domain, the exact same account transfers seamlessly to disjunction also.

[5] For ONE-WINNER, the literature on modal indefinites also often uses the *namely* test; I have used *therefore* because it has also been used in numerals. For further discussion of modal indefinites in German, Spanish, Italian, or Romanian, a.o., see Kratzer and Shimoyama (2002), Săvescu Ciucivara (2007), Alonso-Ovalle and Menéndez-Benito (2010), Aloni and Port (2013), Chierchia (2013), Fălăuş (2014), Alonso-Ovalle and Menéndez-Benito (2015), a.o. For specific discussion of *some* as a modal indefinite see Strawson (1974), who explicitly comments on how, compared to *a*, *some* feels modal; Alonso-Ovalle and Menéndez-Benito (2015), who cite Strawson's comments about *some* as the first discussion of a modal indefinite in the literature; Becker (1999) who points out that both *a* and *some* can give rise to ignorance, just that in *some* it is stronger; Becker (1999: Fns. 1, 5), who comments on how *some* is nevertheless compatible with ONE-WINNER scenarios; and Marty, Picat, and Mascarenhas (work in progress, p.c.), who find that, in ONE-WINNER scenarios exactly like the one shown (which was in fact borrowed from them), *some* is accepted at ceiling (90%). Sources for the examples are as follows: for *un NP oarecare*, cf. Anamaria Fălăuş (p.c.); for *irgendein*, ONE-WINNER is from Chierchia (2013: 257) and ONE-LOSER from Niels Torben Kühler (p.c.); for *some*, ONE-WINNER is from Marty, Picat, and Mascarenhas (work in progress, p.c.).

2.1.4. Extending the solution for IG to numerals

Most of the existing solutions to IG in SMNs try to model the solution on disjunction or, more rarely, indefinites. However, they struggle to justify in what way SMNs are like disjunction or indefinites, and also why things should be any different for CMNs or, for that matter, for BNs. I argue that there is a clear difference between BNs and MNs, and a clear similarity between MNs and indefinites/disjunction: While all of BNs, CMNs, and SMNs make reference in their truth conditions to a numeral, only CMNs and SMNs also make reference to a domain, as shown below and in Fig. 1a. From here onward everything follows as for indefinites/disjunction: By replacing the scalar element with its scalemates, and the domain with its subsets, we naturally obtain SA for both BNs and MNs and DA only for MNs, as shown below and in Fig. 1b.

(8) Three people quit.

a. $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (assertion)

b. $\{\exists x[|x| = m \wedge P(x) \wedge Q(x)] \mid m \in S\}$ (SA)

(9) More/less than 3 people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(3)}^{\{4,\,...\}/\{...,\,2\}}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$ (SA)

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subsetneq [\![\text{much/little}]\!]\,(3)\}$ (DA)

(10) At most/least 3 people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(3)}^{\{...,\,3\}/\{3,\,...\}}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$ (SA)

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subsetneq [\![\text{much/little}]\!]\,(3)\}$ (DA)

The alternatives crucial to IG are the DA (though see §3.1). These are factored in via the silent, contradiction-based O (which asserts the prejacent and says that any true alternatives are entailed, i.e., that non-entailed alternatives are false), (11), in pre-exhaustified form (as if prefixed by O, where O proceeds relative to alternatives of the same size),[6] (12), and obligatorily.

(11) $[\![\text{O}]\!]\,(C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s) = p(w) \wedge \forall q \in C[q(w) \rightarrow p \subseteq q]$

(12) E.g., for $\text{O}_{\text{ExhDA}}(x \vee y \vee z)$: $\text{O}x = x \wedge \neg y \wedge \neg z$ and $\text{O}(x \vee y) = (x \vee y) \wedge \neg(x \vee z) \wedge \neg(y \vee z)$

The result is as below. (Schematic notation cf. Fig. 1b. Only first computation given in full.) First, exhaustification without any intervening operator yields contradiction, (13). However, exhaustification across a modal operator yields a Free Choice effect, (14)-(15).

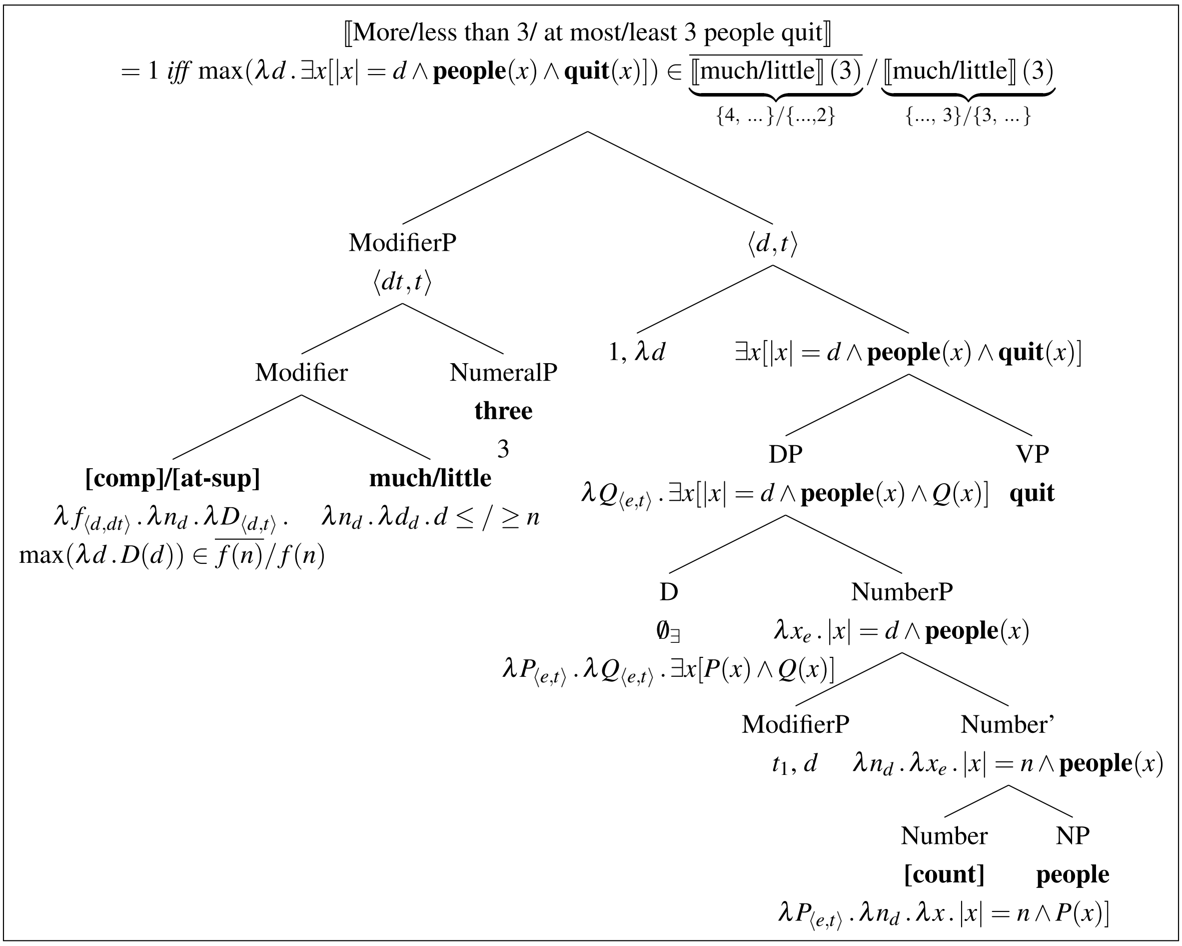
(13) $\text{O}_{\text{ExhDA}}(0 \vee 1) = (0 \vee 1) \wedge \neg \underbrace{\underbrace{\text{O}0}_{0\wedge\neg 1}}_{0\rightarrow 1} \wedge \neg \underbrace{\underbrace{\text{O}1}_{1\wedge\neg 0}}_{1\rightarrow 0}, = \bot$

