Abstract  Questions with an existential modal admit both mention-some (MS) and mention-all (MA) answers. Taking insights from Fox 2013, I argue that MS and MA answers are complete answers derived from the same answerhood, and that the MS/MA-ambiguity comes from structural variations within the question nucleus. However, allowing MS to be complete conflicts with Dayal’s (1996) analysis of uniqueness effects, which derives uniqueness from a presupposition that requires the existence of the strongest true answer. I argue that this dilemma can be solved if exhaustivity is evaluated relative to a subset of the accessible worlds and propose that question interpretations presuppose ‘Relativized Exhaustivity’. This presupposition preserves the merits of Dayal’s presupposition, and moreover, it is advantageous in deriving local uniqueness effects in questions with an existential/universal modal.

Keywords: questions, answers, mention-some, uniqueness, exhaustivity, exclusivity, free choice, modal, modal obviation, higher-order readings

1. Introduction

In most cases, a wh-question calls for an answer that is true and exhaustive relative to the discourse domain. For example, to optimally address the question in (1), the addressee ‘A’ should specify all of the relevant party attendants. Answers as such are called ‘complete answers’.

(1) (A’s belief: Among the relevant individuals, only John and Mary went to the party.)
Q: Who went to the party? A: John and Mary.

If a cooperative addressee believes that the best answer she can provide is non-exhaustive or possibly non-exhaustive, she would mark the incompleteness/ignorance of her answer. For instance, she may explicitly say that I don’t know who else did or I don’t know if anyone else did, or as in (2a), mark the answer with a prosodic rise-fall-rise (RFR) contour (indicated henceforth by ‘.../’; see Wagner et al. 2013). Answers like (2a) are called ‘partial answers’. If a partial answer is not properly marked,
as in (2b) which is uttered with a falling tone (indicated henceforth by ‘\’), it would give rise to an undesired exclusive inference that is misleading to the questioner.

(2) (A’s belief: John went to the party. Unclear who else and if anyone else went to the party.)
Q: Who went to the party? A: ...

a. John did .../  
   l   ]* 1-r/%

b. #John did. \   \~ Only John went to the party.
   l   ]* 1-r/%

Strikingly, \-questions with an existential modal (called ‘\-questions’ henceforth) can be naturally responded to by a non-exhaustive answer (Groenendijk and Stokhof 1984).\ For instance, question (3) can be perfectly addressed by answer (3a), which specifies only one of the chair candidates, and question (4) can be naturally addressed by answer (4a), which names only one nearby coffee place. Crucially, while being non-exhaustive, (3a) and (4a) do not have to carry an ignorance mark — they do not imply an exclusive inference even if uttered with a falling tone. Following Groenendijk and Stokhof (1984), I call these answers ‘mention-some’ (MS) answers. Relatedly, readings in which a question calls for a MS answer are called ‘MS readings’, and questions admitting MS readings are called ‘MS-questions’.

(3) (A’s belief: Only Andy and Billy can chair the committee. Co-chairing is disallowed.)
Q: Who can chair the committee? A: ...

a. Andy. \  \~ Only Andy can chair the committee.

b. Andy and Billy. \  

c. Andy or Billy. \  

(4) (A’s belief: There are two coffee places nearby, namely Starbucks and Peet’s.)
Q: Where can we get coffee around here? A: ...

a. Starbucks. \  \~ Starbucks is the only coffee place around here.

b. Starbucks and Peet’s. \  

c. Starbucks or Peet’s. \  

While tagged as ‘MS-questions’, \-questions also admit ‘mention-all (MA) readings’ in which they call for an exhaustive/MA answer. For example, (3) and (4) can also be addressed by listing all the chair candidates and all the nearby coffee places, respectively. Hence, we say that \-questions are subject to a ‘MS/MA-ambiguity’. The MA answer to a \-question can be conveyed either by a conjunction as in (3b) and (4b), or by a free-choice disjunction as in (3c) and (4c). I henceforth call (3b) and (4b) ‘conjunctive-MA answers’ and the readings in which a question calls for a conjunctive-MA answer ‘conjunctive-MA readings’. Likewise, I call (3c) and (4c) ‘disjunctive-MA answers’ and the readings in which a question calls for a disjunctive-MA answer ‘disjunctive-MA reading’.

a. The sophomores. \  

b. The sophomores, and also a few juniors, seniors, and graduates. \  

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Previous observations with MS readings focused on questions with an existential priority modal. However, example (i) illustrates that questions with an existential epistemic modal may also exhibit a MS/MA-ambiguity. In this example, the uniqueness inference triggered by the singular \-phrase can be read with a narrow scope relative to the modal verb, read as: ‘Only one person is in the room’. The question can be responded by mentioning one possible identity of this person as in (ia), or all possible identities of this person as in (ib).

(i) Which person might/could be in the room?

a. Andy. \  (Intended: ‘The unique person in the room might be Andy.’)

b. Andy or Billy. \  (Intended: ‘The unique person in the room might be Andy and might be Billy.’)
The MS-licensing effect of existential modals is seen not only in matrix wh-questions but also in question-embeddings and many non-interrogative wh-constructions. As for question-embeddings, as seen in (5), knowing a question implies knowing a true MS answer to this question. As for wh-free relatives (FRs), as seen in (6), a wh-FR admits an existential reading only if its interrogative counterpart is a question (Chierchia and Caponigro 2013).

(5) a. Jack knows [who arrived].
   \[\forall x \text{ relevant} \, \text{if } x \text{ arrived, Jack knows that } x \text{ arrived.}\]
b. Jack knows [who can chair the committee].
   \[\forall x \text{ such that } x \text{ can chair the committee, Jack knows that } x \text{ can chair.}\]

(6) a. Jack ate [what Mary cooked for him].
   \[\forall x \text{ such that } x \text{ can chair the committee, Jack knows that } x \text{ can chair.}\]
b. Jack went to [where he could get help].
   \[\forall x \text{ such that } x \text{ can chair the committee, Jack knows that } x \text{ can chair.}\]

The same distributional pattern of MS is observed with wh-conditionals in Mandarin (Liu 2016; Xiang 2020b). A Mandarin wh-conditional is made up of two wh-clauses with the same wh-morphology. In most cases, a wh-conditional expresses an universal inclusion relation between the short answers to the questions expressed by the two wh-clauses: every entity that constitutes a true short answer to the question expressed by the antecedent wh-clause also constitutes a true short answer to the question expressed by the consequent wh-clause. However, as seen in (7), a wh-conditional can be interpreted existentially if the antecedent wh-clause resembles a question.

(7) a. Ni qu-guo nar, wo jiu qu nar.
   \[\text{you go where, I go where}\]
   Intended: ‘I will go to every place where you have been to.’
b. Nar neng mai-dao jiu, wo jiu qu nar.
   \[\text{where can buy-reach liquor, I go where}\]
   Intended: ‘I will go to one of the places where I can buy liquor.’

It remains controversial whether MS answers are partial or complete. Both choices have to address some challenging questions: if MS answers are partial, one has to explain why questions are tolerated of incomplete answers; if MS answers are complete, one has to characterize the answerhood for complete answers with conditions that are compatible with non-exhaustivity.

Previous studies commonly claim that whether a question admits MS is primarily determined by pragmatic factors such as conversation goals. This view is held not only by the earlier pragmatic approaches, which treated MS as a pragmatic phenomenon, but also by many of the recent so-called ‘semantic approaches’, which consider the MS reading as an independent reading on a par with exhaustive readings. Approaches that assume the pragmatic view typically analyze non-exhaustivity in questions with a partiality marker (e.g., Who came, for example?) uniformly.

This paper argues against the pragmatic view. I will show that MS answers are uniquely subject to a ‘mention-one-only’ requirement: each MS answer specifies exactly one option (Sect. 2.2.2). This requirement cannot be explained by pragmatics. Adopting a non-exhaustive characterization of completeness from Fox 2013, I will argue that MS readings are primarily licensed by the presence of an existential modal, and that the MS/MA-ambiguity in questions comes from minimal structural variations within the question nucleus.
However, allowing non-exhaustive answers to be complete leads to a conflict with an influential question interpretation condition, namely ‘Dayal’s presupposition’, according to which a question must have a true exhaustive answer (Dayal 1996). This condition plays a non-trivial role in explaining various semantic effects and constraints in wh-constructions (Dayal 1996), degree constructions (Fox and Hackl 2007; Spector and Abrusán 2011; Abrusán 2014), definite expressions (von Fintel et al. 2014), and so on. For example, with a singular wh-phrase, the question Which student came? can have only one true answer because uttering this question in a context with multiple true answers violates Dayal’s presupposition. To solve this conflict, I propose a ‘Relativized Exhaustivity’ condition, which allows Dayal’s presupposition to be assessed relative to a modal base that introduces only a subset of the accessible worlds. This condition yields similar consequences as Dayal’s presupposition in non-modalized questions but is more advantageous in interpreting questions with modals. It solves the dilemma between the explanations of uniqueness and MS in interpreting wh-questions and helps to overcome the over-generation problem of Fox’s answerhood. Moreover, compared with Dayal’s presupposition, this condition allows to derive a local uniqueness reading that is newly observed by Hirsch and Schwarz (2020).

The rest of this paper is organized as follows. First, after comparing the existing approaches to MS, I will present empirical evidence for a new direction to analyzing MS, which I call as ‘structure-ambiguity approaches’ (Sect. 2). Second, adopting a non-exhaustive answerhood from Fox 2013, I will compositionally derive the three readings of *-questions, including MS, conjunctive-MA, and disjunctive-MA (Sect. 3). Third, section 4 will delve into the dilemma between the explanations of uniqueness and MS and solve this dilemma by Relativized Exhaustivity. Section 5 concludes.

2. Why pursuing a structure-ambiguity approach?

I classify the existing approaches to MS into the following three categories, based on their assumptions in regards to whether MS readings are independent readings and whether the distribution of MS is restricted primarily by grammatical/linguistic factors or contextual/non-linguistic factors.

- **Pragmatic approaches**: Complete answers must be exhaustive. MS answers are special partial answers that are sufficient for the conversation goal behind the question. (Groenendijk and Stokhof 1984; van Rooij 2004; among others)

- **Post-structural approaches** (traditionally referred to as ‘semantic approaches’): MS readings are independent readings on a par with exhaustive/MA readings. The MS/MA-ambiguity is caused by operations outside the question nucleus, primarily constrained by pragmatic factors. (Beck and Rullmann 1999; George 2011: Chap. 2; Caponigro and Davidson 2011; Dayal 2017; Theiler et al. 2018)

- **Structure-ambiguity approaches**: The MS/MA-ambiguity is a result of structural variations within the question nucleus. (George 2011: Chap. 6; Fox 2013; my proposal)

Among these three types of approaches, only structure-ambiguity approaches predict a grammatical restriction in distributing MS. This section will compare the core assumptions and predictions of these approaches and then present evidence for structure-ambiguity approaches.

2.1. Pragmatic approaches and post-structural approaches

Pragmatic approaches and what I call as ‘post-structural approaches’ differ with respect to whether to treat MS readings as independent readings that can be obtained directly from semantic composition.
Despite of this contrast, both types of approaches predict a pragmatic view on the distribution and licensing of MS, that is, MS readings are primarily licensed by pragmatic/non-linguistic factors.

2.1.1. Pragmatic approaches

Pragmatic approaches claim that only exhaustive answers can be complete in semantics. MS answers, which are non-exhaustive, are thought to be special partial answers that are sufficient for the conversation goal behind the question. For instance, for the question Where can we get coffee around here?, if the goal is just to get a coffee, the addressee only needs to name one accessible coffee place; if the goal is to investigate the local coffee market, the addressee needs to list all the nearby coffee places.

This pragmatic treatment was first sketched in Partition Semantics of Groenendijk and Stokhof 1984 and has remained popular in other frameworks of question semantics. van Rooij (2004) develops a utility theory and provides a formal characterization for the circumstances where MS readings are accepted and preferred. A known challenge to the pragmatic view, as pointed out by Groenendijk and Stokhof themselves and reiterated in George 2011, is that pragmatics cannot explain the availability of MS readings in question-embeddings. Pragmatic approaches respond to this challenge by building contextual parameters into the semantic denotations of questions (Ginzburg 1995; Lahiri 2002; van Rooij and Schulz 2004; Schulz and van Rooij 2006). For instance, Lahiri (2002) proposes that interpreting a question-embedding involves picking a sub-question, and that the size of the picked sub-question of the embedded question, compared with the size of the full question, needs to be large enough for the purpose of the speaker.

Ascribing the licensing of MS to contextual conditions, pragmatic approaches predict that MS readings are available in any questions that can have multiple true answers, including those that do not contain an existential modal. This prediction is usually thought to be an advantage of the pragmatic view of MS. For example, in response to the non-modalized questions in (8), unmarked answers do not yield an exhaustive inference when the conversation goal is non-exhaustive.

(8) Who knows Python?/ Who has ever used Python?
   a. (Goal: to find someone to help with programming.)
      i. Andy. \(\not\rightarrow\) Only Andy knows Python.
      ii. Andy and Billy. \(\not\rightarrow\) Only Andy and Billy know Python.
   b. (Goal: to screen the shortlisted job candidates based on their programming skills.)
      i. Andy. \(\rightarrow\) Only Andy knows Python.
      ii. Andy and Billy. \(\rightarrow\) Only Andy and Billy know Python.

However, as I will argue in Sect. 2.2.2, this pragmatic analysis cannot fully account for the characteristics of MS: MS answers to \(\Diamond\)-questions are intrinsically distinct from the non-exhaustive answers in (8a) — they are subject to further linguistic restrictions that cannot be explained by pragmatics.

2.1.2. Post-structural approaches

Post-structural approaches treat the MS reading as an independent reading on a par with exhaustive readings. For this reason, these approaches are commonly referred to as ‘semantic approaches’. I do not call them ‘semantic’ because they predict that the distribution of MS is primarily determined by pragmatic factors; instead, I call them ‘post-structural’, in contrast with the ‘structure-ambiguity approaches’ which I will introduce in Sect. 2.2 and pursue in Sect. 3. Briefly, structure-ambiguity approaches attribute the MS/MA-ambiguity in questions to structural ambiguities within the question.
nucleus, while post-structural approaches attribute this ambigiosity to operations outside the nucleus. The rest of this section reviews two representative post-structural approaches by Beck and Rullmann (1999) and George (2011: Chap. 2).

Beck and Rullmann (1999) account for the MS/MA-ambiguity in terms of answerhood-selection. They define the root of a question unambiguously as a Hamblin-Karttunen intension (i.e., a function that maps a world to the set of true propositional answers to the question in this world). Answers to questions are derived by employing an answerhood (Ans) operator, defined as in (9).

\[
\text{Ans}_{BR1} = \lambda w_3 \lambda Q_{(\langle s, (s,t) \rangle)} \cdot \cap \{ p \mid Q(w)(p) \wedge p(w) \}
\]

\[
\text{Ans}_{BR3} = \lambda w_3 \lambda Q_{(\langle s, (s,t) \rangle)} \lambda P_{(\langle s, (s,t) \rangle)} \cdot \exists p [P(w)(p) \wedge Q(w)(p) \wedge p(w)]
\]

Different Ans-operators yield different forms of exhaustivity. As exemplified in (10), employing Ans_{BR1} returns the conjunction of all the true propositional answers, yielding a MA reading. In contrast, employing Ans_{BR3}, which is encoded with an existential closure, introduces MS. As illustrated in (11), in an embedding of a MS-question, Ans_{BR3} lifts the embedded question into an existential quantifier denoting a family of questions, and this quantifier is raised over the embedding predicate.

