A Plea for Equality:
Commentary on “Truthmaker Semantics for Natural Language”*

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In “Truthmaker Semantics for Natural Language”, Moltmann (2020) develops a unified account of a variety of phenomena of interest to both philosophers of language and linguistic semanticists, such as attitude reports, modals, and intensional transitive verbs, in terms of object-based truthmaker semantics — a version of Fine’s (2017a, 2017b, 2017c) truthmaker semantics, which expands the purview of verification/falsification from sentences to attitudinal and modal objects. As such, Moltmann’s account requires a significant enrichment of the fairly austere ontology standardly assumed by semanticists, but arguably delivers concomitant conceptual and empirical gains.

From a linguistic perspective, there are many reasons to find the notion of content delivered by truthmaker semantics attractive. For example, it provides a natural notion partial content which tracks speaker intuitions, as discussed extensively by Yablo (2014), and touched upon by Moltmann. Here, I’ll take it as a given that an account of, e.g., attitude reports using the apparatus of truthmaker semantics is a worthwhile enterprise. The primary focus here will be on the precise semantics that Moltmann suggests for that-clauses. I’ll argue that a semantics in terms of partial content fails to account for a range of facts concerning attitude reports, as well as clausal modifiers of content nouns. Rather, I’ll suggest that a semantics for that-clauses based on equality of content is preferable.

1 Moltmann’s semantic machinery

Object-based truthmaker semantics is a strict extension of Fine’s sentential truthmaker semantics. In order to keep our intuitions straight about how the underlying semantic machinery works, and the predictions it makes, it will be useful to outline the simplest version of Fine’s truthmaker semantics for a simple propositional fragment, before drawing attention to Moltmann’s contribution.

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In truthmaker semantics, exact verification/falsification plays a privileged role. A state \( s \) (which can be situations or actions on Moltmann's rendering) verifies/falsifies a simple sentence \( \phi \) iff \( s \) is “wholly relevant” to the truth/falsity of \( \phi \), respectively. Here, we’ll identify the semantic value of a simple sentence with the sets of its verifiers and falsifiers.

More formally, we can recursively define two functions (i) a function from a sentence to to its verification set \( \langle \cdot \rangle^+, M \rangle \) relative to a model \( M \), and (ii) a function from a sentence to its falsification set \( \langle \cdot \rangle^-, M \rangle \) relative to a model \( M \). A model is a triple \( \langle S, \sqsubseteq, I \rangle \), where \( \langle S, \sqsubseteq \rangle \) is a state space, and \( I \) a valuation function. \( S \) is a non-empty set of states closed under mereological fusion \( \sqcup \), and \( \sqsubseteq \) is a mereological parthood relation, partially ordering \( S \).

The positive valuation function \( I^+ \) maps simple sentences to their exact verifiers and the negative valuation function \( I^- \) maps simple sentences to their exact falsifiers. Verification/falsification sets for a small propositional fragment are defined below.

**Definition 1.1.** Verification/falsification sets for simple sentences.

- \( \langle \phi \rangle^+ := I^+(\phi) \)
- \( \langle \phi \rangle^- := I^-(\phi) \)

**Definition 1.2.** Verification/falsification sets for negated sentences.

- \( \langle \neg \phi \rangle^+ := I^-(\phi) \)
- \( \langle \neg \phi \rangle^- := I^+(\phi) \)

**Definition 1.3.** Verification/falsification sets for conjunctive sentences.

- \( \langle \phi \land \psi \rangle^+ := \{ t \sqcup u \mid t \in \langle \phi \rangle^+, u \in \langle \psi \rangle^+ \} \)
- \( \langle \phi \land \psi \rangle^- := \langle \phi \rangle^- \cup \langle \psi \rangle^- \)

**Definition 1.4.** Verification/falsification sets for disjunctive sentences.

- \( \langle \phi \lor \psi \rangle^+ := \langle \phi \rangle^+ \cup \langle \psi \rangle^+ \)
- \( \langle \phi \lor \psi \rangle^- := \{ t \sqcup u \mid t \in \langle \phi \rangle^-, u \in \langle \psi \rangle^- \} \)

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1Fine, and derivatively, Moltmann instead take verification (\( \models \)), and falsification (\( \not\models \)) — relations between states and sentences — to be primitive notions. These notions can be easily retrieved in the current setting as follows:

- \( s \models \phi \) if \( s \in \langle \phi \rangle^+ \)
- \( s \not\models \phi \) if \( s \in \langle \phi \rangle^- \)

2We’ll suppress the model parameter in what follows.