(14) $\text{O}_{\text{ExhDA}}\Diamond(0 \vee 1)$ (see §4 for more)
$= \Diamond(0 \vee 1) \wedge \Diamond 0 \leftrightarrow \Diamond 1$

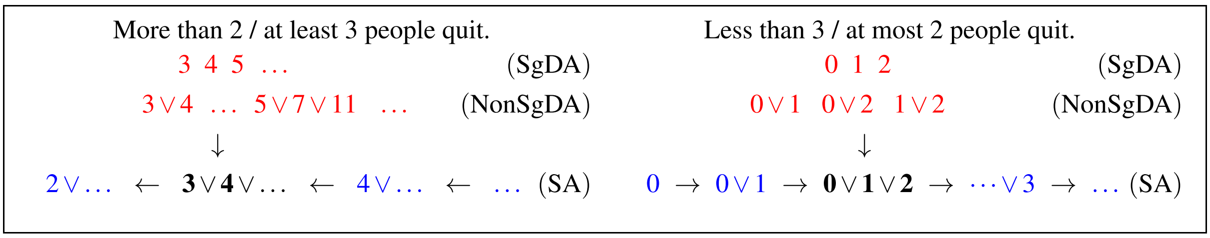
(15) $\text{O}_{\text{ExhDA}}\Box(0 \vee 1)$
$= \Box(0 \vee 1) \wedge \Box 0 \leftrightarrow \Box 1$

Assume now that a contradictory O parse can be rescued via the last resort insertion of a null, speaker-oriented, epistemic necessity modal, $\Box_S$ (akin to the Gricean 'the speaker believes...'). The result is a silent epistemic Free Choice effect—what is commonly called 'ignorance'.

[6]More precisely, as if prefixed by contradiction-*free* O, where O proceeds relative to alternatives of the same size *or smaller*—refinements however necessary only for $\text{O}_{\text{ExhDA}}\Diamond\ldots$ for domains larger than 2 elements.




(a) The syntax and semantics of BNs, CMNs, and SMNs. Those for MNs are shown explicitly, and those for BNs can be obtained by replacing ModifierP near Number' with NumeralP (from higher up). I assume a numeral, NumeralP, denotes a simple degree. In BNs, it combines with Number' to yield a predicative meaning, which then undergoes existential closure; this is the standard existential, 'at least' meaning of BNs cf. Link (1987) (and other literature since). In CMNs/SMNs, it combines with [comp]/[at-sup] and *much/little* (analyzed as positive/negative extent indicators, adapting ideas from Seuren 1984 or Kennedy 1997, 2001), yielding a generalized quantifier over degrees, which cannot combine with Number' but has to move, leaving behind a trace type $d$; this is the meaning of CMNs from Heim (2000)-Hackl (2000) and of CMNs and SMNs from Kennedy (2015), though the assumptions about [comp]/[at-sup] and *much/little* are new. For assumptions about NumberP and NumeralP see Zabbal (2005) or Scontras (2013), and references therein; ModifierP is new.




(b) The truth conditions (bottom center, black, bold) and alternatives (SA bottom left and right, blue; DA top, red) of BNs, CMNs, and SMNs, schematically. Those for MNs are shown explicitly, and those for BNs are the same as for the UE SMNs (e.g., *3* like *at least 3*), except BNs only get the SA. Arrows indicate direction of entailment.

Figure 1: The syntax and semantics (a) and alternatives (b) of BNs, CMNs, and SMNs.

(16) $\mathrm{O}_{\mathrm{ExhDA}} \Box_S(0 \vee 1) = \Box_S(0 \vee 1) \wedge \Box_S 0 \leftrightarrow \Box_S 1$ (ignorance)

Note now that the IG effect thus obtained is compatible with neither ONE-WINNER nor ONE-LOSER—it is only compatible with not knowing that *n* and not knowing that not *n*, for all the *n* in the domain, that is, with what we may call a, total ignorance, NO-WINNER scenario. This captures the patterns for SMNs. However, what about the patterns for CMNs? In analogy to indefinites/disjunction, compatibility with ONE-LOSER and ONE-WINNER in numerals can be obtained if we consider just the SgDA or just the NonSgDA. For a domain with 2 elements we will assume that this is not defined, as there are only SgDA, and removing them would destroy the domain, so all we can get is $\mathrm{O}_{\mathrm{ExhDA}}$, yielding NO-WINNER, as shown above. For a domain with 3 elements or more, however, $\mathrm{O}_{\mathrm{ExhSgDA}}$ yields NO-WINNER or ONE-LOSER and $\mathrm{O}_{\mathrm{ExhNonSgDA}}$ yields NO-WINNER or ONE-WINNER, as can be verified by checking the results below against various models; and $\mathrm{O}_{\mathrm{ExhDA}}$ yields their intersection, that is, only NO-WINNER.