(10) MA reading of ‘x knows Q’:
   a. LF: [x knows [Ans_{BR1}(w)(Q)]]
   b. Meaning: know_{w_3}(x, \cap \{ p \mid Q(w)(p) \wedge p(w) \})

(11) MS reading of ‘x knows Q’:
   a. LF: [Ans_{BR3}(w)(Q) \lambda p_{\langle s,t \rangle} \lambda w_3 [x knows_{w_3} p]]
   b. Meaning: \exists p[\text{know}_{w_3}(x, p) \wedge Q(w)(p) \wedge p(w)]

George (2011) has two accounts of MS. In particular, George 2011: Chap. 2 pursues a post-structural approach, which defines answerhood unambiguously existential and attributes the MS/MA-ambiguity to the absence/presence of a strengthening operator within the question root. As illustrated in (12), a question root is formed in two steps: (i) forming an abstract Ans, which denotes a property, and (ii) shifting this property into a set of propositions by employing a question-forming operator Q. In addition, between these two steps, a strengthening operator X can optionally appear right above Ans. When X is absent, the root denotes a set of non-exhaustified propositions as in (12d), and then applying an existential-closure to this set yields MS or weak exhaustivity. When X is present, the root denotes a set of exhaustified propositions read as ‘Only the members of \( \beta_{\langle x,t \rangle} \) came’ as in (12e), and then applying an existential-closure returns strong exhaustivity.

(12) Who came?
   a. \[\text{Abs} = \lambda w \lambda x. \text{came}_w(x)\]
   b. \[Q = \lambda r_{\langle s,t \rangle} \lambda p_\forall. \exists \beta_\forall [p = \lambda w. a(w)(\beta)]\]
   c. \[X = \lambda s_\forall \lambda \delta_\forall [\delta = \gamma]\]
   d. Without X: MS/weakly exhaustive
      \[Q(\text{Abs}) = \lambda p_{\langle s,t \rangle} \cdot \exists \beta_\forall [p = \lambda w. \text{came}_w(\beta)]\]
      \[= \{ \lambda w. \text{came}_w(\beta) \mid \beta \in D_\forall \}\]
   e. With X: strongly exhaustive
      \[Q(\text{Abs}) = \lambda p_{\langle s,t \rangle} \cdot \exists \beta_{\langle s,t \rangle} [p = \lambda w[\langle \lambda x. \text{came}_w(x) \rangle(\beta)]]\]
      \[= \{ \lambda w. [\langle \lambda x. \text{came}_w(x) \rangle(\beta)] \mid \beta \in D_{\langle x,t \rangle} \}\]
Attributing the MS/MA-ambiguity to operations outside the question nucleus, post-structural approaches do not predict any structure-related licensing requirements for MS. For instance, no grammatical factor may block the use of Beck & Rullmann’s AnsBR3-operator or force the presence of George’s X-operator. Hence, the same as pragmatic approaches, post-structural approaches predict that the distribution of MS is only determined by pragmatics.

2.2. A new direction: Structure-ambiguity approaches

Structure-ambiguity approaches assume that MS and MA answers are both complete answers and are derived via the same answerhood. This assumption requires to define a non-exhaustive definition of completeness (Sect. 3.1.3). Moreover, as illustrated in Figure 1, in contrast with post-structural approaches, which attribute the MS/MA-ambiguity to an operation outside the question nucleus (e.g., the Ans-operation), structure-ambiguity approaches attribute this ambiguity to structural variations within the nucleus. Structural variations are, for example, scope ambiguities between the wh-trace and other scopal elements in the nucleus (Sect. 3.3; see also George 2011: Chap. 6 and Fox 2013) and the presence/absence of an IP-internal operator (Sect. 3.4). Crucially, since modal expressions in ◊-questions are interpreted within the nucleus, only structure-ambiguity approaches may predict a grammatical relation between the presence of an existential modal and the availability of MS.

![Figure 1: Post-structural approaches versus structure-ambiguity approaches](image)

Note that structure-ambiguity approaches do not claim that pragmatics is completely irrelevant in distributing MS — pragmatics may serve as a ‘MS-blocker’, although not sufficiently a ‘MS-licenser’. If a question is semantically ambiguous between MS and MA, it is compatible with the structure-ambiguity approaches that a conversation goal that calls for an exhaustive answer will block MS. For example in (13), given the context, the search committee expects the assistant to list all the candidates who can teach Experimental Semantics. In this case, an answer without an ignorance mark would be understood exhaustively.

(13) (Context: In making the final decision of a job search, the committee decided to consider only candidates who can teach Experimental Semantics or Field Methods.)
Chair: ‘Who can teach Experimental Semantics?’
Assistant: ‘Judy can.’
⇝ Among the candidates, only Judy can teach Experimental Semantics.

The rest of this section will present empirical evidence for structure-ambiguity approaches and against pragmatic and post-structural approaches. First, while pragmatic conditions being equal,
the presence of an existential modal significantly increases the acceptance of MS readings (Xiang and Cremers 2017; see Sect. 2.2.1). Second, in contrast to non-exhaustive answers to questions with a partiality-marker (e.g., Who is on your committee, for example?), MS answers to ◊-questions are subject to a ‘mention-one-only’ restriction which cannot be explained by non-linguistic factors (Sect. 2.2.2). These findings argue that the licensing condition of MS readings must be distinct from that of other non-exhaustive readings, and that this condition must be relevant to grammatical factors within the question nucleus.

2.2.1. Experimental evidence from Xiang and Cremers 2017

Pragmatic approaches and post-structural approaches predict that MS readings are primarily determined by pragmatic factors, especially the conversation goal, not directly related to the presence/absence of an existential modal. However, in Xiang and Cremers 2017, we found experimental evidence against this prediction: while having the same conversation goal, the presence of an existential modal in the question nucleus may significantly increase the availability of MS readings.

In the experiment, participants were first presented with a paragraph describing the background: “Mary is in charge of choosing two children to lead the dance. The only rule is that the children leading the dance should have an accessory in common.” This background ensures that whatever addresses the embedded ◊-question in (14a) also addresses the embedded non-◊-question in (14b), and vice versa. Next, the participants saw a set of pictures illustrating how children were dressed and two sentences describing Mary’s memory. The following presents a MS trail: Mary remembers only one of the two pairs who have an accessory in common and can co-lead the dance.

Finally, the participants were asked to judge the truth value of a question-embedding sentence. This form of the embedded question varied by two conditions: (i) whether the wh-subject is who or which children, and (ii) whether the embedded question is a ◊-question, as in (14a), or not, as in (14b). Regardless of the form of the wh-phrase, fitting data of the MS trails with a logistic mixed effect modal reported a significant effect of \([\pm \text{Modal}]\) \((p < .001)\).

\[(14)\]
\[
\begin{align*}
\text{a. Mary remembers \{who, which children\} can lead the dance.} & \quad ([+\text{Modal}]) \\
\text{b. Mary remembers \{who, which children\} have an accessory in common.} & \quad ([-\text{Modal}])
\end{align*}
\]

2.2.2. The ‘mention-one-only’ requirement of MS answers

MS answers to ◊-questions express a special form of non-exhaustivity: each MS answer specifies exactly one of the options to solve the question under discussion. Therefore, more precisely speaking, MS answers should be called ‘mention-one answers’, in contrast with ‘mention-few answers’ each of which specifies multiple options. To illustrate this ‘mention-one-only’ requirement, I will compare mention-one and mention-few answers to matrix ◊-questions and examine the availability of mention-
Evidence from matrix questions is concerned with the marked-ness of each type of short answers when these answers are read non-exhaustively. In response to a ◊-question, if the context is compatible with (even if not requiring) an exhaustive reading, a non-exhaustive mention-few answer must be ignorance-marked, otherwise it would be read exhaustively by the hearer. For example, in response to the typical MS-question (15), the mention-one answer (15a) does not give rise to an exclusive inference while being unmarked. However, if the addressee specifies more than one chair candidate as in (15b,c) and does not mark her answer, the hearer would read the answer exhaustively.

(15) Who can chair the committee?
   a. Andy. \(\not\rightarrow\) Only Andy can chair.
   b. Andy and Billy. \(\not\rightarrow\) Only Andy and Billy can chair.
   c. Andy or Billy. \(\not\rightarrow\) Only Andy and Billy can chair.

In contrast, in response to questions with a partiality marker (e.g., for example, for instance, give me an example), there is no clear contrast in exhaustivity between mention-one answers and mention-few answers. As seen in (16) and (17), regardless of whether the question carries an existential modal, neither mention-one nor mention-few answers yield an exclusive inference.

(16) Who is on your committee, for example?
   a. Andy. \(\not\rightarrow\) Only Andy is in my committee.
   b. Andy and Billy. \(\not\rightarrow\) Only Andy and Billy are in my committee.

(17) Who can chair the committee, for example?
   a. Andy. \(\not\rightarrow\) Only Andy can chair.
   b. Andy and Billy. \(\not\rightarrow\) Only Andy and Billy can chair.

The contrast in exhaustiveness between (15a) and (15b,c) shows that there is a grammatical distinction between individual answers and con-/dis-junctive answers. As I will argue in Sect. 3.2, when a ◊-question has a MS reading, answers expressed by an (atomic/plural) individual are possibly complete, while answers expressed by an elided conjunction or disjunction are not. Moreover, the contrast between (15) and (16),(17) in admitting non-exhaustive mention-few answers shows that the source of non-exhaustivity in ◊-questions is different from that in questions with a partiality-marker.\(^4\)

Evidence from question-embeddings is concerned with the truth-conditional meanings of these embedding sentences. Embeddings of ◊-questions allow mention-one and mention-all readings but not mention-few readings, even if mention-few fits with the conversation goal better than mention-one/all. For instance in (18), the conversation goal calls for a ‘mention-three answer’ (viz., an answer that specifies three possible chairs). If MS readings were primarily licensed by the conversation goal, (18) should have the mention-three reading (18c), contrary to fact.

(18) (Context: The dean wants to meet with three chair candidates so as to make plans for the committee.) Jill knows who can chair the committee.
   a. Available mention-one reading:
      ‘For one individual \(x\) such that \(x\) can chair the committee, Jill knows that \(x\) can.’

\(^4\) I treat the partiality-marker for example as a discourse-level expression separated from the question root. It signals that the questioner is tolerated of an incomplete true answer and presupposes the existence of such an answer in the answer space of the question (Xiang 2020b).
b. Available mention-all reading:
   ‘For every individual x, if x can chair the committee, Jill knows that x can.’

c. Unavailable mention-three reading:
   ‘For three individuals x,y,z such that x,y,z each can chair the committee, Jill knows that x,y,z each can.’

Note that it is difficult to exclude mention-few readings based on the truth-conditions of sentence (18). For a question-embedding sentence ‘x knows Q’, where Q has a MS reading, it is true if and only if (i) x knows a true MS answer (viz., a mention-one answer) to Q, and (ii) x has no false belief relevant to Q. Knowing a true mention-three answer or knowing three mention-one answers entails condition (i); therefore, sentence (18) is true in a scenario where Jill correctly identifies three chair candidates. In regards to this confound, Seth Cable (pers. comm.) suggests a truth-value-judgement task that allows to determine the availability of mention-few readings in embeddings. This task is to judge the truth value of the question-embedding sentence (19b) in the three subtly different scenarios in (19a). Language consultants judged (19b) true in the mention-one scenario and the mention-all scenario but false in the mention-three scenario.

(19) a. Norvin said to us, “On my exam, you will have to name ... with multiple wh-fronting.”
   i. ... one language ... [mention-one: true]
   ii. ... all languages ... [mention-all: ?true]
   iii. ... three languages ... [mention-three: false]

b. Norvin said that we will have to know where we can find multiple wh-fronting.

In conclusion, MS answers to ◊-questions are subject to a ‘mention-one-only’ restriction. For one thing, in responding to a matrix ◊-question, non-exhaustive mention-few answers, while being more informative than mention-one answers, have to be marked as partial answers. For another, the question-embedding sentence ‘x knows ◊-Q’ implies that x knows a mention-one answer or the mention-all answer to the embedded ◊-Q, but not that x knows a mention-few answer to the embedded ◊-Q’. This ‘mention-one-only’ restriction cannot be explained by pragmatics. I will provide an explanation in Sect. 3.2.

Before ending this section, I would clear a possible confusion: the word ‘one’ in ‘mention-one’ refers to one option, not necessarily one atomic individual/entity. In contrast to the above examples where each option is constituted by one single individual or language, there are also cases where an option is made up of multiple individuals/entities. In (15) and (18), for example, if co-chairing is possible, then a possible group of co-chairs also counts as one single option. This point is more clearly shown in (20). Regardless of whether the predicate that the existential modal can combines with is collective (e.g., form the committee) or distributive (e.g., serve on the committee), a mention-one answer specifies the sum of a full group of individuals who can simultaneously serve on committee, as in (20b,c) (cf. (20a)), and the MA answer specifies both possible groups, as in (20d).

3This footnote explains why example (19), in contrast to (18), can effectively distinguish between mention-one and mention-three readings. In this example, a global scalar implicature associated with the numeral one/three breaks the entailment from knowing three multiple wh-fronting languages to knowing one such language. More specifically, sentence (i) is naturally interpreted as (ia/b): the numeral N itself is read as inclusive (viz, read as ‘at least N’, not ‘exactly N’), but the exhaustification above the necessity modal have to yields a non-monotonic environment relative to N.

   (i) You will have to name (at least) N languages with multiple wh-fronting.
      a. ‘You will only have to name (at least) N languages with multiple wh-fronting.’
      b. ‘The maximal number of languages with multiple wh-fronting that you will have to name is N.’
Who can form the committee? / Who can serve on the committee?

a. # Andy.
b. Andy and Billy.
c. Andy, Billy, and Cindy.
d. Andy, Billy or Andy, Billy, Cindy.

3. Structure-ambiguity approach to composing MS-questions

Structure-ambiguity approaches ascribe the MS/MA-ambiguity to structural variations within the question nucleus. The first structure-ambiguity treatment applicable to \( \diamond \)-questions was made by Fox (2013).\(^6\) This treatment has two major assumptions: first, a true answer is complete as long as it is not logically weaker than any true answer; second, the MS/MA-ambiguity of a \( \diamond \)-question comes from the scope ambiguity between the existential modal and a covert distributivity operator associated with the \textit{wh}-trace. (For a review of Fox 2013, see Xiang 2016: Chap. 2.) My analysis adopts Fox’s answerhood but departs from his analysis on the compositional derivation of this ambiguity.

In the following, I will first lay out my background assumptions on question semantics and introduce Fox’s answerhood (Sect. 3.1). Next, I will propose an analysis to derive MS readings compositionally (Sect. 3.2). This analysis considers both first-order and higher-order readings. Last, I will present two ways to derive MA readings. In particular, conjunctive-MA is derived by interpreting the higher-order \textit{wh}-trace above the existential modal (Sect. 3.3), and disjunctive-MA is derived by employing a free-choice-triggering operator above the existential modal (Sect. 3.4).