3See, e.g., Fine (2017c) for a thorough discussion of the logical properties of state spaces.
A couple of features of the semantics are worth drawing attention to: the verifiers of a conjunctive sentence are given by the pointwise fusion of the verifiers of the individual conjuncts, whereas the verifiers of a disjunctive sentence are simply the union of the verifiers of the individual disjuncts. This will be important in the definition of partial content, which we’ll come to in a moment.

Moltmann’s object-based truthmaker semantics differs from Fine’s sentential semantics in one simple respect: it is not just sentences that are assigned verification/falsification sets, but also certain abstract objects, which in some intuitive sense, bear content — namely, attitudinal objects, which encompass beliefs, thoughts, and claims, as well as modal objects, which encompass and facts, possibilities, and requirements. We’ll refer to the objects for which verification/falsification is defined as “content-bearers”.

One of the most compelling features of truthmaker semantics is that it delivers a natural notion of partial content that tracks speaker intuitions regarding the validity of arguments involving certain attitude verbs. Consider, e.g., the fact that (1) is intuitively valid, whereas (2) is not (see Yablo 2014 for extensive discussion). In order to account for (1), it is natural to say that two individuals x and y agree that φ, iff φ is “part of the content” of both x’s beliefs and y’s beliefs. In possible world semantics, we can only really cash out partial content in terms of logical entailment — this captures (1), since both Sarah’s beliefs and Josie’s beliefs logically entail that it’s raining, but this clearly over-generates for (2), since both Sarah’s beliefs and Josie’s beliefs logically entail that it’s warm or cold.

(1) a. Context: 
Sarah thinks [it’s rainy and cold], Josie thinks [it’s rainy and warm].

   b. Sarah and Josie agree [that it’s rainy].

(2) a. Context:
Sarah thinks [it’s warm], and Josie thinks [it’s cold].

   b. #Sarah and Josie agree that [it’s warm or cold].

In truthmaker semantics, a more adequate notion of partial content can be defined formally in terms of conjunctive parthood, ψ is part of the content of φ iff ψ is a conjunctive part of φ:

Definition 1.5 (Conjunctive parthood). Given two content-bearers φ, ψ, ψ is a conjunctive part of φ iff:

- ∀s’ ∈ [ψ]⁺, ∃s ∈ [φ]⁺ [s’ ⊑ s]

[Moltmann (2020) is not very explicit about this (at least in this paper), but it is natural to think that the domain of attitudinal/modal objects is closed under mereological fusion ⊔, and furthermore that verification/falsification sets for attitudinal/modal objects should be constrained such that, given two attitudinal/modal objects x, y:

\[
[x \cup y]^+ = \{ t \cup u \mid t \in [x]^+ \land u \in [y]^+ \} \\
[x \cup y]^− = [x]^− \cup [y]^−
\]
∀s ∈ [ϕ]⁺, ∃s′ ∈ [ψ]⁺ [s′ ⊑ s]

This guarantees that Josie and Sarah agree that it’s rainy — due to the recipe for conjunctive sentences in truthmaker semantics, “it's rainy” is part of the content of what they both think in (1). On the other hand, due to the recipe for disjunctive sentence, this doesn't hold in (2). This is easy to see — assume that I⁺(it's rainy) = {r}, I⁺(it's cold) = {c₁, c₂}, I⁺(it's warm) = {w₁, w₂}:

- [it's rainy and cold]⁺ = {r ⊔ c₁, r ⊔ c₂}
- [it's rainy and warm]⁺ = {r ⊔ w₁, w ⊔ c₂}
- [it's warm or cold]⁺ = {w₁, w₂, c₁, c₂}

2 Moltmann’s semantics for embedded clauses

The notion of partial content plays an important role in Moltmann’s semantics for clausal embedding, since on Moltmann’s view an embedded clause that S denotes the property of having S as a conjunctive part. Following Moulton (2009), I’ll use a subscript c to indicate a variable ranging over content-bearers.5

(3) [that S] = λx_c . ∀s′ ∈ [S]⁺, ∃s ∈ [x_c]⁺ [s′ ⊑ s] ∧ ∀s [x_c]⁺ , ∃s′ ∈ [S]⁺ [s′ ⊑ s]

How does this integrate into a compositional semantics for attitude reports? Moltmann adopts a version of neo-Davidsonian event semantics (Castañeda 1967, a.o.), where certain kinds of eventualities, such as belief states, thinking events, and claiming events, are associated with a unique attitudinal object via a thematic function, which we’ll call att.6 Embedded clauses are predicated of the unique attitudinal object associated with an eventuality. To illustrate, consider the Logical Form of a simple attitude report, given in (4b):7

(4) a. Josie thinks that Sarah is cute.
   b. ∃e[thinking(e) ∧ agent(e) = josie ∧ [that Sarah is cute] (att(e))]

5Moltmann’s final proposal is a little more complicated than this, in order to distinguish between necessity and possibility. These complications are, as far as I can see, irrelevant to the points made here, and therefore I’ll stick to the simpler semantics in (3).

6I depart somewhat from Moltmann’s presentation here in drawing a parallel between att and thematic roles such as agent and theme.

7Moltmann isn't explicit about how this works compositionally, but one possibility is to complicate the semantics of that-clauses such that they compose with an eventuality and retrieve the unique attitudinal object associated with it. This will work for attitude reports, but it’s not immediately clear how to reconcile this with CP modifiers of content nouns.

8There’s an obvious question as to the status of the Davidsonian Logical Forms Moltmann posits in truthmaker semantics, given that the semantic value of a sentence should be properly understood as its verification/falsification set.
Spelling out this Logical Form explicitly, we end up with the following:

\[
\exists e \left[ \text{thinking}(e) \land \text{agent}(e) = \text{josie} \land \forall s' \in I^+(\text{Sarah cute}), \exists s \in I^+(\text{att}(e))[s' \sqsubseteq s] \land \forall s \in I^+(\text{att}(e)), \exists s' \in I^+(\text{Sarah cute})[s' \sqsubseteq s] \right]
\]

In more informal terms, this says that “Sarah is cute” is a conjunctive part of Josie’s thought (understood as an attitudinal object). This seems like a fairly weak requirement, but it nevertheless captures the intuitions concerning agreement that motivated the notion of partial content naturally stated in truthmaker semantics, since conjunctive parthood is transitive. I will suggest however, that this requirement will turn out to be too weak, once we consider the interaction with certain content nouns ranging over modal objects, such as fact.

Following Moulton (2009, 2015), Moltmann (2020) extends the predicational semantics for that-clauses outlined here to constructions involving an apparent CP complement to a noun, such as (5a) Moltmann (2020) assumes that content nouns such as claim denote properties of attitudinal objects. Consequently, (5a) denotes an attitudinal object, a conjunctive part of which is “Sarah is cute”.

(5) a. The claim that Sarah is cute.
   b. \( \forall x_c[\text{claim}(x_c) \land [\text{that Sarah is cute}] (x_c)] \)

This seems reasonable, but it fails to capture a definiteness restriction associated with certain content nouns such as fact (this pattern was initially discussed by Elliott 2016, 2017). In general, DPs can be constructed out of content nouns using any determiner. With, fact specifically however, when it appears with a that-clause modifier, we observe a definiteness restriction — it is only felicitous with the definite determiner, as in (6a).