(17) $\mathrm{O}_{\mathrm{ExhSgDA}}(\Box_S(0 \vee 1 \vee 2))$
$= \Box_S(0 \vee 1 \vee 2) \wedge$
$(\Box_S 0 \rightarrow \Box_S 1 \vee \Box_S 2) \wedge$
$(\Box_S 1 \rightarrow \Box_S 0 \vee \Box_S 2) \wedge$
$(\Box_S 2 \rightarrow \Box_S 0 \vee \Box_S 1)$
(compatible with ONE-LOSER)

(18) $\mathrm{O}_{\mathrm{ExhNonSgDA}}(\Box_S(0 \vee 1 \vee 2))$
$= \Box_S(0 \vee 1 \vee 2) \wedge$
$(\Box_S(0 \vee 1) \rightarrow \Box_S(0 \vee 2) \vee \Box_S(1 \vee 2)) \wedge$
$(\Box_S(0 \vee 2) \rightarrow \Box_S(0 \vee 1) \vee \Box_S(1 \vee 2)) \wedge$
$(\Box_S(1 \vee 2) \rightarrow \Box_S(0 \vee 1) \vee \Box_S(0 \vee 2))$
(compatible with ONE-WINNER)

If both CMNs and SMNs undergo obligatory $\mathrm{O}_{\mathrm{ExhDA}}$ but CMNs can ignore either the SgDA or its NonSgDA while SMNs can do neither, this captures all the patterns in IG.

## 2.2. POL1: A matter of obligatory $\mathrm{O}_{\mathrm{ExhDA}}$ ± proper strengthening

### 2.2.1. POL1 in numerals is richer and also includes considerable similarity

As mentioned, Geurts and Nouwen (2007) showed that in a DE environment such as the scope of negation CMNs are fine but SMNs are degraded, (19a), though in a DE environment such as the first argument of a conditional or a universal they are both fine, (19f)-(19g). These patterns, supported experimentally, have been used, like IG, to argue that SMNs are fundamentally different from CMNs (cf., in very different ways, Geurts and Nouwen, 2007, Cohen and Krifka, 2014, Spector, 2015). Consistently, SMNs are also degraded in further DE environments, for example, the scope of *nobody*, *without*, *few*, *rarely*, etc., (19b)-(19e). However, SMNs are also fine in further DE environments, for example, the scope of a negated factive or of *only*, (19h)-(19i).

(19)
a. Jo did**n't** call ✓**more than 2** / # **at least 3** people. (85%/55%)[7]
b. **No**body called ✓**more than 2** / # **at least 3** people.
c. Jo managed **without** calling ✓**more than 2** / # **at least 3** people.
d. **Few** of the participants called ✓**more than 2** / # **at least 3** people.
e. Jo **rarely** called ✓**more than 2** / # **at least 3** people.
f. **If** Jo called ✓**more than 2** / ✓**at least 3** people, she won. (95%/95%)[7]
g. **Every**one who called ✓**more than 2** / ✓**at least 3** people won. (95%/95%)[7]
h. Tim does**n't** know that Jo called ✓**more than 2** / ✓**at least 3** people. (89%/89%)[7]
i. **Only** kids aged ✓**more than 2** / ✓**at least 3** can attend.[8]

[7]Cf. Mihoc and Davidson (2021). The DE modifiers were generally rated lower, but they compared similarly.
[8]Cf. `https://macaonews.org/only-kids-aged-at-least-3-can-attend-kindergarten-classes/`

### 2.2.2. POL1 in numerals is similar to POL1 in disjunction and indefinites

All the existing literature on POL1 in SMNs ties it in one way or another to IG. But neither POL1 nor the relation between IG and POL1 are truly considered in broader perspective. Having shown that IG in disjunction/indefinites varies just as in CMNs and SMNs, below I point out that the same is true of POL1, and also, crucially, that IG and POL1 vary independently. This is already visible from disjunction: English *or* can embed freely, but French *ou* is degraded under *not* (cf. below), *nobody*, *without*, ..., though fine under *few*, *rarely*, *if*, and *every*.

(20) a. Jo didn't call Alice **or** Bob.

b. Marie **n**'a **pas** invité # Léa **ou** Jean à dîner.
Mary not has not invited Leah OU John to dinner

And modal indefinites show the same: Romanian *un NP oarecare* or Italian *un NP qualsiasi/qualunque* are degraded under all of *not*, *nobody*, *few*, *rarely*, though fine under *if*, *every*, *only*, exactly like SMNs, though Italian *un qualsiasi/qualunque NP* embeds freely (though see Fn. 9). German *irgendein* or Italian *un qualche NP* embed freely (though Fn. 9), but Spanish *algún* is degraded under *not* or *nobody* but fine under *few*, *if*, or *every*. And English *some*, is degraded under *not*, *nobody*, *without*, but fine under *few* or *rarely*, as well as under *if* or *every*, exactly like *ou* and partly like SMNs.[9]

### 2.2.3. The solution for POL1 in disjunction and indefinites

Just as for IG, many of the most recent solutions to POL1 in disjunction/indefinites rely on the obligatory use of disjunct-based / subdomain-based alternatives, with or without pre-exhaustification—let's call them (Exh)DA. Moreover, all of these argue that the reason why some items get # under *not*, *nobody*, ..., comes from the fact that $O_{(Exh)DA}$ across these operators does not lead to a properly stronger (PS) meaning, and the reason the same items always get ✓under *if*, *every*, ..., and sometimes also under *few*, *rarely*, ..., comes from the fact that, due to the fact that these operators contain an UE presupposition / implicature, $O_{(Exh)DA}$ here does lead to a PS meaning. However, these approaches differ in whether they assume a contradiction-free variant of O (e.g., Nicolae 2017) or rather the contradiction-based O we used for IG (Chierchia, 2013). This doesn't affect POL1 but it affects IG and especially the predicted relation between IG and POL1—the former predicts them to co-vary whereas the latter predicts them to vary independently. Since our data support the latter, I will maintain the contradiction-based O solution to IG and add to it the contradiction-based O solution to POL1.