3.1. General assumptions on question semantics

3.1.1. Questions as topical properties

In line with categorial approaches to questions, I assume that the root denotation of a question is a function which I refer to as a ‘topical property’. A \textit{wh}-question, as exemplified in (21a,b), denotes a function that maps a meaning in the quantificational domain of the \textit{wh}-phrase (viz., a short answer) to a proposition in the answer space (viz., a propositional answer). In other words, as in (21c), the answer space of a question is the image of the topical property of this question. I henceforth write topical property as ‘\([Q]\)’ and answer space (i.e., Hamblin set) as ‘\(Q\)’.

\[
(21) \text{Which books did John read? Harry Potter.}
\]

\[
\text{a. } [Q] = \lambda x : x \in \text{books}_{\text{\textsc{en}}} \lambda w [\text{read}_{w}(j, x)]
\]

\[
\text{b. } [Q]([\text{HP}]) = \text{hp} \in \text{books}_{\text{\textsc{en}}} \lambda w [\text{read}_{w}(j, \text{hp})]
\]

\(^6\)Earlier than Fox (2013), George (2011: Chap. 6) provided the first structure-ambiguity approach to the MS/MA-ambiguity. This approach, however, only tackles questions like (i) which carries an existential indefinite \textit{one of the hosts}. George assumes that the existential indefinite undertakes quantifier raising, yielding scope ambiguity relative to the \textit{wh}-trace and causing an ambiguity. This approach does not extend to \( \diamond \)-questions since modals are not (known to be) able to move.

(i) Who did one of the hosts invite?

The non-exhaustive reading of (i) is more commonly considered as a choice reading involving quantifying-into a question (Groenendijk and Stokhof 1984; Chierchia 1993; among others). In Xiang 2019, 2020d, I propose to derive choice readings and other quantifying-into question readings uniformly by quantifying-into \textit{wh}-dependencies.
In regards to the issues concerned in this paper, there are three reasons for defining questions as topical properties. First, with this assumption, the relation between questions and short answers can be analyzed as a simple function–argument relation. This analysis is convenient for tracking the contrast among different forms of short answers, especially the contrast between conjunctive-MA and disjunctive-MA answers. Second, in Sect. 4.4 on solving the dilemma between uniqueness and MS, I will propose a condition called ‘Relativized Exhaustivity’, which cannot be defined without retrieving the short answers (see f.n. 18). Categorial approaches allow short answers to be semantically retrieved from question denotations, in contrast with frameworks that define questions as sets of propositions (e.g., Hamblin–Karttunen Semantics, Partition Semantics). Third, as seen in Sect. 1, MS readings are also available in nominal/predicative constructions such as wh-FRs and Mandarin wh-conditionals. Defining questions as topical properties, a treatment for MS readings of wh-questions also extends to existential readings of these non-interrogative wh-constructions. For further arguments for pursuing a categorial approach and details of the formal theory, see Xiang 2020b.

3.1.2. Higher-order denotations of wh-questions

Example (21) has illustrated the case where a wh-question has a first-order reading and denotes a first-order function. In addition, a wh-question may alternatively have a higher-order reading, in which it denotes a function defined for higher-order meanings such as generalized quantifiers.

Spector (2007, 2008) argues for the existence of higher-order readings to account for cases like (22). To derive the free-choice reading of the disjunctive answer, the elided disjunction should be treated as a Boolean disjunction (i.e., the union of two Montagovian individuals, of type \langle et, t \rangle), and the LF of the question should involve semantic construction across the modal (namely, the wh-phrase binds a higher-order trace \pi (of type \langle et, t \rangle) across the modal verb). This LF yields the higher-order topical property in (22b), where \( ^{\text{wh}} \text{books@} \) stands for a set of generalized quantifiers ranging over a set of books.

(22) Which books does John have to read?
The French novels or the Russian novels. The choice is up to him.  
\begin{align}
\text{a. } & \langle \text{ct} \rangle \text{ which-books } \lambda \pi \langle \langle et, t \rangle \rangle \text{ [if have-to [ } \pi \lambda x [\text{ [vt] John read } x ] ] ] \\
\text{b. } & \llbracket Q \rrbracket = \lambda \pi \langle \langle et, t \rangle \rangle : \pi \in ^{\text{wh}} \text{books@} \boxdot \lambda w [\pi (\lambda x . \text{read}_w (j, x))] \\
\text{c. } & \llbracket F \text{ or } R \rrbracket = f^\otimes \cup r^\otimes \quad (x^\otimes \text{ stands for the Montagovian individual yielded from } x.) \\
\text{d. } & \llbracket Q \rrbracket (\llbracket F \text{ or } R \rrbracket) = f^\otimes \cup r^\otimes \in ^{\text{wh}} \text{books@} \boxdot \lambda w [\{f^\otimes \cup r^\otimes \} (\lambda x . \text{read}_w (j, x))] \\
& = \{t, r\} \subseteq \text{books@} \boxdot \lambda w [\text{read}_w (j, t) \lor \text{read}_w (j, t)]
\end{align}

More precisely, not all generalized quantifiers can serve as semantic answers to wh-questions.\footnote{Spector (2007, 2008) argues that only increasing quantifiers can serve as semantic answers to wh-questions. In Xiang 2020c, observing variations among non-monotonic quantifiers, I argue for a weaker constraint which only requires the quantifiers in the domain of a question denotation to be ‘homogeneously positive’. Any \( \pi \) of type \langle et, t \rangle is homogeneously positive if and only if existence is ensured with respect to the quantification domain of \( \pi^+ \) and \( \pi^- \), where \( \pi^+ \) and \( \pi^- \) are the strongest increasing and decreasing quantifiers retrieved from \( \pi \), respectively.}

For the interests of this paper, it only matters that the domain of a higher-order question denotation includes at least Montagovian individuals and their Boolean coordinations.
### 3.1.3. Answerhood

Dayal (1996) assumes that only the ‘strongest true answer’ (namely, the true exhaustive answer which entails all the true answers) may completely address a question. As defined in (23), applying the Ans\textsubscript{Dayal} operator to a world \( w \) and an answer space \( Q \) returns the unique strongest proposition among the propositions in \( Q \) that are true in \( w \), defined only if this strongest proposition exists.

\begin{equation}
\text{Ans}\textsubscript{Dayal}(w)(Q) = \exists p[w \in Q \land \forall q[w \in q \rightarrow p \subseteq q]],
\end{equation}

\begin{equation}
\iota p[w \in Q \land \forall q[w \in q \rightarrow p \subseteq q]]
\end{equation}

The presupposition carried by this answerhood, which I refer to as ‘Dayal’s presupposition’, has played a crucial role in explaining various semantic effects and constraints in \( \text{wh} \)-constructions, degree constructions, and definite expressions. In regards to \( \text{wh} \)-questions, it is especially advantageous in accounting for the uniqueness effects of singular \( \text{wh} \)-phrases (Sect. 4.1). However, this answerhood predicts that only exhaustive answers can be complete, leaving no space for MS.

To account for MS readings and to derive MS and MA with a uniform answerhood, Fox (2013) weakens the characterization of completeness. He proposes that any true answer that is not asymmetrically entailed by any true answers is complete, and he calls such answers ‘maximally (max-)informative true answers’. The answerhood-operator for complete answers is defined as in (24): \( \text{Ans}\textsubscript{Fox}(w)(Q) \) returns the set of max-informative propositions among those in \( Q \) that are true in \( w \). Each proposition in this set counts a complete true answer in \( w \).

\begin{equation}
\text{Ans}\textsubscript{Fox}(w)(Q) = \{ p \mid w \in p \in Q \land \forall q[w \in q \rightarrow q \not\subset p]\}
\end{equation}

The following illustrates the difference between Dayal’s and Fox’s answerhood-operators:

\begin{equation}
\begin{aligned}
\text{Ans}\textsubscript{Dayal}(w)(Q_1) &= p \land q, \text{ while } \text{Ans}\textsubscript{Fox}(w)(Q_1) = \{ p \land q \}; \\
\text{Ans}\textsubscript{Dayal}(w)(Q_2) &= \text{undefined, while } \text{Ans}\textsubscript{Fox}(w)(Q_2) = \{ p, q \}.
\end{aligned}
\end{equation}

In contrast to Dayal’s answerhood, Fox’s answerhood allows complete answers to be non-exhaustive and allows a question to have multiple complete true answers. It predicts the following distribution of MS: a question with an answer space \( Q \) has a MS reading only if there is a world \( w \) such that \( \text{Ans}\textsubscript{Fox}(w)(Q) \) is a non-singleton set (as in the case of \( Q_2 \)). If \( Q \) is closed under conjunction (as in the case of \( Q_1 \)) or if the propositions in \( Q \) are all mutually exclusive, \( \text{Ans}\textsubscript{Fox}(w)(Q) \) remains a singleton set for every \( w \), and this question has a MA reading.

This paper adopts Fox’s answerhood so as to allow for MS readings. Adapting this answerhood to a categorial approach, I define the answerhood-operators as follows, where these operators combine with topical properties, not Hamblin sets. \( \llbracket Q \rrbracket_w (=_{df} \{ \alpha \mid \alpha \in \text{Dom}(\llbracket Q \rrbracket) \land w \in \llbracket Q \rrbracket(\alpha)\}) \) abbreviates the set of true short answers to \( Q \) in \( w \).

\begin{equation}
\begin{aligned}
\text{Ans}^5(w)(\llbracket Q \rrbracket) &= \{ \alpha \mid \alpha \in \llbracket Q \rrbracket_w \land \forall \beta[\beta \in \llbracket Q \rrbracket_w \rightarrow \llbracket Q \rrbracket(\beta) \not\subset \llbracket Q \rrbracket(\alpha)]\}; \\
\text{Ans}^5(w)(\llbracket Q \rrbracket(\alpha)) &= \{ \llbracket Q \rrbracket(\alpha) \mid \alpha \in \text{Ans}^5(w)(\llbracket Q \rrbracket)\}
\end{aligned}
\end{equation}
This definition does not consider presuppositions that are needed to account for uniqueness effects in questions and to avoid over-generating MS-readings. I will return to this issue in Sect. 4 and add a presupposition called ‘Relativized Exhaustivity’ to the definition.

3.2. Deriving MS readings

I propose that MS readings of $\Diamond$-questions are compositionally derived with the following LFs:

(27) Who can chair the committee?

<table>
<thead>
<tr>
<th></th>
<th>First-order MS reading</th>
<th>Higher-order MS reading ($\Diamond \gg \pi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP (Q-root)</td>
<td>who $\lambda x e$ C'</td>
<td>who $\lambda \pi_{(et,t)}$ C'</td>
</tr>
<tr>
<td>IP (Q-nucleus)</td>
<td>can $O_C$ VP</td>
<td>can $\pi \lambda x e$ O_C VP</td>
</tr>
<tr>
<td></td>
<td>$x_{[+v]}$ c.t.c.</td>
<td>$x_{[+v]}$ c.t.c.</td>
</tr>
</tbody>
</table>

These derivations have two main assumptions. First, an exhaustification operator $O$ ($\approx$ only) appears under the existential modal and is associated with the individual $wh$-trace $x$ (of type $e$). Second, in the LF for the higher-order MS reading, the fronted $wh$-phrase binds a narrow scope higher-order trace $\pi$ (of type $\langle et, t \rangle$) across the existential modal. With this assumption, the topical property yielded by the composition is a higher-order function which can take higher-order arguments such as Boolean conjunctions and disjunctions. By contrast, in the LF for the first-order MS reading, the fronted $wh$-phrase directly binds an $e$-type trace, and yielded topical property is a first-order function which can only take entities/individuals as arguments.

In the following, I will explain the reasons for assuming local exhaustification (Sect. 3.2.1) and then show how MS-readings are derived with the above LFs (Sect. 3.2.3 and Sect. 3.2.2).

3.2.1. Local exhaustification

The local $O$-operator is assumed to account for the local exhaustivity effects and mutual independence of MS answers. To see how local exhaustivity manifests itself, compare the three answers to the $\Diamond$-question (28). Although it is true that Andy can serve on the committee, answer (28a) is not a good MS answer, in contrast to (28b,c) each of which specifies a full possible committee. This contrast suggests that MS answers are subject to local exhaustivity: a good MS answer to (28) must specify all the members of a possible committee. For example, answer (28b) is read as the following: ‘It can be the case that only Andy and Billy serve on the committee.’

(28) Who can serve on the committee?

(Context: The committee can be formed in two ways: either by two people Andy and Billy, or by three people Andy, Billy, and Cindy.)

a. # Andy.

b. Andy and Billy.

c. Andy, Billy, and Cindy.

Inserting an O-operator under the existential modal captures this local exhaustivity effect. This operator has a meaning akin to the exclusive focus particle *only*: as defined in (29), the O-operator affirms the prejacent proposition and negates the alternatives of the prejacent that are not entailed by the prejacent (Chierchia et al. 2012; among others). The domain variable C carried by the O-operator denotes a contextually relevant set of alternatives of the prejacent.8

\[
\|O_C\| = \lambda p \lambda w. p(w) = 1 \land \forall q \in C[p \not\subseteq q \rightarrow q(w) = 0]
\]

(For any proposition p, \(\|O_C\|(p)\) is true if and only if p is true and any proposition in C that is not entailed by p is false.)

Other than accounting for local exhaustivity, assuming local exhaustification also captures the mutual independence of MS answers and solves an under-generation problem with Fox’s answerhood. In (28), since the predicate *serve on the committee* (abbr: *serve.on*) is distributive, if the answers (28b) and (28c) are interpreted as \(\diamond \lambda w. \text{serve.on}_w(a \oplus b)\) and \(\lambda \lambda w. \text{serve.on}_w(a \oplus b \oplus c)\), respectively, (28b) would be asymmetrically entailed by (28c) and would not count as a good MS answer according to Fox’s answerhood, contrary to fact. The local O-operator, as a non-monotonic operator, breaks the entailment relation between the two propositions and allows them both to be max-informative.

I assume that the local O-operator is preferred in parsing but not required. As seen in (30), the local implicature is cancellable and suspend-able. Hence, the presence of the O-operator should only matter to cancellable inferences such as local exhaustivity; in contrast, mandatory effects such as the uniqueness effects of singular *wh*-phrases and the MS-licensing effects of existential modals should be independent from the presence of the O-operator. In Sect. 4.4, I will account for these mandatory effects without requesting a local O-operator.

(30) Who can serve on the committee?

---

8In the expanded Alternative Semantics of Chierchia (2006, 2013), alternatives are activated by lexically encoded grammatical features such as the focus feature [+f], the scalar feature [+s], and the domain feature [+d]. Feature-activated alternatives of terminal expressions are defined as follows (Chierchia 2013; Xiang 2020a).

(i) Focus-alternatives: for any basic expression \(\alpha\),

\[
\text{F-Alt}(\alpha) = \begin{cases}
D_{\text{type}}([\alpha]) & \text{if } \alpha \text{ carries a [+f] feature} \\
\{[\alpha]\} & \text{otherwise}
\end{cases}
\]

(ii) Scalar-alternatives: for any basic expression \(\alpha\),

\[
\text{S-Alt}(\alpha) = \begin{cases}
\{[\alpha_1], ... , [\alpha_n]\} & \text{if } \alpha \text{ carries a [+s] feature and } [\alpha]\text{ is part of a scale } ([\alpha_1], ... , [\alpha_n]) \\
\{[\alpha]\} & \text{otherwise}
\end{cases}
\]

(iii) Domain-alternatives

a. for the disjunctive connective or,

i. \(\text{D-Alt}(\sigma_{or}) = \{\lambda b \lambda a.a \cup b, \lambda b \lambda a.a, \lambda b \lambda a.b\}\)

ii. \(\text{D-Alt}(\sigma_{and}) = \{\lambda b \lambda a.a \cup b\}\)

b. for any quantifier \(\Theta\) with a syntactic domain variable \(D\),

\[
\text{D-Alt}(\Theta_D) = \begin{cases}
\{[\alpha]D, [\alpha]^{D \cap [D']} \mid D' \subseteq g(D)\} & \text{if } \alpha \text{ carries a [+d] feature} \\
\{[\alpha]D\} & \text{otherwise}
\end{cases}
\]

These alternatives grow point-wise till being used by an associated operator. The association relation between an exhaustivity operator and a c-commanded expression is realized via a domain restriction condition.