(6) a. Josie mentioned the fact (that it’s raining).
   b. Josie mentioned a fact (*that it’s raining).

(7) a. Josie mentioned the claim/rumour (that it’s raining).
   b. Josie mentioned a claim/rumour (that it’s raining).

Why should this be? On Moltmann’s view, facts are modal objects, which are of the same sort of attitudinal objects (they are content bearers). As such “a fact that it’s raining” is predicted to denote an existential quantifier ranging over modal objects — specifically, facts which have “that it’s raining” as a conjunctive part. There could, of course, be many such facts.

A reader familiar with the literature on the semantics definite vs. indefinite articles will immediately be reminded of the data motivating Heim’s (1991) Maximize Presupposition! constraint; some examples adapted from Heim are given in (8).

(8) a. The/*a weight of our tent is under 4lb.
   b. I interviewed the/*a biological father of the victim.
A detailed discussion of Maximize Presupposition! is outside of the scope of this short note. What is important is that Maximize Presupposition! predicts that usage of the indefinite article gives rise to an (obligatory) anti-uniqueness inference, which can lead to oddness in cases where uniqueness is contextually entailed, such as the cases in (8). The generalization constraining the distribution of the indefinite article can be stated as follows:

(9) In utterance situations where the presupposition of \([\text{the } \rho] \psi\) is already known to be satisfied, it is not permitted to utter \([a \rho] \psi\).

In order to capture the oddness of “a fact that it’s raining” in line with (9), I’d like to suggest that that-clauses don’t denote the property of having \(S\) as a conjunctive part, but rather equate the content of an attitudinal/modal object with the content of \(S\). This significantly simplifies the semantics of embedded clauses, while leaving the central features of Moltmann’s theory intact:

(10) Embedded clauses (equality semantics)

\[[\text{that } S] = \lambda x_c . [x_c]^+ = [S]^+\]

This straightforwardly accounts for the definiteness restriction we observed with “fact that \(P\)”. In Moltmann’s ontology, facts are modal objects, whose verification set contains actual states. The noun fact therefore ranges over modal objects whose verifiers are actual. Naturally, there can (and will be) many such modal objects, which explains why, sans a that-clause, the noun fact is compatible with the indefinite article. When fact composes with a that-clause however, the result is the property in (11b) (on the revised semantics) — namely, a property that holds of a content-bearer \(x_c\) iff the content of \(x_c\) is the content of the embedded clause, and whose content is actual:

(a) fact that \(S\)

(b) \(\lambda x_c . [x_c]^+ = [S]^+ \land \forall s \in [x_c]^+[s \subseteq w]\)

The final piece of the puzzle is the assumption that facts, unlike, e.g., claims and rumours are individuated exclusively by their content. From this, alongside the equality semantics, it follows that there can only ever be a unique fact that \(S\). It’s important to note that this result doesn’t only hold for facts, but we find similar evidence for an equality semantics from other content nouns ranging over modal objects, such as possibility, as in (13).11

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9This echoes the proposals made in Moulton (2009, 2015) and Elliott (2017), which are framed in terms of the Kratzerian approach to clausal embedding.

10One potential worry which deserves further investigation is whether this simplification will be incompatible with Moltmann’s account of possibility vs. necessity.

11Interestingly, content nouns ranging over deontic necessities don’t seem to exhibit a definiteness restriction.

(12) Josie mentioned the/a requirement that we stay indoors.

Plausibly, this is due to how requirements are individuated — not just by content, but also, e.g., by which authority they are imposed. I leave a more careful consideration of these cases to future work.
Josie considered the possibility that it’s raining.

The revised semantics for *that*-clauses has another potential advantage, although the issue is somewhat obscured by the lack of any explicit assumptions regarding the compositional semantics. Much like Kratzer (2006), Moulton (2009, 2015), Moltmann (2020) treats *that*-clauses as predicates of content bearers. This leads to the straightforward prediction that, *that*-clause modifiers of content nouns should be stackable, much like relative clauses. This prediction is not borne out, as illustrated by the contrast in (14):

(14) a. the claim [that Josie made] [that Sarah believed].
   b. *The claim [that it’s raining] [that it’s cold].