### 2.2.4. Extending the solution for POL1 to numerals

As for disjunction and indefinites, $O_{ExhDA}$ across *not* / other DE operators does not lead to PS.

(21) $O_{ExhDA}(\neg(0 \vee 1)) = \neg(0 \vee 1) \wedge \neg \underbrace{(\neg 0 \wedge \neg\neg 1)}_{\text{already excluded}} \wedge \neg \underbrace{(\neg 1 \wedge \neg\neg 0)}_{\text{already excluded}}$ (does not lead to PS)

[9]For Italian *un NP qualsiasi/qualunque*, *un qualsiasi/qualunque NP*, *un qualche NP*, or German *irgendein*, see Chierchia (2013: 260-2) or Aloni and Port (2015: 121). For Romanian *un NP oarecare*, cf. Anamaria Fălăuş (p.c.). For Spanish *algún*, cf. Sara Sanchez-Alonso (p.c.). For English *some*, see Szabolcsi (2004); Nicolae (2012). Caveat: As acknowledged in the sources, many of the items that we said embedded freely are actually bad under sentential negation. This however is argued to be due to independent reasons–for example, competition with an NPI or a negative concord item. The fact that they are fine under *nobody* or *without*—that is, strongly negative DE operators—however suffices to make the point.

However, *if/every* carry a positive presupposition. If we factor this in, $O_{ExhDA}$ does lead to PS.

(22) $O_{ExhDA}(\forall w[(0\vee 1)_w\ldots]\wedge\exists\mathbf{w}[(\mathbf{0}\vee\mathbf{1})_{\mathbf{w}}])$
$=\forall w[(0\vee 1)_w\ldots]\wedge\exists\mathbf{w}[(\mathbf{0}\vee\mathbf{1})_{\mathbf{w}}]\wedge$
$(\ldots\wedge\exists\mathbf{w}[\mathbf{0}_{\mathbf{w}}])\leftrightarrow(\ldots\wedge\exists\mathbf{w}[\mathbf{1}_{\mathbf{w}}])$
(leads to PS as in (14))

(23) $O_{ExhDA}(\forall x[(0\vee 1)_x\ldots]\wedge\exists\mathbf{x}[(\mathbf{0}\vee\mathbf{1})_{\mathbf{x}}])$
$=\forall x[(0\vee 1)_x\ldots]\wedge\exists\mathbf{x}[(\mathbf{0}\vee\mathbf{1})_{\mathbf{x}}]\wedge$
$(\ldots\exists\mathbf{x}[\mathbf{0}_{\mathbf{x}}])\leftrightarrow(\ldots\exists\mathbf{x}[\mathbf{1}_{\mathbf{x}}])$
(with $\Box_S$, leads to PS as in (16))

If both CMNs and SMNs undergo obligatory $O_{ExhDA}$, and both allow it to take into account presuppositions, but CMNs are fine with an outcome that does not lead to PS whereas SMNs are not, this captures all the patterns in POL1.

## 3. SI and POL2: A matter of scalar alternatives and O(nly) / E(ven)

Building on the previous section, in this section I address SI and POL2. In the existing literature, SI has been used to conclude fundamental difference between BNs, on the one hand, and CMNs and SMNs, on the other, and POL2 has been used to conclude a fundamental split within SMNs. I reject the conclusion for SI and show that it helps us reject the conclusion for POL2 also.

### 3.1. SI: A matter of obligatory $O_{SA}$ and how it interacts with $O_{ExhDA}$

#### 3.1.1. SI in numerals is richer and also includes considerable similarity

As mentioned, Krifka (1999) showed that, while in a plain UE environment BNs give rise to an exact meaning, as predicted by Horn (1972), CMNs and SMNs do not, (24a-i). This has been used to argue that BNs give rise to classic, Horn-style SI based on classic, Horn-style SA, but CMNs or SMNs do not (Krifka, 1999 and most of the literature since; see §3.1.4). However, in the UE environment defined by the scope of a necessity modal all of BNs, CMNs, and SMNs give rise to the predicted SI (cf. Fox and Hackl, 2006 for CMNs, Mayr 2013 for both CMNs and SMNs), (24a-ii). And in the DE environment defined by the scope of negation none do (cf., e.g., Spector, 2013 for BNs, Mayr, 2013 for CMNs and SMNs), (24b-i). Finally, in the DE environment defined, for example, by the antecedent of a conditional (but not only), all do again (cf., e.g., Spector, 2013: 279ff. for BNs), (24b-ii). Thus, all of BNs, CMNs, and SMNs sometimes do and sometimes do not give rise to both direct and indirect SI.[10] Moreover: All the cases where we get SI-gaps are, first, cases where we can still get SI if only we considered, instead of the immediately strongest SA, some other stronger SA (cf., e.g., Spector, 2014: 42; experimental evidence, cf. Cummins et al. 2012), (24c), and, second, cases where the numeral gives rise to ignorance—witness being compatibility with ignorance, which under negation shows up for BNs also, (24d). (Under negation, # immediately before the SMN marks POL1.)

(24) a. (i) Jo called **3 / more than 2 / at least 3** people.
⇝ not ✓4 / # more than 3 / # at least 4 (predicted meaning: exactly 3)
(ii) Jo is **required** to call **3 / more than 2 / at least 3** people.
⇝ not required ✓4 / ✓more than 3 / ✓at least 4
b. (i) Jo didn't call **3 / more than 2 / # at least 3** people.
⇝ not not # 2 / # more than 1 / # at least 2 (predicted meaning: exactly 2)
(ii) If Jo called **3 / more than 2 / at least 3** people, she won.
⇝ not if ✓2 / ✓more than 1 / ✓at least 2

[10] Cf. Chierchia (2004: 59), direct SI = SI based on a SA stronger by virtue of its position in a scale, and indirect SI = SI based on a SA stronger by virtue of its position in a scale reversed by a higher negative element.

c. (i) Jo called **3 / more than 2 / at least 3** people.
$\rightsquigarrow$ not ✓4 / ✓e.g., more than 4 / ✓e.g., at least 5
(ii) Jo didn't call **3 / more than 2 / # at least 3** people.
$\rightsquigarrow$ not not ✓1 / ✓e.g., more than 0 / ✓e.g., at least 1
d. (i) I'm not sure how many ... but it was # **3 / ✓more than 2 / ✓at least 3**.
(ii) I'm not sure how many ... but it wasn't **✓3 / ✓more than 2 / ✓# at least 3**.