(iv) Domain restriction condition (Xiang 2020a; expanding on Rooth 1996 on focus association)

for any operator \(\Theta\) quantifying over a domain C and combining with an expression \(\delta\), if \(\Theta\) agrees with an alternative-activating feature [+s], \(\Theta_C(\delta)\) is defined only if \(C \subseteq \text{x-Alt}(\delta)\).
a. Andy and Billy ... maybe also Cindy.
b. Andy and Billy. I don’t know whether we have to add a third person.

3.2.2. First-order MS-denotation

Return to the derivation of MS readings. The composition for the first-order reading of a $\Diamond$-question is pretty straightforward. The illustration in (31) is concerned with the $\Diamond$-question (28), to which a MS answer names the sum of multiple individuals. In the LF, an $O$-operator appears under the modal verb *can* and is associated with the $e$-type $wh$-phrase $x$. This trace carries a [+v] feature, which activates a set of variable-alternatives that are meanings of type $e$. This set of alternatives grows point-wise, yielding the set of alternatives of the VP as in (31a). The domain variable $C$ carried by the $O$-operator denotes a subset of $Alt(VP)$, and its value is restricted by the quantification domain of the frontal $wh$-phrase (i.e., $\text{hmn}_{\text{@}}$, a set of [contextually relevant] human individuals in the actual world $\Box$), as in (31b). The composition yields the first-order topical property (31c). Clearly, the propositions in the yielded answer space are all logically independent. Applying Fox’s answerhood yields a MS reading and returns a set with two complete true answers as in (31d).

(31) Who can serve on the committee? (First-order MS. Context: The same as in (28).)
[cp who $\lambda x_c \{ \text{can} [ O_C \{ \text{serve on the committee} \} ] ] \}]

a. $Alt(VP) = \{ \lambda w.\text{serve-on-the-comm}_w(x) \mid x \in D_c \}$ (Abbreviated as: $\{ \phi_x \mid x \in D_c \}$)
b. $C = \{ \phi_x \mid x \in \text{hmn}_{\text{@}} \}$
c. $\|Q\| = \lambda x_c : x \in \text{hmn}_{\text{@}} \circ O_C \phi_x$
d. i. $\text{Ans}^S(w)(\|Q\|) = \{ a \oplus b, a \oplus b \oplus c \}$
   ii. $\text{Ans}^P(w)(\|Q\|) = \{ \Diamond O_C \phi_{a \oplus b}, \Diamond O_C \phi_{a \oplus b \oplus c} \}$

3.2.3. Higher-order MS-denotation

Example (32) illustrates the derivation of a higher-order MS reading. Compared to the derivation of a first-order reading, the only difference is that here the $wh$-phrase undertakes an IP-internal movement from $x$ to $\pi$ before reaching [Spec, CP]. The higher-order trace $\pi$ is above the local $O$-operator and under the existential modal *can*. The composition yields the higher-order topical property (32b), where $\text{hmn}_{\text{@}}$ stands for a set of generalized quantifiers that range over a set of human individuals.

(32) Who can chair the committee? (Higher-order MS)
[cp who $\lambda \pi_{(\text{at},t)} \{ \text{can} [ \pi \text{can} [ O_C \{ \text{chair the committee} \} ] ] \}]

a. $Alt(VP) = \{ \lambda w.\text{chair-the-comm}_w(x) \mid x \in D_c \}$ (Abbreviated as: $\{ \phi_x \mid x \in D_c \}$)
b. $\|Q\| = \lambda \pi_{(\text{at},t)} : \pi \in \text{hmn}_{\text{@}} \circ \pi(\lambda x_c, O_C \phi_x)$

The answer space yielded from (32b) is illustrated in Figure 2. Here I only consider the three types of answers related to the two individuals Andy ($a$) and Billy ($b$) and their sum ($a \oplus b$), yielded by

---

9Adding to the ontology of alternatives and alternative-triggering features, I propose that $A$-traces optionally carry a [+v] feature which activates a set of variable-alternatives ($v$-$Alt$). As schematized in (i), variable-alternatives are defined in the same way as focus-alternatives (see fn. 8), but they differ from focus-alternatives in that they are specific to variable-denoting expressions and do not require focus-marking.

(i) Variable-alternatives: for any trace and pronoun $a$,

$v$-$Alt(a) = \begin{cases} 
D_{\text{type}[=1]} & \text{if a carries a [+v] feature} \\
\{a\} & \text{otherwise}
\end{cases}
applying the topical property (32b) to Boolean conjunctions (e.g., $a \cap b$), Montagovian individuals (e.g., $a^{□}$), and Boolean disjunctions (e.g., $a \cup b$). For example, $\Diamond O_C \phi_a$ is derived based on $a^{□}$ and is read as ‘There is a world in which [among the relevant individuals] only $a$ chairs the committee’. Arrows indicate entailment relations among the propositions in the answer space, shading marks the true answers, and underwaving marks the max-informative true answers. In the considered world, the question has two max-informative true answers, namely $\Diamond O_C \phi_a$ and $\Diamond O_C \phi_b$.

(Context: Only Andy and Billy can chair the committee. Only single-chairing is allowed.)

![Figure 2: The answer space of who can chair the committee? (MS)](image)

The following considers the semantics of the three types of answers:

- **Row 1-2**: Due to the presence of a local $O$-operator, the conjunctive answers are all contradictory. For example, $\Diamond [O_C \phi_a \land O_C \phi_b \land O_C \phi_{a \oplus b}]$ is read as ‘There is a world in which only $a$ chairs the committee and only $b$ chairs the committee’.
- **Row 3**: The individual answers can be true. Moreover, due to the local $O$-operator, the individual answers are logically independent from each other, no matter whether an answer names an atomic individual or a sum of individuals and whether the predicate $chair the committee$ is read distributive (i.e., ‘chair the committee separately’) or collective (i.e., ‘chair the committee together’). Hence, any true individual answer is also a max-informative true answer.
- **Row 4-5**: The disjunctive answers can be true but can never be complete.

To see why the disjunctive answers are partial, consider the following simplified answer space which ignores the co-chairing options. (In other words, answers related to the sum $a \oplus b$ are removed.) The disjunctive symbol ‘$\lor$’ in the middle stands for the disjunction of the two individual answers, namely $\Diamond O_C \phi_a \lor \Diamond O_C \phi_b$.

(Context: Only Andy and Billy can chair the committee. Only single-chairing is allowed.)

![Figure 3: The (simplified) answer space of who can chair the committee? (MS)](image)
In this answer space, the disjunctive answer at the bottom is logically equivalent to the disjunction of the two individual answers. Hence, the disjunctive answers yielded from the MS topical property can never be max-informative: whenever a disjunctive answer is true, there is a true individual answer that asymmetrically entails this disjunctive answer.

3.2.4. Consequences

The proposed analysis explains why MS answers to \(\Diamond\)-questions are all ‘mention-one’. In responding to a \(\Diamond\)-question with a MS reading, only individual answers, each of which names one single full option, are possibly max-informative. Conjunctions and disjunctions do not express mention-few answers; conjunctive answers are by default parsed as conveying a contradiction, and disjunctive answers are always partial and are read with an ignorance inference.\(^{10}\) Hence, in response to a matrix \(\Diamond\)-question, if the addressee uses an elided conjunction or disjunction to convey a mention-few answer, she must be interpreting the question with a MA reading which calls for an exhaustive answer. This mention-few answer, if is non-exhaustive, should be ignorance-marked as a partial answer. For the derivations of MA readings, see the next two subsections.

Moreover, this analysis partially explains the MS-licensing effects of existential modals. With the answerhood adopted from Fox 2013, a question has a MS reading if and only if there is a world in which the answer space of this question has multiple max-informative true propositions. This condition is met only if both of the following sub-conditions are met:

(A) the semantically independent answers are not mutually exclusive.

(B) the answer space is not closed under conjunction;

On sub-condition (A), the insertion of a local O-operator makes the individual answers logically independent (i.e., not logically weaker or stronger than the others), and further, the presence of an existential modal above the O-operator ensures that these answers are not mutually exclusive and allows multiple individual answers to be simultaneously true. In comparison, while other conditions being equal, if the existential modal is dropped as in (33b) or replaced with a universal modal as in (33c), the individual answers would be mutually exclusive.

\[(33)\] Let \(C = \{\phi_x \mid x \in D\}\). For any \(a\) and \(b\) such that \(a \in D, b \in D,\) and \(\phi_a \neq \phi_b,\) we have:

\[a. \Diamond O_C \phi_a \land \Diamond O_C \phi_b \neq \perp\]
\[b. O_C \phi_a \land O_C \phi_b = \perp\]
\[c. \Box O_C \phi_a \land \Box O_C \phi_b = \perp\]

\(^{10}\)In the following example, the conjunctive answers do not appear contradictory in a narrow scope reading. However, each such answer should be understood as providing one single option, just like any MS answer. For example, answer (ib) should be parsed as \(\Diamond O_C [\phi_a \land \phi_b \land \phi_{c|d}],\) in which the exhaustivity-operator is applied above the conjunction. LFs for the MS readings of (i) are as in (ii).

(i) (Context: The players will be grouped into two or three teams, and each team needs one or two leaders.)
Q: Who can lead a team? A: ...
   a. Andy and Billy.
      (Intended: ‘It can be the case that that Andy and Billy each lead a team.’)
   b. Andy, Billy, and Cindy,Danny.
      (Intended: ‘It can be the case that Andy, Billy, and Cindy+Danny each lead a team.’)

(ii) Who can lead a team?
   a. \(\{x \mid \lambda x.P_{(t,t)} [\rho \text{ can } O_C [ P_{(v,v)} \lambda x. [\rho x \text{ lead a team }]]]]\)
   b. \(\{x \mid \lambda x.P_{(t,t,t,t)} [\rho \text{ can } [ P_{(v,v)} O_C [ P_{(v,v)} \lambda x. [\rho x \text{ lead a team }]]]]\)
On sub-condition (B), the answer space of a $\Diamond$-question with a higher-order reading is not closed under conjunction if the higher-order $wh$-trace $\pi$ takes scope under the existential modal. As seen in the illustration of the answer space in Figure 2/3, for any two distinct individual answers, their conjunction is not in the answer space. In contrast, for a $wh$-question without a modal or with a universal modal, the answer space yielded in a higher-order reading is closed under conjunction. For example, in each answer space in Figure 4a-c obtained from these questions without employing exhaustification, although the answers are not mutually exclusive, the conjunctive answer at the top is semantically equivalent to the conjunction of the two individual answers.

![Figure 4: Answer space of a question without a modal or with a universal modal](image)

Note that the current analysis does not fully explain why MS readings are only available in $\Diamond$-questions. Consider a non-modalized question for instance. The illustration in Figure 4a considers only the higher-order reading, in which the answer space includes conjunctive propositions obtained through Boolean conjunctions (e.g., $a \land b$) and violates condition (B). However, if the extension of the predicate that the $wh$-phrase combines with is not closed under sum, the answer space yielded in a first-order reading (i.e., $\{\phi_x \mid x \in D\}$) does satisfy both sub-conditions. Hence, the current analysis cannot rule out MS readings for non-modalized questions. This over-generation problem also applies to questions with a universal modal. I will return to this problem in Sect. 4.2 and solve it in Sect. 4.4: roughly, interpreting a non-$\Diamond$-question with reading that allows for multiple max-informative true answers causes a violation of ‘Relativized Exhaustivity’.

### 3.3. Deriving conjunctive-MA readings

Recall that the MA answer to a $\Diamond$-question can be expressed either as an elided conjunction or as an elided free-choice (FC) disjunction. I argue that the two forms of MA answers come from two different derivations which differ in the structure of the question nucleus.

(34) Who can chair the committee?

(Context: Only Andy and Billy can chair the committee; only single-chaining is allowed.)

a. Andy.\ (MS)

b. Andy and Billy.\ (Conjunctive-MA)

c. Andy or Billy.\ (Disjunctive-MA)

Conjunctive-MA arises if the higher-order $wh$-trace takes scope above the existential modal, as illustrated in (35b). Compared with (35a), repeated and simplified from (27), this LF only differs in the scope of the higher-order $wh$-trace $\pi$ relative to the existential modal $\Box$.

(35) Who can chair the committee?
The answer space yielded from the LF in (35a/b) is illustrated in Figure 5a/b. (Figure 5a is identical to Figure 3.) Legends and abbreviations are the same as in Figure 3.

(Context: Only Andy and Billy can chair the committee. Only single-chairing is allowed.)

The answer space in Figure 5b is closed under conjunction: the conjunctive answer is not contradictory and is logically equivalent to the conjunction of the two individual answers. Hence, the yielded reading is a MA reading. In the described multi-choice scenario, while the answer space in Figure 5a has multiple max-informative true answers, the one in Figure 5b has only one max-informative true answer $\diamond O_C \phi_a \land O_C \phi_b$, which comes from the Boolean conjunction $a \cap b$. The individual answers $\diamond O_C \phi_a$ and $\diamond O_C \phi_b$ and the disjunctive answer $\diamond O_C \phi_a \lor \diamond O_C \phi_b$ are asymmetrically entailed by this conjunctive answer and thus are all partial answers.

3.4. Deriving disjunctive-MA readings

MA answers to $\diamond$-questions are more commonly expressed in the form of an elided disjunction. As shown in (36), an elided disjunction can be interpreted with either an ignorance inference or a FC inference, used as a partial answer and a mention-few/all answer, respectively.

(36) Who can chair the committee?

a. Andy or Billy .../ (Ignorance: partial)
   \[\sim \text{Either Andy or Billy can chair the committee. I don't know which.}\]

b. Andy or Billy. (FC: mention-few/all)
   \[\sim \text{Both Andy and Billy can chair the committee.}\]

I argue that the MS/MA-ambiguity in $\diamond$-questions has the same source as the ignorance/FC ambiguity in $\diamond$-disjunctives. In a $\diamond$-question, a disjunctive-MA reading arises if the higher-order $\text{wh}$-trace is
associated with a FC-triggering operator, which turns plain disjunctions into FC-disjunctions. In the following, I will first motivate this analysis, drawn on evidence from the Mandarin multi-functional particle *dou*. Then I will show how the absence/presence of a *dou*-like operator in the question nucleus can account for the MS/MA-ambiguity in ◇-questions.

### 3.4.1. Two uses of *dou*: Exhaustivity-marker and FC-licenser

The Mandarin particle *dou* has various uses. These uses can be disambiguated by the structure and the prosodic pattern of the occurring environment and the meaning of *dou*’s associate. In particular, two of the uses in ◇-questions and in ◇-declaratives exhibit an interesting parallel. In a ◇-question, as seen in (37), associating *dou* with the *wh*-phrase across the existential modal blocks MS. Underlining marks the expression associated with *dou*. Following Beck and Rullmann (1999), I call *dou* in this use descriptively an ‘exhaustivity-marker’.

(37) a. (Dou) *shui* jiao jichu *hanyu*?
   
   (dou) who can teach Intro Chinese
   
   Without *dou*: ‘Who can teach Intro Chinese?’ (✓MS, ✓MA)
   
   With *dou*: ‘Who all can teach Intro Chinese?’ (√MS, √MA)
   
   b. Mali (dou) keyi *nali* mai dao *kafei*?
   
   Mary (dou) can at where buy coffee
   
   Without *dou*: ‘Where can Mary get coffee?’ (✓MS, ✓MA)
   
   With *dou*: ‘Where all can Mary get coffee?’ (√MS, √MA)

---

11 Exhaustivity markers also include the English adverbial *all* in several southern dialects and the German floating particle *alles* (and its variants such as *überall*). In the following, the (a)-questions demand an exhaustive list of individuals who can teach Introduction to Linguistics, and the (b)-questions call for an exhaustive list of nearby coffee places.