(14b) is predicted to have a perfectly sensible Logical Form:

\[
\text{claim}(x_c) \land \text{[that it’s raining]}(x_c) \land \text{[that it’s cold]}(x_c)
\]

In other words, the DP picks out an attitudinal object — a claim — which has “it’s raining” and “it’s cold” as conjunctive parts. The equality semantics fares better:

\[
\text{claim}(x_c) \land [x_c]^+ = \text{[it’s raining]}^+ \land [x_c]^+ = \text{[it’s cold]}^+
\]

The predicted Logical Form is attributes contradictory properties to the attitudinal object. The equality semantics therefore accounts for the impossibility of stacking by virtue of the semantics of *that*-clauses. A similar point can be made with regards to attitude reports, where stacking is of course disallowed.

12 Equality semantics also makes a seemingly bad prediction — conjoining *that*-clauses under attitude verbs is predicted to lead to a contradictory logical form, whereas this does not seem to be borne out by the data.

(15) Josie claims [that it’s raining] and [that it’s cold].

Bassi & Bondarenko (2020) argue extensively that this prediction is in fact borne out in Russian. Elliott (2017) suggests one way out of the dilemma for English — we can locate the semantics attributed to the overt complementizer in some higher, silent functional head \(\mathcal{C}\).

(16) Josie claims [\(\mathcal{C}\] that it’s raining and that it’s cold)]
(18) Fiona wants to catch a fish.

Moltmann’s semantics directly accounts for underspecification, since the attitude report only conveys that there is a wanting eventuality of which Fiona is the experiencer, and “Fiona catches a fish” is a conjunctive part of the associated attitudinal object.

How to reconcile underspecification with the equality semantics will remain an open question, but it’s worth considering whether (18) might not have a pragmatic explanation. Note that catching some fish is a precondition on catching an edible fish, especially if Fiona is unable to discriminate between edible and inedible fish. In fact, (18) is a rather odd thing to assert in a context in which we know that Fiona is able to discriminate between edible and inedible fish, and can exert some influence on which she catches. This kind of context-sensitivity is not predicted straightforwardly by Moltmann’s account of underspecification.\(^{13}\)

3 Conclusion

The observations made in the previous section, pertaining to stackability of embedded clauses, should sound a cautionary note. In adopting an account of that-clauses as predicates of content-bearers, there is an implicit reliance on a restrictive syntactic component, which must be carefully motivated for the semantics to have linguistic plausibility. The same point applies to the approach to clausal embedding initiated by Kratzer (2006), and developed further by Moulton (2009, 2015), Bogal-Allbritten (2016), and Elliott (2017), among others, according to which embedded clauses express properties of contentful entities.

A prime example of where a more articulated syntactic story is necessary, is Moltmann’s account of response-stance verbs. For example, Moltmann analyzes agree as a three-place predicate, taking an (often implicit) attitudinal object \(y_c\) alongside the attitudinal object associated with the agreeing event — we may as well call this the theme. The resulting Logical Form is illustrated below:

\[
\exists e \exists e [\text{agent}(e) = \text{josie} \land \text{theme}(e) = y_c \land \text{that Sarah is cute} \land \text{att}(e)]
\]

In support of this Logical Form, Moltmann notes that \(y_c\) can be overtly realized.

(21) Josie agrees with [the claim that Sarah is cute].

The obvious question to ask is why the theme of agree can’t co-occur with an embedded clause, since the embedded clause saturates a distinct argument position. The sentence in (22) should express something perfectly sensible, on Moltmann’s rendering, namely that Josie agrees with the claim whose content has “Sarah is cute and funny” as a conjuctive part, and the attitudinal

\(^{13}\)Note furthermore that the following discourse is totally coherent:

(19) Fiona wanted to catch a fish. He desires were met, but she was disappointed to find that it was inedible.
The puzzle remains even if an equality semantics for embedded clauses is adopted.