3.1.2. SI in numerals is similar to SI in disjunction and indefinites
Since Horn (1972), most of the existing literature on SI in BNs assumes that it is similar to SI in disjunction and indefinites. Indeed, disjunction/indefinites give rise to direct SI (crucial to controlling their IG) and participate in indirect SI (triggered by their scalemates) also. Based on all the discussion so far, I will argue that SI in MNs is similar to SI in BNs, disjunction, and indefinites: All these categories entail one bound and implicate another bound via classic SI.

3.1.3. The solution for SI in disjunction and indefinites
Just as for BNs, Horn (1972) proposed that SI in disjunction and indefinites comes from the fact that these items belong to natural lexical scales: ⟨or, and⟩, ⟨some, every⟩, etc. Thus, they naturally activate scalar alternatives—their scalemates. These are factored into meaning via Gricean reasoning and give rise to SI. Following Chierchia et al. (2012), I adopt the view that SI actually arise in the grammar, via $O_{SA}$. Following Chierchia (2013) for modal indefinites, I also assume that for all the disjunctions/ indefinites reviewed above, $O_{SA}$ is computed obligatorily.[11]

3.1.4. Extending the solution for SI to numerals
Since Krifka (1999), because of (24a-i), most of the literature abandons the classic, Horn-style notion of SI and SA for MNs. What I mean is the following: The Horn (1972) view of SA and SI involved an alternative set that was totally ordered and SI that endowed one-bounded meanings with a second bound. Also, it allowed the granularity of the SA scales to be contextual. While the approaches to MNs since then sometimes do assume some form of SA and SI for MNs, they are no longer classical in the sense that they depart from either one or both of these features.

This has important conceptual consequences—the notion of SI in numerals ends up being very different from the notion of SI in other categories.[12] It also has important empirical consequences—they can capture only some of the SI patterns, usually failing to do so for CMNs, or indirect implicatures, or the direct and indirect implicatures based on non-immediately stronger SA. In contrast, the classical view is conceptually general and empirically adequate—as we just saw, the only problem are a few gaps which are however remarkably systematic and occur not just in MNs but also in BNs.

I will thus uphold the classical, Horn-view of SI for all of BNs, CMNs, and SMNs: The alternatives crucial to SI are the classic, Horn-style SA. However, in analogy to the proposal for disjunction and modal indefinites just now, I update it with the following features: These

[11]A reason for this is because, with just $O_{ExhDA}$, *or*, for example, is predicted to mean *and* and also to be compatible with ONE-WINNER. Following Chierchia (2004: 66ff.), I regard the felicity of continuations such as *in fact, ... and...* as a form of backtracking and note that they can correct *or* not just to *and* but also to ONE-WINNER: *Jo called Alice or Bob. ✓In fact, both. / ✓In fact, Alice.*
[12]As Mayr (2013: Fn. 16) notes, accounts that depart from totally ordered scales "contradict Matsumoto (1995)'s assumptions that the fundamental condition on Horn-sets is that the alternatives are ordered by monotonicity".

alternatives are factored into meaning via the silent, contradiction-based O, and obligatorily.

Regarding the SI-gaps in numerals, I believe they can be explained as follows: In both disjunction/indefinites and numerals, IG and SI, or more generally $O_{ExhDA}$ and $O_{SA}$, interact with one another. However, while in disjunction/indefinites they merely constrain one another, in numerals, in some contexts, due to the nature of the domain/scale, they always *contradict* one another, and this clash is resolved in favor of IG, by weakening SI. For reasons of space, we will not be able to discuss this idea any further here, but I believe it can be defended.[13] As for why, even in SI-gap contexts, some form of SI are always intuitively available, I believe this is because such cases of invariable contradiction trigger a repair mechanism whereby the scale granularity becomes coarser, based on context.

### 3.2. POL2: A matter of $\pm$ obligatory $E_{ExhSA}$

#### 3.2.1. POL2 in numerals is richer and also includes considerable similarity

As mentioned, Cohen and Krifka (2014) (and refs. therein) argued that, even in those DE environments where SMNs are reported to be fine, this is in fact not always so: In a positive/negative antecedent of a conditional or restriction of a universal, SMNs with a pragmatically neutral predicate, P1, are actually degraded if the predicate in the continuation, P2, is pragmatically negative/positive (lose/win), (25a). They used this to argue that SMNs are different not just from CMNs but also from SMNs: That is, SMNs have one lexical meaning that is sensitive to DE environments, responsible for POL1, and another that is not sensitive to DE environments but sensitive instead to some notion of evaluativity and responsible for POL2. Now, experimental results do indeed support some form of POL2. However, the ratings are sometimes different than assumed, and sometimes also differ sharply between *at least* and *at most* (see esp. underlined judgments/percentages). My intuitions about patterns with a pragmatically marked P1 similarly suggest a sharp split within SMNs by motonicity, and that CMNs might exhibit a light form of the POL2 effect detected in their same-monotonicity SMN counterparts also, (25b). Finally, as Cohen and Krifka (2014) already acknowledge, SMNs under negation continue to be degraded regardless of the pragmatic polarity of their P1, (25c).