(i) English *all* (Texan English)
   a. Who *all* can teach Introduction to Linguistics?
   b. Where *all* can we get coffee around here?

(ii) German *alles*
   a. Wer kann *alles* Einführung in die Sprachwissenschaft unterrichten?
   Who can all introduction into Linguistics teach
   ‘Who all can teach Introduction to Linguistics?’
   b. Wo kann ich hier *überall* Kaffee bekommen?
   where can I here everywhere get
   ‘Where all can we get coffee around here?’

In (37), I translate *dou* as ‘all’. However, despite the similarity between *dou* and English *all* in questions, *dou* should not be analyzed as a universal distributor (contra Lin 1998, Jie Li 1995, Xiaoguang Li 1997). Xiaoguang Li (1997) assumes that, in the exhaustivity-marker use, *dou* is associated with a covert adverbial denoting multiple events and quantifies over events. This analysis cannot predict the unavailability of MS in ◇-questions like (iii). If here *dou* were associated with a covert quantificational adverbial over events, then (iii) should admit pair-list MS or individual MS readings, as observed in (iiib). For example, if Starbucks is always accessible to John while Peet’s is only sometimes accessible to John, ‘*Starbucks*’ is a proper answer to (iiib) but not to (iiia).

(iii) a. Yuehan dou keyi *qu nali* mai *kafei*?
    John *dou* can go where buy coffee?
    ‘Where all can John buy coffee?’ (MA)

   b. Yuehan *mei-ci* dou keyi *qu nali* mai *kafei*?
    John each-time *dou* can go where buy coffee?
    ‘Each time, where can John can buy coffee?’ (Pair-list MS)
    ‘John always can buy coffee from where?’ (Individual MS)
The following discusses where *dou* is positioned in the structure of a question when it functions as an exhaustivity-marker. First, as seen in (37b-b'), the exhaustivity-marker *dou* must appear on the right side of the subject unless the subject is an interrogative *wh*-item. Given that a non-interrogative subject stays within IP, I conjecture that *dou* is also within IP.

(37)  b'. (*Dou) Mali  keyi zai nali, mai dao kefei?  
(*dou) Mary can nat where buy get coffee

Second, the exhaustivity-marker *dou* must be associated with the *wh*-item and appear to the left of its *wh*-associate. As seen in (38), *dou* functions as an exhaustivity-marker when appearing before *shenme* ‘what’ and as a universal distributor when appearing after *shenme*.

(38)  (Context: John can give Mary either all the apples or some (but not all) of the cookies; the choice is up to him.)

a. Yuehan dou keyi ba shenme gei Mali?  
John dou can ba what give Mary  
‘What all is John allowed to give to Mary?’  
(Exhaustivity-marker)  
Proper reply: ‘The apples or some of the cookies.’

b. Yuehan keyi ba shenme dou gei Mali?  
John can ba what dou give Mary  
‘What x is such that John can give all of x to Mary?’  
(Universal distributor)  
Proper reply: ‘The apples.’

Considering that Mandarin is *wh*-in-situ and assuming that *wh*-items in questions undertake covert movement at LF (Huang 1982), I argue that the exhaustivity-marker *dou* is an IP-internal operator that c-commands the *wh*-trace. The surface structure (SS) and LF of (37a/b) are thus as in (39)/(40).

(39)  Dou shui keyi jiao jichu hanyu?  
*dou* who can teach Intro Chinese  
SS: [cp [ip douC [vp who can teach Intro Chinese]]]  
LF: [cp whoC [u douC [u vp tij+v] can teach Intro Chinese]]

(40)  Mali dou keyi zai nali mai dao kefei?  
Mary dou can at where buy get coffee  
SS: [cp [ip Maryj douC [vp tj can get coffee where]]]  
LF: [cp whereC [u Maryj douC [vp tj can get coffee tij+v]]]

Interestingly, in parallel to the exhaustivity-marker use in ◊-questions, associating *dou* with a pre-verbal disjunction in a ◊-declarative evokes a universal FC inference, as exemplified in (41).

(41)  a. Yuehan huozhe Mali (dou) keyi jiao jichu hanyu.  
John or Mary (dou) can teach Intro Chinese  
Without *dou*: ‘Either John or Mary can teach Intro Chinese.’  
(Ignorance)  
With *dou*: ‘Both John and Mary can teach Intro Chinese.’  
(Universal FC)

b. Mali zai Xingbake huozhe Maidanglao (dou) keyi mai dao kefei.  
Mary at Starbucks or McDonalds (dou) can buy get coffee  
Without *dou*: ‘From either SB or MD, Mary can get coffee.’  
(Ignorance)  
With *dou*: ‘From both SB and MD, Mary can get coffee.’  
(Universal FC)
In Xiang 2020a, I define *dou* as a pre-exhaustification exhaustifier that operates on sub-alternatives, as schematized in (42). \( \text{Sub}(p, C) \) stands for a set of ‘sub-alternatives’ of \( p \) in \( C \).

(42) The meaning of *dou* (Xiang 2020a)
\[
[dou_C] = \lambda p \lambda w : \exists q \in \text{Sub}(p, C). p(w) = 1 \land \forall q \in \text{Sub}(p, C)[O_C(q)(w) = 0]
\]

a. **Non-vacuity presupposition**: The prejacent has at least one sub-alternative.
b. **Prejacent assertion**: The prejacent is true.
c. **Anti-exhaustification assertion**: The exhaustification of each sub-alternative is false.

The FC-trigger use of *dou* arises when sub-alternatives are defined in terms of innocent (I)-excludability, as in (43a). As schematized in (43b), an alternative is innocently (I)-excludable if and only if it is included in every maximal set of alternatives \( A \) such that affirming the prejacent is consistent with negating all the alternatives in \( A \). In particular, relative to a disjunctive sentence, the disjuncts are not I-excludable: affirming the disjunction and negating both/all of its disjuncts yield a contradiction. (Formally: \( \{ \neg p, \neg q \} \cup \{ p \lor q \} \) is inconsistent, because \( p \lor q \land \neg p \land \neg q = \bot \).

(43) a. Sub-alternatives (Based on I-excludability)
\( \text{Sub}(p, C) = (C - \text{IEXcl}(p, C)) - \{ p \} \)
(The set of alternatives that are not I-excludable and are distinct from the prejacent)
b. Innocently (I)-excludable alternatives (Fox 2007)
\( \text{IEXcl}(p, C) = \cap \{ A \mid A \text{ is a maximal subset of } C \text{ s.t. } \{ \neg q \mid q \in A \} \cup \{ p \} \text{ is consistent} \} \)
\( (q \text{ is I-excludable to } p \text{ if and only if } q \text{ is included in every maximal set of alternatives of } p \text{ such that the exclusion of this set is consistent with } p. ) \)

The FC-inference of (41a) is computed as in (44). The disjunctive *or* carries a [+d] feature which activates a set of domain-alternatives (for the definition, see fn. 8). Agreeing with the [+d] feature of *or*, *dou* quantifies over a set of the domain-alternatives of its prejacent. Employing *dou* affirms the prejacent disjunctive sentence and negates the exhaustification of each disjunct, yielding a conjunctive inference. (This computation does not consider modal obviations effects, see details in Sect. 4.5.)

(44) John or Mary *dou* can teach Intro Chinese.
\[
\begin{align*}
\text{a. } [\text{dou}_C\{\text{John-or}_{[+d]}\text{-Mary can teach Intro Chinese }\}] \\
b. \{S\} = \phi_1 \lor \phi_m \quad (\phi_x \text{ abbreviates } 'x \text{ teach Intro Chinese}') \\
c. C = \text{D-Alt}(S) = \{ \phi_1, \phi_m, \phi_1 \lor \phi_m \} \\
d. \text{Sub}([S], C) = \{ \phi_1, \phi_m \} \\
e. [\text{dou}_C(S)] \iff [\phi_1 \lor \phi_m] \land \neg O_C \phi_1 \land \neg O_C \phi_m \\
\quad \iff [\phi_1 \lor \phi_m] \land [\phi_1 \rightarrow \phi_m] \land [\phi_m \rightarrow \phi_1] \\
\quad \iff [\phi_1 \lor \phi_m] \land [\phi_1 \leftrightarrow \phi_m] \\
\quad \iff \phi_1 \land \phi_m
\end{align*}
\]

\(^{12}\)Sub-alternatives are primarily logically weaker alternatives, but in particular syntactic and prosodic environments, the ordering source in defining sub-alternatives gets shifted from logical strength to innocent excludability or likelihood, yielding different functions of *dou*. In Xiang 2016: Chap. 7 and Xiang 2020a, I have shown that this semantics can account for the three basic uses of *dou* in declaratives, including the universal quantifier-distributor use, the FC-licenser use, and the even-like scalar-marker use.
3.4.2. Deriving disjunctive-MA

Drawing on the parallel uses of the Mandarin particle *dou* in ◇-declaratives and ◇-questions, I propose that disjunctive-MA readings are derived by employing a covert *dou*-like operator above the existential modal. I define a covert *dou*-operator as in (45), which has the same semantics as the asserted meaning of the FC-triggering particle *dou*.\(^\text{13}\)

\[
\llbracket \text{dou}_C \rrbracket = \lambda p \lambda w. p(w) = 1 \land \forall q \in \text{Sub}(p, C)[O_C(q)(w) = 1]
\]
where \(\text{Sub}(p, C) = (C - \text{IExcl}(p, C)) - \{p\}\)

(For any proposition \(p\), \(\llbracket \text{dou}_C \rrbracket (p)\) is true if and only if the prejacent \(p\) is true, and the exhaustification of each sub-alternative of \(p\) in \(C\) is false. Sub-alternatives are alternatives that are not I-excludable and are distinct from the prejacent.)

In the derivation of MS and the derivation of conjunctive-MA, the question nucleus is as the \(\delta\)-node in (46a) and in (46b), respectively. These two \(\delta\)-nodes only differ in the scope of the higher-order \(wh\)-trace \(\pi\) relative to the existential modal \(\text{can}\). In both (46a) and (46b), a *dou*-operator is applied to the \(\delta\)-node and is associated with the trace \(\pi\). The domain variable \(C'\) carried by *dou* denotes a contextually determined set of alternatives of \(\delta\) (or equivalent, the answer space derived in the absence of *dou*), defined in the same way as the domain variable \(C\) carried by the local \(O\)-operator. As I will show next, in both cases, applying a *dou*-operator to \(\delta\) gives rise to a reading that calls for a disjunctive-MA answer.

(46) **Who can chair the committee?** (Disjunctive-MA)

a. ◇ \(\gg\) \(\pi\) (with *dou*)

b. \(\pi\ \gg\) ◇ (with *dou*)

Figure 7a/b illustrates the answer space yielded by the LF (46a/b). The corresponding answer space yielded in the absence of *dou* is repeated in Figure 6a/b. Legends and abbreviations are the same as in Figure 3.

\(^{13}\)I assume that the covert *dou* does not carry a non-vacuity presupposition. This presupposition comes from the economy condition that an overt operator cannot be used vacuously and hence it only applies to overt expressions. In Sect. 4.5, I will add a ‘Relativized Exclusivity’ presupposition to the semantics of *dou/dou* to account for modal obviation effects in licensing universal FC and to explain the unavailability of a disjunctive-mention-few answers in non-modalized questions and ◇-questions.
In both answer spaces in Figure 7, the application of $\text{dou}$ to the individual answers and to the conjunctive answers are truth-conditionally vacuous. The individual answers have no sub-alternative. The conjunctive answers, while having sub-alternatives, each entail the anti-exhaustivity inference (underlined) with respect to its sub-alternatives: in (47b), the prejacent denotes a contradiction which entails any inference, and in (48b), the prejacent denotes a conjunctive proposition which entails the anti-exhaustification of its conjuncts. In both answer spaces, however, the application of $\text{dou}$ strengthens the disjunctive answer into a FC statement that is logically equivalent to the conjunction of the two individual answers, as computed in (47c) and (48c). Hence, in the considered scenario (i.e., only Andy and Billy can chair, and co-chairing is disallowed), the FC-disjunctive answer is a max-informative true answer, while the individual answers are asymmetrically entailed by this disjunctive answer and are partial answers.

(47) For $\text{dou} \gg \Diamond \gg \pi$ (answer space in Figure 7a)

a. $C = \{\lambda x. O_C \phi_x \mid \pi \in hmn_\pi\}$

b. Computing the conjunctive answer

i. $\text{Sub}(\Diamond [O_C \phi_a \land O_C \phi_b], C') = C' - \{\bot\}$

ii. $\text{dou}_{C'} \Diamond [O_C \phi_a \lor O_C \phi_b]$

\[\iff \Diamond [O_C \phi_a \lor O_C \phi_b] \land \forall \phi \in (C' - \{\bot\}) \neg O_C \phi\]

\[\iff \Diamond [O_C \phi_a \lor O_C \phi_b] \land \bot\]

iii. $\text{Computing the disjunctive answer}$

i. $\text{Sub}(\Diamond [O_C \phi_a \lor O_C \phi_b], C') = \{\Diamond O_C \phi_a, \Diamond O_C \phi_b\}$

ii. $\text{dou}_{C'} \Diamond [O_C \phi_a \lor O_C \phi_b]$

\[\iff \Diamond [O_C \phi_a \lor O_C \phi_b] \land \neg O_{C'} \Diamond O_C \phi_a \land \neg O_{C'} \Diamond O_C \phi_b\]

\[\iff \Diamond [O_C \phi_a \lor O_C \phi_b] \land [\Diamond O_C \phi_a \rightarrow \Diamond O_C \phi_b] \land [\Diamond O_C \phi_b \rightarrow \Diamond O_C \phi_a]\]
Adopting Fox’s (2013) non-exhaustive answerhood and adapting it to a categorial approach, this presupposition to be assessed relative to a modal base that introduces only a subset of the accessible worlds (more precisely, a minimal set of accessible worlds that verifies the truth of a true answer).

4. Relativized Exhaustivity: Solving the dilemma

My analysis of the MS/MA-ambiguity adopts Fox’s non-exhaustive answerhood so as to allow for MS. However, this answerhood has a conflict with ‘Dayal’s presupposition’, which is crucial in accounting for uniqueness effects in questions (Sect. 4.1). Moreover, this answerhood over-predicts MS readings for a variety of questions (Sect. 4.2).

Hirsch and Schwarz (2020) novelly observe that uniqueness may be interpreted locally, contrary to the prediction of Dayal’s presupposition. They abandon Dayal’s presupposition and propose that the uniqueness presupposition in a singular which-question comes from the lexicon of the determiner which. However, this account faces many problems (Sect. 4.3).