(25) a. (i) If Jo called **✓more than 2 / ✓at least 3** people, she won. (95%/95%)[14]
If Jo called **✓more than 2 / <u>#</u> at least 3** people, she lost. (93%/<u>95</u>%)[14]
If Jo didn't call **✓more than 2 / # at least 3** people, she won. (78%/55%)[14]

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[13]Regarding plain UE contexts: This can be seen by computing $O_{ExhDA}\Box_S O_{SA}(\dots)$ for disjunction/indefinites and MNs. A challenge is to make sure that the ability of CMNs to accommodate ONE-WINNER does not affect this result, though if it can be argued that the default for them too is $O_{ExhDA}$, and that DA can't be removed to accommodate $O_{SA}$, this solves the problem. Regarding the scope of negation: The data patterns we saw earlier suggest the explanation might have to do with a clash between SI and IG also. However, it is not obvious how to derive IG here. I see two possibilities: (1) $O_{SA}$ across negation actually proceeds relative to SA both with and without negation, which leads to contradiction, which triggers the last resort insertion of $\Box_S$, yielding IG. A potentially serious issue might be that a sentence such as *Jo didn't read 3 papers* can be interpreted as $\neg O_{SA}3$ but still carry an IG effect in spite of the fact that the SA have been used already. Alternatively: (2) The shape of the domain in MNs suggests that domains can be *derived* sorts of objects. Now, under negation, both a BN and a MN yield a domain of sorts—*not 3 / more than 2 / at least 3* = $\overline{\{3,\dots\}}$, = $\{0,1,2\}$. This suggests that perhaps there is a form of $O_{(Exh)DA}$ in this context also. A potentially serious issue might be that we will have to keep this $O_{(Exh)DA}$ well apart from the one that yielded POL1, via PS. However, an attractive feature is that it allows us to explain all cases of SI-gaps as the result of a tension between a pressure to use the scale and a pressure to use the domain.

[14]Cf. Mihoc and Davidson (2021), who test configurations exactly like this, with a neutral P1 and a marked P2.

If Jo didn't call ✓**more than 2** / ✓**at least 3** people, she lost. (80%/88%)[14]
(ii) If Jo called ✓**less than 4** / ✓**at most 3** people, she won. (95%/75%)[14]
If Jo called ✓**less than 4** / # **at most 3** people, she lost. (92%/23%)[14]
If Jo didn't call ✓**less than 4** / # **at most 3** people, she won. (50%/20%)[14]
If Jo didn't call ✓**less than 4** / ✓ **at most 3** people, she lost. (45%/23%)[14]

b. (i) If Jo solved ✓**more than 3** / ✓**at least 3** problems, she passed.
If Jo solved ? **more than 3** / # **at least 3** problems, she failed.
If Jo didn't solve ? **more than 3** / # **at least 3** problems, she passed.
If Jo didn't solve ✓**more than 3** / ✓**at least 3** problems, she failed.
If Jo made ? **more than 3** / # **at least 3** mistakes, she passed.
If Jo made ✓**more than 3** / ? **at least 3** mistakes, she failed.
If Jo didn't make ✓**more than 3** / ? **at least 3** mistakes, she passed.
If Jo didn't make ? **more than 3** / # **at least 3** mistakes, she failed.
(ii) If Jo solved ? **less than 4** / # **at most 3** problems, she passed.
If Jo solved ✓**less than 4** / ✓**at most 3** problems, she failed.
If Jo didn't solve ✓**less than 4** / ? **at most 3** problems, she passed.
If Jo didn't solve ? **less than 4** / # **at most 3** problems, she failed.
If Jo made ✓**less than 4** / ✓**at most 3** mistakes, she passed.
If Jo made ? **less than 4** / # **at most 3** mistakes, she failed.
If Jo didn't make ? **less than 4** / # **at most 3** mistakes, she passed.
If Jo didn't make ✓**less than 4** / ? **at most 3** mistakes, she failed.

c. (i) Jo did**n't** solve ✓**more than 2** / # **at least 3** problems.
Jo did**n't** make ✓**more than 2** / # **at least 3** mistakes.
(ii) Jo did**n't** solve ✓**less than 4** / # **at most 3** problems.
Jo did**n't** make ✓**less than 4** / # **at most 3** mistakes.

3.2.2. POL2 in numerals is similar to POL2 in minimizers

Cohen and Krifka (2014: 77) (and refs. therein) note that POL2 in numerals resembles similar facts in NPIs, especially strong NPIs. Here we note that it also resembles facts discussed by Crnič (2011: 49ff.) (and refs. therein) in minimizers, for example, *lift a finger*. First, *lift a finger* also exhibits POL1, though in the opposite direction than SMNs. Second, they also exhibit POL2, which is similar to SMNs.

(26) a. (i) Jo ✓**helped** / # **lifted a finger** to help.
(ii) Jo did**n't** ✓**help** / ✓**lift a finger** to help.
b. (i) **Every**one who ✓**helped** / ✓**lifted a finger to help** was rewarded.
(ii) **Every**one who ✓**helped** / # **lifted a finger to help** was punished.

3.2.3. The solution for POL2 in minimizers

Chierchia (2013: 148ff.) (and refs. therein; also Crnič 2011 and refs. therein) identifies the source of POL1 in minimizers with the fact that minimizers make reference in their truth conditions to a scalar element. This naturally gives rise to SA. These have to be factored into meaning via the silent exhaustivity operator E(ven). E carries a scalar condition that requires that the prejacent be less likely than all its SA. Now, a minimizer is by definition the lowest in its scale. Thus, $E_{SA}$ in a UE context fails—all the SA entail the prejacent, so the prejacent cannot be less likely than them—but in a DE environment it succeeds—the prejacent entails all the SA, so it can easily satisfy the requirement of being less likely than them. This captures POL1. Following Crnič (2012), I assume that in some items E may use the scalar in both the prejacent and the SA in an exact sense, making them non-monotonic, and causing the scalar condition to be assessed relative not to logic but rather context. This captures POL2.

#### 3.2.4. Extending the solution for POL2 to numerals

Cohen and Krifka (2014) propose a solution on which POL2 comes from a second meaning of SMNs, one that does not mind being in a DE environment but simply requires that the predicate that the SMNs combines with most immediately, P1, be pragmatically positive. However, this does not capture all the patterns in SMNs, nor their light/strong parallels in CMNs/minimizers.

Building on our discussion of SI earlier, I argue that there is a clear similarity between SMNs and minimizers: They both make reference in their truth conditions to a scalar element. Thus, we should be able to extend the account from minimizers to SMNs also. As for minimizers, the alternatives relevant to POL2 are the SA. These are factored in via the silent exhaustivity operator E. Following Chierchia (2013: 148)'s concrete discussion of minimizers, I assume that E asserts the prejacent and adds that any alternatives (different from it; see Crnič, 2012: 542) are contextually more likely than it, that is, that the prejacent is the least likely.

However, at this point we are faced with a conundrum. Minimizers are, by definition, end-of-scale items. They stand in the same relation with respect to all their SA. Thus, there was a clear sense in which they could satisfy the scalar condition of E. However, numerals are typically not end-of-scale. They have SA that entail them and SA that are entailed by them, in both UE and DE environments. Thus, they can never satisfy the scalar condition of E, whether by logical or by contextual reasoning. I propose that, quite generally, the scalar condition of E is really just about the *entailed* SA, (27). (See (38) in §4 for further refinements.)

(27) $[\![\mathrm{E}]\!]\,(C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s) = p(w) \wedge \forall q \in C[\underline{p \subseteq q} \rightarrow p \prec_\mu q]$ (further thoughts in §4)

Finally, following Crnič (2012)'s solution for POL2, mentioned above, I assume that both the prejacent and the SA are used by E on an exhaustive interpretation, that is, following Crnič (2012), as if strengthened at some level via O, (28).

(28) E.g., $\mathrm{E}_{\mathrm{SA}}$(*If* $\mathrm{O}_{\mathrm{SA}}$*(at least 3 P1), P2*) is in a sense $\mathrm{E}_{\mathrm{ExhSA}}$(*If at least 3 P1, P2*), with the prejacent *(If* $\mathrm{O}_{\mathrm{SA}}$*(at least 3 P1), P2)* and the SA *(If* $\mathrm{O}_{\mathrm{SA}}$*(at least m P1), P2)*.

The result is as below. (First example shown explicitly, subsequent examples abbreviated to first line—giving the intuitive judgments—and last lines—giving the predictions from $\mathrm{E}_{\mathrm{SA}}$.)

(29)
a. If Jo solved ✓**at least 3** problems, she passed.
$\underbrace{\mathrm{E}_{\mathrm{ExhSA}}(\text{If } \underbrace{\mathrm{O}_{\mathrm{SA}}(\text{Jo solved ✓}\textbf{at least 3}\text{ problems})}_{\text{exactly 3 solutions}}\text{, she passed.}}_{\text{exactly 3 solutions} \rightarrow \text{pass} \prec_\mu \text{exactly 4/5/... solutions} \rightarrow \text{pass}}$
b. If Jo solved # **at least 3** problems, she failed.
\# exactly 3 solutions → fail $\prec_c$ exactly 4 solutions → fail
c. If Jo didn't solve # **at least 3** problems, she passed.
\# exactly 3 solutions → pass $\prec_c$ exactly 2 solutions → pass
d. If Jo didn't solve ✓**at least 3** problems, she failed.

✓exactly 3 solutions → fail $\prec_c$ exactly 2 solutions → fail

e. If Jo made # **at least 3** mistakes, she passed.
# exactly 3 mistakes → pass $\prec_c$ exactly 4 mistakes → pass

f. If Jo made ? **at least 3** mistakes, she failed.
✓exactly 3 mistakes → fail $\prec_c$ exactly 4 mistakes → fail

g. If Jo didn't make ? **at least 3** mistakes, she passed.
✓exactly 3 mistakes → pass $\prec_c$ exactly 2 mistakes → pass

h. If Jo didn't make # **at least 3** mistakes, she failed.
# exactly 3 mistakes → fail $\prec_c$ exactly 2 mistakes → fail

(30) a. If Jo solved # **at most 3** problems, she passed.
# exactly 3 solutions → pass $\prec_c$ exactly 2 solutions → pass

b. If Jo solved ✓**at most 3** problems, she failed.
✓exactly 3 solutions → fail $\prec_c$ exactly 2 solutions → fail

c. If Jo didn't solve ? **at most 3** problems, she passed.
✓exactly 3 solutions → pass $\prec_c$ exactly 4 solutions → pass

d. If Jo didn't solve # **at most 3** problems, she failed.
# exactly 3 solutions → fail $\prec_c$ exactly 4 solutions → fail

e. If Jo made ✓**at most 3** mistakes, she passed.
✓exactly 3 mistakes → pass $\prec_c$ exactly 2 mistakes → pass

f. If Jo made # **at most 3** mistakes, she failed.
# exactly 3 mistakes → fail $\prec_c$ exactly 2 mistakes → fail

g. If Jo didn't make # **at most 3** mistakes, she passed.
# exactly 3 mistakes → pass $\prec_c$ exactly 4 mistakes → pass

h. If Jo didn't make ? **at most 3** mistakes, she failed.
✓exactly 3 mistakes → fail $\prec_c$ exactly 4 mistakes → fail

If both CMNs and SMNs can undergo $E_{SA}$, but SMNs must in fact do so obligatorily, this captures all the patterns for POL2 with marked P1. As for the patterns for POL2 with neutral P1, I believe they should work the same—once we figure out the strategy for interpreting P1.

## 4. Predictions and extensions

### 4.1. Prediction: $O_{ExhDA}$–$O_{SA}$–$E_{ExhSA}$

Given the above, we expect $O_{ExhDA}$, $O_{SA}$, and $E_{ExhSA}$ to interact. Two examples are as below.

First, SMNs in plain UE contexts. Here, as discussed, SMNs give rise to IG, via $O_{ExhDA}$, and to SI, via $O_{ExhDA}$. However, they also give rise to an evaluative judgment—*at least 3/at most 3* suggests that 3 is many/few. Note that this can be derived from $E_{SA}$.

(31) Jo solved at least 3 problems.
exactly 3 $\prec_\mu$ exactly 2 $\rightsquigarrow$ 3 is many!

(32) Jo solved at most 3 problems.
exactly 3 $\prec_\mu$ exactly 4 $\rightsquigarrow$ 3 is few!