In Sect. 4.4, I will propose a ‘Relativized Exhaustivity’ presupposition, which allows Dayal’s presupposition to be assessed relative to a modal base that introduces only a subset of the accessible worlds (more precisely, a minimal set of accessible worlds that verifies the truth of a true answer).

\[
\begin{align*}
\forall \phi_a, \phi_b & \iff \lozenge [O_C \phi_a \lor O_C \phi_b] \land [\lozenge O_C \phi_a \iff \lozenge O_C \phi_b] \\
& \iff \lozenge O_C \phi_a \land \lozenge O_C \phi_b \\
& \iff \text{dou}_C \lozenge O_C \phi_a \land \text{dou}_C \lozenge O_C \phi_b \\
\end{align*}
\]

(48) For \text{dou} \gg \pi \gg \lozenge (answer space in Figure 7b)

a. \(C = \{\phi_x \mid x \in \text{hmn}_@\}, C' = \{\pi (\lambda x. \lozenge O_C \phi_x) \mid \pi \in \text{hmn}_@\}\)

b. Computing the conjunctive answer

i. \text{Sub}(\lozenge O_C \phi_a \land \lozenge O_C \phi_b, C') = \{\lozenge O_C \phi_a, \lozenge O_C \phi_b\}

ii. \text{dou}_C(\lozenge O_C \phi_a \land \lozenge O_C \phi_b)

\[
\begin{align*}
& \iff \lozenge [O_C \phi_a \land O_C \phi_b] \land \neg O_C \lozenge O_C \phi_a \land \neg O_C \lozenge O_C \phi_b \\
& \iff \lozenge O_C \phi_a \land \lozenge O_C \phi_b \\
& \iff \text{dou}_C \lozenge O_C \phi_a \land \text{dou}_C \lozenge O_C \phi_b \\
\end{align*}
\]

c. Computing the disjunctive answer

i. \text{Sub}(\lozenge [O_C \phi_a \lor O_C \phi_b], C') = \{\lozenge O_C \phi_a, \lozenge O_C \phi_b\}

ii. \text{dou}_C(\lozenge [O_C \phi_a \lor O_C \phi_b])

\[
\begin{align*}
& \iff \lozenge [O_C \phi_a \lor O_C \phi_b] \land \neg O_C \lozenge O_C \phi_a \land \neg O_C \lozenge O_C \phi_b \\
& \iff \lozenge [O_C \phi_a \lor O_C \phi_b] \land [\lozenge O_C \phi_a \iff \lozenge O_C \phi_b] \\
& \iff \lozenge O_C \phi_a \land \lozenge O_C \phi_b \\
& \iff \text{dou}_C \lozenge O_C \phi_a \land \text{dou}_C \lozenge O_C \phi_b \\
\end{align*}
\]

3.5. Interim summary

Adopting Fox’s (2013) non-exhaustive answerhood and adapting it to a categorial approach, this section has discussed the compositional derivations of first-order and higher-order MS-readings of \(\lozenge\)-questions. Moreover, I have argued that the MS/MA-ambiguity in \(\lozenge\)-questions can be explained in terms of minimal structural variations within the question nucleus. MA readings arise if one of the following two conditions is met: (i) the higher-order \(wh\)-trace takes scope above the existential modal, and (ii) a FC-triggering operator (= the Mandarin particle \text{dou}) appears above the existential modal and is associated with the higher-order \(wh\)-trace. In particular, condition (i) yields conjunctive-MA, and condition (ii) yields disjunctive-MA.

3.5.1. Interim summary

Adopting Fox’s (2013) non-exhaustive answerhood and adapting it to a categorial approach, this section has discussed the compositional derivations of first-order and higher-order MS-readings of \(\lozenge\)-questions. Moreover, I have argued that the MS/MA-ambiguity in \(\lozenge\)-questions can be explained in terms of minimal structural variations within the question nucleus. MA readings arise if one of the following two conditions is met: (i) the higher-order \(wh\)-trace takes scope above the existential modal, and (ii) a FC-triggering operator (= the Mandarin particle \text{dou}) appears above the existential modal and is associated with the higher-order \(wh\)-trace. In particular, condition (i) yields conjunctive-MA, and condition (ii) yields disjunctive-MA.
This presupposition solves the dilemma between uniqueness and MS, overcomes the over-generation problem of Fox’s answerhood, and nicely explains the local uniqueness effects in modalized questions.

Section 4.5 will address the additional complications in deriving uniqueness effects from disjunctive-MA interpretations. I will propose that dou/dou presupposes ‘Relativized Exclusivity’, defined in analogy with ‘Relativized Exhaustivity’. This presupposition also explains the modal obviation effects in licensing universal FC.

4.1. A dilemma with Dayal’s presupposition

Wh-questions in which the _wh_-complement is singular-marked or numeral-modified (called ‘singular _wh_-questions’ and ‘numeral-modified _wh_-questions’ henceforth) are subject to uniqueness and can have only one true answer.

(49)  
\begin{align*}
\text{a. Which child came?} & \quad \leadsto \text{Only one of the children came.} \\
\text{b. Which two children came?} & \quad \leadsto \text{Only two of the children came.} \\
\text{c. Which two children formed a team?} & \quad \leadsto \text{Only one pair of the children formed any team.}
\end{align*}

This uniqueness effect is standardly analyzed as a result of ‘Dayal’s presupposition’, according to which a question is defined only if it has a strongest true answer. This requirement is introduced as a presupposition of the answerhood-operator, as seen in Sect. 3.1.3, repeated below:

(50) \[ \text{ANS}_{\text{Dayal}}(w)(Q) = \exists p[w \in Q \land \forall q[w \in q \Rightarrow p \subseteq q]]. \]

With this assumption, Dayal argues that the contrast between singular and plural _wh_-questions with respect to uniqueness comes from the semantic contrast between singular and plural nouns. The ontology of individuals assumes that a singular noun denotes a set of atomic entities, while a plural noun denotes a set consisting of both atomic and sum entities (Sharvy 1980; Link 1983). Incorporating this contrast into _wh_-questions, Dayal (1996) argues that the answer space of a plural _wh_-question includes sum-based propositions which are not available in the answer space of the corresponding singular _wh_-question. For example, in a context where multiple children came, the plural _wh_-question (51a) has a unique strongest true answer derived based on the sum of the children who came (viz., \( a \oplus b \)). In contrast, the singular _wh_-question (51b) does not have a strongest true answer in the described context, violating Dayal’s presupposition; thus, (51b) is only acceptable in contexts where exactly one of the children came.

(51) \( \text{(Context: Among the considered children, only Andy and Billy came. The speaker knows that more than one child came.)} \)
\begin{align*}
\text{a. Which children came?} & \quad Q_w = \{ \lambda w.\text{came}_w(a), \lambda w.\text{came}_w(b), \lambda w.\text{came}_w(a \oplus b) \} \\
\text{b. Which child came?} & \quad Q_w = \{ \lambda w.\text{came}_w(a), \lambda w.\text{came}_w(b) \}
\end{align*}

Dayal (1996) focuses on the uniqueness effects of singular _wh_-phrases and considers only first-order readings of questions. In Xiang 2020c, drawing on evidence from questions with collective predicates and questions with modals, I further explore the semantics of _wh_-phrases in higher-order readings and extends Dayal’s analysis of uniqueness to numeral-modified _wh_-questions. I argue
that questions with a simplex plural or number-neutral wh-phrase admit answers naming a Boolean conjunction, while those with a singular or numeral-modified wh-phrase do not. The argumentation is as follows. First, the deviance of the declarative (52a) and its embedding (52b) in a multi-team context shows that the predicate formed a team admits only a collective reading. (In Xiang 2020c, I call such predicates ‘stubbornly collective predicates’.) However, the question-embedding (53a) is felicitous in a multi-team context, and it implies that Jill knows a conjunctive inference which specifies the members of each team. Given the contrast between (52a,b) and (53a), I argue that the embedded wh-question in (52c) can be interpreted with a higher-order reading, in which the strongest true answer is a conjunctive proposition derived based on the Boolean conjunction \((a \oplus b)^\# \cap (c \oplus d)^\#\).

(52) (Context for (52)-(54)): The considered children formed exactly two teams: \(a + b\) formed one, and \(c + d\) formed the other.

a. # The children formed a team. [False]
b. # Jill knows that the children formed a team. [Infelicitous]

(53) Jill knows \([Q\text{ which children formed a team}].\)
\(\sim \# \text{Jill knows that } a + b\text{ formed a team and } c + d\text{ formed a team.}\)

a. First-order reading:
\[Q_w = \{\lambda w. f.a.tm_w(a \oplus b), \lambda w. f.a.tm_w(c \oplus d)\}\]
b. Higher-order reading:
\[Q_w = \{\lambda w. f.a.tm_w(a \oplus b) \land f.a.tm_w(c \oplus d),\]
\[\lambda w. f.a.tm_w(a \oplus b),\lambda w. f.a.tm_w(c \oplus d),\]
\[\lambda w. f.a.tm_w(a \oplus b) \lor f.a.tm_w(c \oplus d)\}\]

Next, in contrast to (53), the embedding of a numeral-modified wh-question in (54) is infelicitous in a multi-team context. To explain this uniqueness effect, I propose that the domain of a numeral-modified (or singular) wh-phrase in a higher-order reading includes Montagovian individuals (e.g., \((a \oplus b)^\#\)) and some Boolean disjunctions (e.g., \((a \oplus b)^\# \cup (c \oplus d)^\#\)) but no Boolean conjunction. In a multi-team context, the embedded question in (54), no matter whether having a first-order reading or a higher-order reading, does not have a strongest true answer, violating Dayal’s presupposition.

(54) # Jill knows \([Q\text{ which two children formed a team}].\)
\(\sim \# \text{Only one pair of the children formed any team.}\)

a. First-order reading:
\[Q_w = \{\lambda w. f.a.tm_w(a \oplus b), \lambda w. f.a.tm_w(c \oplus d)\}\]
b. Higher-order reading:
\[Q_w = \{\lambda w. f.a.tm_w(a \oplus b),\lambda w. f.a.tm_w(c \oplus d),\]
\[\lambda w. f.a.tm_w(a \oplus b) \lor f.a.tm_w(c \oplus d)\}\]

A dilemma arises between the explanations to uniqueness and to MS. Dayal’s presupposition predicts that a question is undefined when it does not have a strongest true answer. In contrast, as seen in Sect. 3.1.3, Fox’s generalization of MS predicts that MS reading is only available in questions where Dayal’s presupposition is not trivially satisfied, namely, there is a world in which this question has multiple max-informative true answers instead of a unique strongest true answer. Hence, on the one hand, Dayal’s presupposition explains uniqueness but it is too strong to allow for MS; on the other hand, Fox’s generalization allows for MS but it is too weak to explain uniqueness effects.
4.2. The over-generation problem

The generalization of distributing MS drawn from Fox’s answerhood, henceforth called ‘Fox’s generalization’, also faces an over-generation problem: without further amendment, this generalization over-predicts MS readings for a variety of questions.

This over-generation problem is partially a consequence of the dilemma between uniqueness and MS. For any singular or numeral-modified wh-question, its individual answers are logically independent from each other and are stronger than the disjunctive answers (if any); thus, Fox’s generalization incorrectly predicts that these questions are MS-questions and that their individual answers are possible MS answers. This problem also applies to alternative questions.

(55) Which child came? / Which two children formed a team? / Did you invite Andy, Billy, or Cindy?
   a. Reading predicted by Dayal’s presupposition: MA with uniqueness ✓
   b. Reading predicted by Fox’s generalization: MS without uniqueness ✗

For questions with a non-divisive collective predicate (e.g., formed a team, presented a paper together, worth $10 in total), the individual answers are also logically independent regardless of the form of the wh-phrase; Fox’s generalization incorrectly predicts that these answers are possible MS answers when the question is interpreted with a first-order reading.14

(56) Which children formed a team?
   a. Reading predicted by Dayal’s presupposition: MA without uniqueness ✓
   b. Reading predicted by Fox’s generalization: Ambiguous between MS and MA ✗

The over-generation problem also applies to questions with an existential quantifier. Wh-questions with an existential quantifier (called ‘∃-questions’ henceforth) can be interpreted with a choice reading and an individual reading.15 As exemplified in (57), while the choice reading calls for an answer that specifies a boy-movie(s) pair, the individual reading calls for an answer that specifies some movie(s) and does not request to specify the boy(s) who watched these movie(s). In regards to exhaustivity, while the choice reading only requests to specify the movie(s) that a specific boy watched, the individual reading requests to list all the movies that any of the relevant boys watched, as in (57b-i).

(57) (Context: Among the relevant boys, Andy watched Ironman and Spiderman, Billy watched only Hulk, and Clark didn’t watch any movies.)
Which movie or movies did one of the boys watch?
   a. Choice reading
      ‘Name any/one boy x, and tell me which movie(s) y was watched by x.’
   b. Individual reading
      ‘List all movies that any of the relevant boys watched.’

14Fox (2018, 2020) himself has noticed this problem. In contrast to my assumption that plural and number-neutral wh-phrases can quantify over Boolean conjunctions, he proposes that these phrases can quantify over higher-order pluralities. For example, in responding to the question (56), the conjunctive answer ‘a + b formed a team and c + d formed the other one’ is derived based on the higher-order plurality \{\{a, b\}, \{c, d\}\}. As long as one assumes that such higher-order pluralities are available in the non-quantificational/first-order reading, the over-generation problem for questions with a collective predicate can be avoided. I think this solution is also plausible and straightforward.

15Wh-questions with a quantificational subject may also have a functional reading which calls for a pronominal answer interpreted as being bound by the quantificational subject, as in (ib,c). However, this reading is not available in questions with an existential quantifier.

(i) a. ‘Which movie did one of the boys watch?’ ‘#His favorite superhero movie.’
   b. ‘Which movie did each of the boys watch?’ ‘His favorite superhero movie.’
   c. ‘Which movie did most of the boys watch?’ ‘Their favorite superhero movie.’
i. Andy watched *Ironman* and *Spiderman*.
ii. Billy watched *Hulk*.

b. Individual reading
‘For which movie(s) y, is it the case that y was watched by any/one of the boys?’
i. *Ironman*, *Spiderman*, and *Hulk*.
ii. # *Ironman* and *Spiderman*./ *Hulk*.

The deviance of the non-exhaustive individual answers in (57b-ii) shows that existential quantifiers do not license MS readings, in contrast to existential modals. This contrast is quite puzzling: without further restrictions, the proposed derivation of MS readings together with Fox’s answerhood predicts that any existential expression that can occur within the question nucleus is a MS-licenser. For example, in parallel to ◊-questions, ∃-questions in principle can have an LF as follows, which yields an answer space not closed under conjunction:

(58) Which movie or movies did one of the boys watch?
   a. \[ \lambda w : \lambda y : [\exists y \ [\text{one-boy}_w(x) \land O_C[\text{invited}_w(x, y)] | x_e \in \text{movies}_@] ] \]
   b. \{ \lambda w : \lambda y : [\exists y \ [\text{one-boy}_w(x) \land O_C[\text{invited}_w(x, y)] | x_e \in \text{movies}_@] ] \}

Some authors (George 2011; Fox 2013; Nicolae 2013; among others) treat the choice reading as a MS reading and thus claim that existential quantifiers can license MS readings. However, if the existential quantifier were interpreted locally within IP as in (58a,b), there should be no need for a choice answer to specify the agent boy; for example, the question (57) should admit the fragment answers such as ‘*Ironman* and *Spiderman*’ and ‘*Hulk*’, contrary to fact. By contrast, I argue that the choice reading is derived in the same way as the pair-list reading of a question with a universal quantifier (e.g., *Which movie did every boy watch?*). In this derivation, the smallest IP denotes an open sentence expressing a functional dependency relation between the trace of the quantifier and the trace of the *wh*-phrase, and the quantifier is interpreted above a null predication operator over this IP. For details, see Xiang 2020d.

4.3. Local uniqueness and the presuppositional-*which* account

Hirsch and Schwarz (2020) novelly observe that uniqueness effects in *wh*-questions are not always global. In a singular ◊-question, the uniqueness inference triggered by the singular *wh*-phrase can be interpreted under the scope of the existential modal. For example, the questions in (59) are acceptable in a multi-choice context, and the elided disjunctive answer ‘A or r’ can be interpreted with a FC inference. Hirsch and Schwarz argue that Dayal’s presupposition, globally applied to the answer space as a whole, cannot account for local uniqueness. In addition, this reading also cast doubt on the recent account of Fox (2018, 2020) which assumes a question-partition-matching principle.