Second, SMNs under possibility modals. Here, as discussed, SMNs give rise to a Free Choice effect, via $O_{ExhDA}$. However, they also give rise to an interesting bound—*possible at most 3* mysteriously suggests *not possible more*. I argue this comes from $O_{SA}$ being computed atop of $O_{ExhDA}$. However, this bound is attested only for *at most*. I suggest this might be because the $E_{ExhSA}$ built atop of all this yields a sensible interpretation for *at most* but not for *at least*.

(33) Jo may drink ✓**at least** 3 beers.

$$\underbrace{\mathrm{E}_{\mathrm{ExhSA}}(\underbrace{\mathrm{O}_{\mathrm{SA}}(\underbrace{\mathrm{O}_{\mathrm{ExhDA}}(\Diamond(3\vee 4\vee\dots))}_{(\Diamond 3\wedge\Diamond 4\wedge\dots)\ \text{Free Choice}})}_{\neg(\Diamond 2\wedge\Diamond 3\wedge\dots)\ \text{not possible less}})}_{\Diamond 3\prec_{\mu}\Diamond 4\ \text{does not fit common assumptions}}$$

(34) Jo may drink ✓**at most** 3 beers.

$$\underbrace{\mathrm{E}_{\mathrm{ExhSA}}(\underbrace{\mathrm{O}_{\mathrm{SA}}(\underbrace{\mathrm{O}_{\mathrm{ExhDA}}(\Diamond(\dots\vee 2\vee 3))}_{(\dots\wedge\Diamond 2\wedge\Diamond 3)\ \text{Free Choice}})}_{\neg(\dots\wedge\Diamond 3\wedge\Diamond 4)\ \text{not possible more}})}_{\Diamond 3\prec_{\mu}\Diamond 2\ \text{fits common assumptions}}$$

### 4.2. Extension: O–*only*, E–*even*, and O/*only*–E/*even*

Horn (1972: 37ff.) notes contrasts between what we now call instances of covert O and overt *only*—O is fine with *more* but bad with *less*, while *only* is the opposite. Building on comments from an anonymous reviewer, I similarly note contrasts between what we now call instances of covert E and overt *even*—E with *at least [positive P1]* is fine with a positive P2 but bad with a negative P2, which we said is because E pitches the prejacent up against its entailed SA, but *even* is fine with either a positive or a negative P2, which by our reasoning must come because *even* can pitch the prejacent up against either its entailed or its non-entailed SA.

(35) {O / Only} 60% of the electorate will be fooled, if not ✓/# more / #/✓less.

(36) {E / Even} if you solve at least 3 problems, you ✓/✓pass / #/✓fail.

Building on suggestions from Horn (1972) and literature since, I suggest that *only*/O are as follows: (1) They both convey the same set of inferences. However: (2) They differ in the status of some of these inferences (whether it is asserted, $\alpha$, or presupposed, $\pi$). Moreover: (3) Their overt/covert status likely matters also. Analogously, and very tentatively, I suggest that *even*/E are as follows: (1) They both convey the same set of inferences. Above I said that, in light of non-end-of-the-scale items, the scalar inference cannot be about *all* the SA, and I suggested it is about the *entailed* SA. Here I suggest, in light of *even*, that it is restricted to the *true* SA, where a third commonly assumed existential inference ensures that at least one true alternative (different from the prejacent) is always available. Then: (2) They might also differ in the status of some of these inferences. With the literature, for now I will assume that they do not. However, crucially: (3) Their overt/covert status matters, as follows: With $\mathrm{O}_{\mathrm{SA}}$ at the same site as $\mathrm{E}_{\mathrm{SA}}$, the set of true alternatives are the *entailed* SA (see (11), §3.1), which constrains the scalar inference such that sometimes it ends up clashing with context. Without $\mathrm{O}_{\mathrm{SA}}$at the same site as $\mathrm{E}_{\mathrm{SA}}$, the set of true alternatives can in principle be either the entailed or the non-entailed $\mathrm{O}_{\mathrm{SA}}$, which ensures that the scalar inference can adjust to context. Now, I assume that overt *even* can suspend $\mathrm{O}_{\mathrm{SA}}$ at the same site, thus accessing the non-entailed SA, but covert E cannot. This affects their set of true alternatives, thus capturing their patterns.

(37) $[\![\text{only/O}]\!]\,(C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s)$

a. $p(w)$ $(\pi/\alpha)$

b. $\forall q\in C[q(w)\rightarrow p\subseteq q]$ $(\alpha/\alpha)$

(38) $[\![\text{even/E}]\!]\,(C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s)$

a. $p(w)$ $(\alpha/\alpha)$

b. $\forall q\in C[\underline{q(w)}\rightarrow p\prec_{\mu} q]$ $(\pi/\pi)$

c. $\exists q\in C[\overline{q\neq p}\wedge q(w)]$ $(\pi/\pi)$

If this is on the right track, we can also draw conclusions about the role of the exhaustivity operators O-*only* and E-*even* more generally. When an assertion has multiple alternatives that are true, as is the case with the entailed SA of scalar items or with the additional true SA added by E-*even*, these operators clarify why the assertion was chosen over them: It entails any other true alternative (O-*only*) and/or it is more noteworthy than any other true alternative (E-*even*).

## 5. Conclusion and outlook

IG, SI, POL1, and POL2 have all been used to conclude fundamental difference—between BNs and MNs, within MNs, and within SMNs. We have revisited these effects and argued instead for fundamental similarity—within numerals and between numerals and disjunction, indefinites, and minimizers—offering also a unified account. This account revisits the connection between IG and POL1, suggesting they both come from DA, via O. This account also argues for a connection between SI and POL2, suggesting they both come from SA, the former via O and the latter via E. The proposal predicts some free variation, that is, that one might find, for example SMNs compatible with ONE-WINNER, or items where a clash between IG and SI is resolved in favor of SI, or CMNs with POL2. I believe all these are indeed attested—the former, for example, in Chinese *zhi-shao* (vs. *zui-shao*; Nouwen, 2015: 255), and the latter two both, possibly, in *no more than n* (vs. *not more than*; Nouwen, 2008: 77). The proposal also tentatively tackles the connection between O and *only*, E and *even*, and O/*only* and E/*even*, suggesting that they all come from the same source: a pressure to justify the choice of the assertion in light of its true alternatives.

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