(59) a. Which letter could we add to *f*om (to form a word)?
   A or r. (Intended: ‘The unique letter that we add to *f*om could be a and could be r.’)
   b. Which letter could be missing in *f*om?
   A or r. (Intended: ‘The unique letter missing in *f*om could be a and could be r.’)

To account for local uniqueness effects, in line with Rullmann and Beck (1998), Hirsch and Schwarz propose that the uniqueness presupposition comes from the lexical meaning of the determiner *which,*
as defined in (60), and assume that which is interpreted within the question nucleus.  

\[(60) \quad \llbracket \text{which} \rrbracket = \lambda x . \lambda m \llbracket \text{e.st} \rrbracket \lambda g \llbracket \text{e.st} \rrbracket \lambda w : \exists y[ f(y)(w) \land g(y)(w) \land f(x)(w) \land g(x)(w)] \]

They further argue that global and local uniqueness arise if which takes scope above and below could, respectively. The LFs for (59a) and the derived Hamblin sets are as in (61). In the Hamblin sets, the propositions all carry a global or local uniqueness presupposition (underlined).

(61) Which letter could we add to f_om?
   a. which \textit{\Rightarrow} could: global uniqueness
      i. [\text{cp} ? \lambda 1 [\text{ip} \lambda t \lambda 2 [\text{vp} \lambda w \lambda 2 \llbracket \text{could} \rrbracket \lambda t_2 \llbracket \text{f_om} \rrbracket]][]
      ii. \{\lambda w : \exists y[\text{wh\textunderscore w'}(y) \land \text{add\textunderscore w'}(y)] \land \text{add\textunderscore w'}(a) \mid x \in D_e\}
   b. could \textit{\Rightarrow} which: local uniqueness
      i. [\text{cp} ? \lambda 1 [\text{ip} \lambda t \lambda 2 [\text{vp} \lambda w \lambda 2 \llbracket \text{could} \rrbracket \lambda t_2 \llbracket \text{f_om} \rrbracket]][]
      ii. \{\lambda w : \text{wh\textunderscore w'}(y) \land \text{add\textunderscore w'}(y) \land \text{add\textunderscore w'}(a) \mid x \in D_e\}

Finally, Hirsch and Schwarz adopt Fox’s of answerhood and assume that a question must have a true answer. When uniqueness is not satisfied in any accessible world, a singular question is deviant due to the lack of a defined (and true) answer.

The above illustration considers only singular \textit{wh}-questions. To uniformly account for the lack of uniqueness effects in plural \textit{wh}-questions, Hirsch and Schwarz further re-define the presuppositional determiner which as follows:

\[(62) \quad \llbracket \text{which} \rrbracket = \lambda x . \lambda m \llbracket \text{e.st} \rrbracket \lambda g \llbracket \text{e.st} \rrbracket \lambda w : \exists y[ f(y)(w) \land g(y)(w) \land \forall z[f(y)(w) \land g(y)(w) \rightarrow z \leq y]]

Accordingly, the presupposition of which is trivially satisfied if and only if the intersection between the extension of the \textit{wh}-complement and the extension of the predicate that the \textit{wh}-phrase combines with is closed under sum formation. For example, this presupposition is trivially satisfied in Which students came?: for any world w, the set of (atomic or plural) students who came in w is closed under sum. In contrast, Which student came? is subject to uniqueness: the set of atomic students who came in w is closed under sum if and only if exactly one student came in w.

Compared with the analysis of uniqueness using Dayal’s presupposition, the presuppositional-which account is advantageous in deriving local uniqueness in \textit{\diamond}-questions. Moreover, this account is compatible with Fox’s generalization of MS.

Despite of these advantages, the presuppositional-which account faces many problems. First, this account only deals with uniqueness effects and does not help to solve the over-generation problem of Fox’s generalization in interpreting questions with a collective predicate and in questions with an existential quantifier.

Second, the predicted local uniqueness presupposition is too weak. In (61b), the presupposition only requires the existence of an option that satisfies uniqueness, read as “We have options of adding one single letter”, and it does not ensure that the chosen MS answer satisfies uniqueness. For example
in (63), with the option of assigning only chapter 1, Hirsch and Schwarz predict that local uniqueness is satisfied and that any answer naming one single chapter of the book is a good MS answer. However, sentence (63a) is clearly unacceptable: uniqueness is not satisfied in the accessible worlds where we assign chapter 3. Moreover, sentence (63b) is also deviant despite that uniqueness is satisfied in worlds where we assign chapter 1. It is more natural to express the intended meaning with a number-neutral question as in (63c), or as in (63d) in which the \textit{wh}-complement is modified by \textit{SINgle} (capitalization marks stress). (For an explanation on the contrast between (63b) and (63d), see the end of Sect. 4.4.2.)

(63) (Context: The relevant book has three chapters. The speaker knows that we can assign either chapter 1, or chapter 2, or both chapters 2-3 to the students next week.)

a. #I know which chapter we can assign to the students... chapter 3.

b. ?I know which chapter we can assign to the students... chapter 1.

c. I know what we can assign to the students... chapter 1.

d. I know which \textit{SINgle} chapter we can assign to the students... chapter 1.

In contrast to Hirsch and Schwarz, I argue that the uniqueness requirement in a singular \textit{◇}-question must be met in every accessible world that verifies a true answer. Dialogue (64) illustrates this requirement. In an informal survey, native speakers all reported that TA1’s utterance sounded unnatural or sly as she was unintentionally assuming or intentionally suggesting that only one paper should be assigned next week, conflicting with what the instructor said. In comparison, dialogue (65) is perfectly natural. TA2 clearly didn’t imply that the reading for next week can only be a journal article; what she asked is simply a sub-question of the more inclusive question ‘Which journal article or book chapter could we assign to the students next week?’

(64) Instructor: “We should assign one or two papers to the students each week.”

TA1: “Got it. Which paper could we assign to the students next week?” [Unnatural]

(65) Instructor: “We should assign a journal article or a book chapter to the students each week.”

TA2: “Got it. Which journal article could we assign to the students next week?” [Natural]

Third, the presuppositional-\textit{which} account does not fully account for the distribution of uniqueness effects in questions where the extension of the predicate that the \textit{which}-phrase combines with is not divisive. For example, the stubbornly collective predicate solved a problem together is not divisive: ‘\(a + b + c\) solved a problem together’ does not entail ‘\(a + b\) solved a problem together’.

\[\text{(66)}\quad \text{A predicate } P \text{ is divisive if and only if } \forall x[P(x) \rightarrow \forall y \leq x [y \in \text{Dom}(P) \rightarrow P(y)]]\]

(Whenever \(P\) holds of something \(x\), it also holds of every subpart of \(x\) defined for \(P\).)

As seen in (67b,c), embeddings of a numeral-modified \textit{wh}-question are unacceptable in a uniqueness-violating context. The presuppositional-\textit{which} account does not predict the sensitivity to uniqueness in (67c): in the given context, the intersection between the extension of the \textit{wh}-complement (viz., two-or-three-students\(w\)) and the extension of the nucleus predicate (viz., s.a.p.t\(w\)) is closed under sum, and thus the presupposition of \textit{which} assumed in (62) is satisfied.

(67) (Context: The students solved three problems in total. In particular, \(a + b\) together solved one, \(b + c\) together solved one, and \(a + b + c\) together solved one.)

a. Jill knows which students solved a problem together.
b. # Jill knows which two students solved a problem together.
\[
\text{two-students}_w \cap \text{s.a.p.t}_w = \{ a \oplus b, b \oplus c \}
\]
c. # Jill knows which two or three students solved a problem together.
\[
\text{two-or-three-students}_w \cap \text{s.a.p.t}_w = \{ a \oplus b, b \oplus c, a \oplus b \oplus c \}
\]

More generally, Dayal’s presupposition is concerned with the entailment relation among the propositional answers, while the presupposition of which assumed in (62) is concerned with the part-of relation among the short answers. When the predicate that the wh-phrase combines with is not divisive, satisfying the part-of relation does not ensure satisfying the entailment relation. The uniqueness effects in cases like (67c) argue that uniqueness comes from a constraint in regards to propositions, not a constraint in regards to individuals.

Last, in questions with multiple singular wh-phrases, the uniqueness presupposition assumed for the higher/subject wh-phrase is too strong to allow for a pair-list reading. In the pair-list reading, as described in (68b), uniqueness is required point-wise to each boy who watched any movie, and there is no uniqueness requirement with respect to boys. To allow for non-uniqueness with respect to the subject, it is inevitable to assume a non-presuppositional meaning for the subject which boy, which clearly conflicts with Hirsch and Schwarz’s explanation of uniqueness.

(68) Which boy watched which movie?

a. Single-pair reading:
   ‘Which unique boy-x-movie-y pair is such that x watched y?’
   ‘Andy watched Spiderman.’

b. Pair-list reading:
   ‘[Each boy watched at most one movie, tell me:] which boy-x-movie-y pairs are such that x watched y?’
   ‘Andy watched Ironman, Billy watched Spiderman, Clark watched Hulk.’

By contrast, point-wise uniqueness in pair-list readings can be derived by Dayal’s presupposition. See Dayal 1996, 2017 for details, and see Fox 2012 and Xiang 2019, 2020b,d for two alternative accounts assuming Dayal’s presupposition.

4.4. A new solution: Relativized Exhaustivity

I assume that Fox’s answerhood and Dayal’s presupposition are both employed in question semantics. In \( \Diamond \)-questions, the conflict between these two conditions is resolved if Dayal’s presupposition is assessed relative to a different modal base, more precisely, a modal base that introduces a minimal set of accessible worlds that verifies the truth of a true answer. I propose that question interpretations presuppose ‘Relativized Exhaustivity’. This condition carries forward the merits of Dayal’s presupposition and is even superior to Dayal’s presupposition in deriving the local uniqueness effects in modalized questions. Moreover, this condition solves the over-generation problem of Fox’s answerhood.

4.4.1. Allowing for MS: Relativizing Dayal’s presupposition

Following Kratzer (1977, 1991), I assume that a modal expression is interpreted relative to a modal base \( M_w \), which restricts the quantification domain of modal expressions to \( M_w \) given the evaluation world \( w \). Other parameters such as ordering source are omitted. Modal sentences are interpreted as follows:
I propose that question interpretations must satisfy Relativized Exhaustivity (RelExh). As defined in (71), this condition requires Dayal’s presupposition (70) to be satisfied relative to every modal base that introduces a minimal set of accessible worlds that verifies the truth of a true answer.\(^{17\text{,}18}\) 

\[ \forall M'[M'_w \in \min\{M''_w \mid M''_w \subseteq M_w \land \exists \alpha \in [\phi]^M_w \land \alpha \in [\phi]^M'_w\}] \rightarrow DP(w, [\phi]^M'_w) \]

(70) Dayal’s presupposition (adapted from Dayal 1996)
Given modal base M, a question Q is defined in w only if
\[ \exists \alpha \in [\phi]^M_w \land \forall \beta \in [\phi]^M_w \rightarrow [\phi]^M_w(\beta) \subseteq [\phi]^M_w(\alpha) \] 
[abbreviated as DP(w, [\phi]^M_w)]

(71) Relativized Exhaustivity
Given modal base M, a question Q is defined in w only if
\[ \forall M'[M'_w \in \min\{M''_w \mid M''_w \subseteq M_w \land \exists \alpha \in [\phi]^M_w \land \alpha \in [\phi]^M'_w\}] \rightarrow DP(w, [\phi]^M'_w).\]
(For every modal base M’, where M’ is a minimal subset of M, such that some true short answer to Q in w given M is also a true short answer to Q in w given M’, the interpretation of Q relative to M’ satisfies Dayal’s presupposition in w.) [abbreviated as REP(w, [\phi]^M_w)]

In interpreting a modalized question, the consequence of the RelExh condition can be different from that of Dayal’s presupposition. For illustration, consider the ∘-question in (72). Assume that only Andy chairs in w₁ and only Billy chairs in w₂, and that the modal base M maps w to \{w₁, w₂\}. In a first-order MS reading, the question has two true answers (∘O₃Φ₃ and ∘O₃Φ₄) but no strongest true answer in w. However, if interpreted relative to a different modal base M′/M₂ which maps w to \{w₁\}/\{w₂\}, this question does have a strongest true answer in w, namely ∘O₃Φ₃/∘O₃Φ₄. Hence, MS interpretations lead to violations of Dayal’s presupposition but may satisfy RelExh. This idea also applies when the question nucleus is parsed without local exhaustification, and moreover, it extends to higher-order MS-interpretations.

\(^{17}\)Minimization is defined as follows, which allows a set of sets to have more than one minimal set:

(i) \( \min \triangleq \lambda E \in \mathcal{P}(X) \mid X \in E \land \forall Y \subseteq X \). Evaluated for any set of sets E, X is a minimal member of E iff X is in E, and no member of E is a proper subset of X.

For example, assume that only Andy comes in w₁, only Billy comes in w₂, and only Andy and Billy come in w₃. The following pairs each sentence with the minimal sets of accessible worlds that verify the truth of this sentence (Φ₃ stands for 'x come'):

(iii) a. \( \forall \phi₃: \{w₁\}, \{w₃\} \)
    b. \( \forall \phi₃: \{w₂\}, \{w₃\} \)
    c. \( \forall \phi₃ \land \forall \phi₄: \{w₁, w₂\}, \{w₃\} \)
    d. \( \forall \phi₃ \lor \forall \phi₄: \{w₁\}, \{w₂\}, \{w₃\} \)

(iv) a. \( \forall \phi₃: \{w₁\}, \{w₃\} \)
    b. \( \forall \phi₃: \{w₂\}, \{w₃\} \)
    c. \( \forall \phi₃ \land \forall \phi₄: \{w₁, w₂\}, \{w₃\} \)
    d. \( \forall \phi₃ \lor \forall \phi₄: \{w₁\}, \{w₂\}, \{w₃\} \)
    e. \( \forall \phi₃ \forall \phi₄: \{w₁\}, \{w₂\}, \{w₃\} \)

\(^{18}\)Note that RelExh cannot be defined without retrieving the short answers. Consider the definition in (i), schematized in parallel to (71). ([Φ₃]′ stands for a set of true propositional answers to Q in w given M.)

(i) \[ \forall M'[M'_w \in \min\{M''_w \mid M''_w \subseteq M_w \land \exists p \in [\phi]^M_w \land p \in [\phi]^M'_w\}] \rightarrow DP(w, [\phi]^M'_w) \]

This definition is problematic because the two underlined parts might yield a conflict for modalized questions. For example, let M₂ = \{w₁, w₂\} and assume that the non-modalized question ‘?’Φ₃’ has a unique true answer ‘?’Φ₃’ in w₁ and ‘?’Φ₃’ in w₂, then the corresponding ∘-question ‘?’Φ₃’ satisfies Dayal’s presupposition in w relative to M’ where M₂’ = \{w₁\}. Evaluated with different modal bases, the two sets of true propositions \{[Φ₃]′, [Φ₄]′\} and \{[Φ₃]′\} have no member in common. In contrast, defining the condition in terms of short answers avoids this mismatch. In the same example, the two sets of true short answers \{a, b\} and \{a\} have a common member a.
(72) Who can chair the committee? (First-order MS reading)
\[ [Q]^M = \lambda x_r : \text{hmn}_g(x) . \exists w' \in M_w[w' \in O_C \varphi_x] \]
Let c.t.c = \[
\begin{bmatrix}
 w_1 \to \{a\} \\
 w_2 \to \{b\} \\
... \\
 M_w = \{w_1, w_2\} \\
 M_w^1 = \{w_1\} \\
 M_w^2 = \{w_2\} 
\end{bmatrix}
\]
we have: \[
\begin{align*}
[Q]^M_w &= \{a, b\} \\
[Q]^M_w^1 &= \{a\} \\
[Q]^M_w^2 &= \{b\}
\end{align*}
\]

More generally, in any \(\Diamond\)-question interpreted with a MS reading, RelExh stands in the following relation with Dayal’s presupposition. Accordingly, if ’Wh-A P?’ trivially satisfies Dayal’s presupposition, the MS-interpretations of ‘Wh-A can P?’ trivially satisfy RelExh.

(73) Generalization of RelExh for MS-interpretations
The MS-interpretations of a \(\Diamond\)-question ’Wh-A can P?’ satisfy the RelExh presupposition if and only if the interpretation of the non-modal counterpart ’Wh-A P?’ satisfies Dayal’s presupposition in every accessible world where \(P\) holds for a member of \(A\).

Modifying from (26), I finalize the definitions of the answerhood-operators as follows. The RelExh presupposition and the max-informativity condition are abbreviated as \(\text{REP}(w, [Q]^M)\) and \(\text{MaxI}(a, w, [Q]^M)\), respectively. In (75a), the answerhood-operators for complete true answers has two effects: (i) the same as Fox’s answerhood, it requires that these answers are max-informative among those that are true in \(w\), and (ii) it assigns a RelExh presupposition to each of these answers.\(^{19}\)

In (75b), I add the option of requiring Dayal’s presupposition to be also satisfied relative to the original modal base, which only matters for deriving global uniqueness readings of \(\Diamond\)-questions.

(74) \(\text{MaxI}(a, w, [Q]^M) = 1\) if and only if \(a \in [Q]^M_w\) and \(\forall \beta \in [Q]^M_w [Q]^M(\beta) \not\subseteq [Q]^M(a)\).

(75) Answerhood-operators (final version)
\[ a. \text{With relativized exhaustivity only} \]
\[ \text{Ans}_{\text{RE}}(w)([Q]^M) = \{\text{REP}(w, [Q]^M) , \text{MaxI}(a, w, [Q]^M)\} \]
\[ \text{Ans}_{\text{RE}}^E(w)([Q]^M) = \{\text{REP}(w, [Q]^M), [Q]^M(a) \mid \text{MaxI}(a, w, [Q]^M)\} \]
\[ b. \text{With relativized and non-relativized exhaustivity} \]
\[ \text{Ans}_{\text{NRE}}(w)([Q]^M) = \{\text{REP}(w, [Q]^M) \land \text{DP}(w, [Q]^M) , \text{MaxI}(a, w, [Q]^M)\} \]
\[ \text{Ans}_{\text{NRE}}^E(w)([Q]^M) = \{\text{REP}(w, [Q]^M) \land \text{DP}(w, [Q]^M), [Q]^M(a) \mid \text{MaxI}(a, w, [Q]^M)\} \]

4.4.2. Predicting uniqueness effects

Uniqueness effects in non-modalized questions For questions without a modal expression, changing the modal base does not affect their interpretations. Hence, RelExh carries forward the merits of Dayal’s presupposition in accounting for the uniqueness effects in (55).

(55′) a. Which child came?
   b. Which two children formed a team?
   c. Did you invite Andy, Billy, or Cindy?

\(^{19}\) I assume that the RelExh presupposition is point-wise carried by each complete true answer, not as the following where the presupposition is carried by the set of complete answers as a whole. The assumed definition is compatible with Uegaki 2018, 2020 in accounting for the projection of uniqueness in question-embeddings. See fn. 16 and Uegaki 2020: Appendix A.
Moreover, the analysis of uniqueness assuming RelExh presupposition extends to cases that are challenging to the presuppositional-which account of Hirsch and Schwarz (2020): it predicts a desired uniqueness effect in (67) and allows for a pair-list reading with point-wise uniqueness in (68).

(67') Which two or three students solved a problem together?

(68') Which boy watched which movie?

**Uniqueness effects in 2-questions** For 2-questions interpreted with a first-order reading, RelExh yields the same consequence as Dayal’s presupposition. For example in (76), in a first-order reading, the question calls for an individual answer naming an atomic chapter and presupposes that there is only one chapter that we have to assign to the students. Dayal’s presupposition is violated if there are multiple chapters such that we assign them to the students in every accessible world. Since uniqueness is violated in all the accessible worlds, the violation of Dayal’s presupposition cannot be salvaged by interpreting the question relative to a subset of the accessible worlds.

(76) Q: Which chapter do we have to assign to the students? (First-order reading)

\[ \therefore \text{There is a unique chapter that we have to assign to the students.} \]

A: Chapter 1.

However, when the same singular 2-question is interpreted with a narrow-scope higher-order reading, its uniqueness inference can only be properly explained by RelExh. As seen in example (77), this question can be exhaustively addressed by a narrow-scope disjunction, read as ‘There is no particular chapter that we have to assign to the students; we just need to chose between chapter 1 and chapter 2.’ (Spector 2007, 2008; Xiang 2020c; see Sect. 3.1.2) The existence of this exhaustive answer sufficiently ensures the satisfaction of Dayal’s presupposition.

(77) Q: Which chapter do we have to assign to the students? (Higher-order reading)

\[ \therefore \text{We should assign only one chapter to the students.} \]

A: Chapter 1 or chapter 2, either is good. (\(\square [\phi_{c_1} \lor \phi_{c_2}]\))

However, similar to what Hirsch and Schwarz (2020) observe with singular 2-questions, this singular 2-question has a local uniqueness inference, namely, we should not assign more than one chapter to the students. Given the modal base \(M\) in (78), this local uniqueness inference is satisfied in \(w'\) but not in \(w\) — a uniqueness-violating world \(w_3\) is accessible to \(w\). Dayal’s presupposition is satisfied in both \(w\) and \(w'\) and thus cannot account for this local uniqueness effect.

(78) \[
\text{assign} = \begin{cases} 
    w_1 \rightarrow \{c_1\} \\
    w_2 \rightarrow \{c_2\} \\
    w_3 \rightarrow \{c_1, c_2\} \\
    \vdots 
\end{cases} \
\quad M = \begin{cases} 
    w \rightarrow \{w_1, w_2, w_3\} \text{ (with uniq.-violation)} \\
    w' \rightarrow \{w_1, w_2\} \text{ (without uniq.-violation)} 
\end{cases}
\]

In contrast, RelExh is satisfied in \(w'\) but violated in \(w\). The truth of the true answer \(\square [\phi_{c_1} \lor \phi_{c_2}]\) in \(w\) can be verified by three minimal sets of accessible worlds, namely \(\{w_1\}, \{w_2\}, \{w_3\}\). The RelExh presupposition is satisfied in \(w\) only if this question has a strongest true answer in \(w\) when interpreted relative to any modal base \(M'\) such that \(M'_{w_3} = \{w_3\}\). This requirement cannot be satisfied: if the universal modal quantifies over \(\{w_3\}\), question (77) has two true answers (\(\square \phi_{c_1}\) and \(\square \phi_{c_2}\)) but no strongest true answer in \(w\).
In sum, while both RelExh and Dayal’s presupposition predict the global uniqueness effects in singular □-questions, only RelExh can explain the local uniqueness effects in higher-order readings. This finding argues that RelExh is a mandatory condition, not just a salvaging strategy for Dayal’s presupposition.

**Uniqueness effects in ◇-questions with a MS reading**  According to the generalization in (73), if a non-modalized question ‘Wh-A P?’ has a true answer in a world \( w' \) accessible to \( w \) but violates Dayal’s presupposition in this \( w' \), then the MS-interpretations of ‘Wh-A can P?’ violate RelExh in \( w \). This relation is demonstrated in (79a,b).

(79) a. Which chapter did we assign to the students?  (Uniqueness)
    \( \rightsquigarrow \) We assigned exactly one chapter to the students.

b. Which chapter can we assign to the students?  (Universal local uniqueness)
    \( \rightsquigarrow \) We are only allowed to assign exactly one chapter to the students.

For concrete illustration, interpret (79b) relative to the following modal base \( M \), repeated from (78). In both \( w \) and \( w' \), this question has the set of true MS-answers, namely \( \{ \Diamond \phi_{c_1}, \Diamond \phi_{c_2} \} \).

(80) assign = 
\[
\begin{bmatrix}
    w_1 \to \{ c_1 \} \\
    w_2 \to \{ c_2 \} \\
    w_3 \to \{ c_1, c_2 \} \\
    \ldots
\end{bmatrix}, \quad \text{and} \quad M = 
\begin{bmatrix}
    w \to \{ w_1, w_2, w_3 \} \text{ (with uniq.-violation)} \\
    w' \to \{ w_1, w_2 \} \text{ (without uniq.-violation)}
\end{bmatrix}
\]

The truth of the true answer \( \Diamond \phi_{c_2} \) in \( w \) can be verified with two minimal sets of accessible worlds, namely \( \{ w_2 \} \) and \( \{ w_3 \} \), and uniqueness is violated in \( w_3 \). The first-order MS-interpretation of (79b) relative to \( M \) does not satisfy RelExh in \( w \): when interpreted relative to a modal base \( M' \) such that \( M'_w = \{ w_3 \} \), this question has two true answers (\( \Diamond \phi_{c_1} \) and \( \Diamond \phi_{c_2} \)) but no strongest true answer in \( w \). In contrast, since \( w' \) has no uniqueness-violating accessible world like \( w_3 \), the first-order MS-interpretation of (79b) satisfies RelExh in \( w' \). This idea also applies when this question has a higher-order MS-interpretation.

In question (81), where the singular \( wh \)-complement is modified by SINgle, the local uniqueness presupposition appears to be non-universal (see also (63d)). This question requests to name a chapter which can be the unique chapter that we assign to the students, but it does not deny the possibility of assigning more than one chapter.

(81) Which SINgle chapter can we assign to the students?  (‘Existential’ local uniqueness)
\( \rightsquigarrow \) We are allowed to assign exactly one chapter to the students.

I argue that in (81) the question nucleus is parsed with local exhaustification, which weakens the local uniqueness inference yielded from RelExh. In contrast to the non-exhaustified inference \( \Diamond \phi_{c_2} \), the truth of \( \Diamond \Diamond \phi_{c_2} \) in \( w \) is verified with \( \{ w_2 \} \) but not with \( \{ w_3 \} \); in other words, the non-modalized question *Which chapter \( x \) is such that that we only assign \( x \) to the students?* has no true answer in \( w_3 \). Hence, when the \( \Diamond \)-question is parsed with local exhaustification, whether Dayal’s presupposition is satisfied relative to a modal base introducing \( \{ w_3 \} \) does not matter to RelExh; the uniqueness inference in (81) thus appears non-universal.

In interpreting ◇-questions, the RelExh presupposition predicts only local uniqueness effects. To account for readings with global uniqueness effects, I assume that question interpretations optionally carry the non-relativized Dayal’s presupposition, on top of mandatorily presupposing RelExh. In
this case, the local uniqueness effects from RelExh collapse under the global uniqueness effects from Dayal’s presupposition.

4.4.3. Solving the over-generation problem

Section 4.4.2 has shown that the RelExh presupposition solves the over-generation problem of Fox’s answerhood in interpreting questions with uniqueness effects. This condition also addresses the over-generation problem for non-modalized questions and □-questions. As first raised in Sect. 3.2.4 and reiterated in Sect. 4.2, for questions like (82a,b) where the extension of the predicate that the wh-phrase combines with is not closed under sum, the answer space yielded in a first-order reading is not closed under conjunction, and Fox’s answerhood wrongly predicts a MS reading. The RelExh presupposition avoids predicting MS readings for these non-◇-questions: in a multi-team context, in contrast to the case of the ◇-question (82c), interpreting (82a,b) with a first-order reading yields a violation of RelExh, the same as it yields a violation of Dayal’s presupposition.

(82) a. Which children formed a team?
   b. Which children must form a team?
   c. Which children can form a team?

What’s more, my analysis explains why existential quantifiers do not have a MS-licensing effect, in contrast to existential modals: relativizing exhaustivity weakens Dayal’s presupposition only in the presence of an existential modal. In question (58) with an existential quantifier, RelExh yields the same consequence as Dayal’s presupposition and forces the individual reading of this question to be globally exhaustive. To allow the individual reading of (58) to be globally non-exhaustive, we would need to allow Dayal’s presupposition be assessed relative to a smaller discourse domain that includes only one of the boys, not relative to a modal base that introduces a subset of the accessible worlds.20

(58’) Which movie or movies did one of the boys watch?

4.5. Relativized Exclusivity: Modal obviation and uniqueness in disjunctive-MA

Section 4.4.2 has considered the uniqueness effects in MS-interpretations. In those cases, the RelExh presupposition predicts a local uniqueness effect. However, when singular ◇-questions are interpreted with a MA reading, further complications arise.

Hirsch and Schwarz (2020) observe that singular ◇-questions also admit mention-few/all readings. In Xiang 2020c, I further discover a conjunction–disjunction asymmetry: as seen in (83), the MA answer can be expressed by an elided disjunction, but not by an elided conjunction. This asymmetry is consistently observed in singular □-questions. To explain this asymmetry, I argue that singular wh-phrases may have a higher-order quantification domain, and that this domain consists of only disjunctions that range over a set of atomic individuals.

(83) Which chapter can we assign to the students?

20I leave it open whether there is any possibility to relativize exhaustivity by tuning the discourse domain or some other interpretation parameter. If other ways of relativizing exhaustivity are possible, the proposed analysis may extend to the following question, which is also treated as a MS-question in some literature (e.g., van Rooij 2004):

(i) Who has got a light?
Chapter 1 or chapter 2. (Disjunctive-MA)

# Chapter 1 and chapter 2. (Conjunctive-MA)

Incorporating these findings to the proposed derivation of disjunction-MA (Sect. 3.4.2), I argue that a singular $\Diamond$-question has a MA reading only if a $\text{dou}$-operator appears in the question nucleus and is associated with the higher-order $\text{wh}$-trace $\pi$. The scopal relation between $\pi$ and the modal does not matter. The answer spaces yielded are as follows (cf. Figure 7). I also remove the local $O$-operator, which was assumed to account for local exhaustivity effects of MS-answers: as argued in Sect. 3.2.1, local uniqueness presuppositions should be derived independently from the presence of the $O$-operator because they are more robust than local exhaustivity implicatures.

Figure 8: Answer space of (83) yielded in a disjunctive-MA reading

Next, consider whether RelExh predicts local uniqueness effects in disjunctive-MA interpretations. Consider the local-uniqueness-violating context in (80), repeated below:

(84) Which chapter can we assign to the students? (Disjunctive-MA)

\[ \Rightarrow \text{We are only allowed to assign exactly one chapter to the students.} \]

\[
\text{assign} = \begin{bmatrix}
    w_1 \rightarrow \{c_1\} \\
    w_2 \rightarrow \{c_2\} \\
    w_3 \rightarrow \{c_1, c_2\} \\
    \ldots
\end{bmatrix}
\]

and $M_w = \{w_1, w_2, w_3\}$

Unfortunately, the current analysis is insufficient: in the above example, the violation of local uniqueness does not lead to a violation of RelExh. Again, the truth of the true answer $\Diamond c_2$ in $w$ can be verified with $\{w_2\}$ and $\{w_3\}$. In (79b), I showed that the MS-interpretations of this question violate RelExh in $w$: Dayal’s presupposition is not satisfied when the question is interpreted relative to any $M'$ such that $M'_w = \{w_3\}$. However, in a disjunctive-MA reading, the answer space includes also the FC-disjunctive proposition $\text{dou}_C[\Diamond \phi_{c_1} \lor \Diamond \phi_{c_2}]$ (or $\text{dou}_C[\Diamond (\phi_{c_1} \lor \phi_{c_2})]$), which is the strongest true answer to in $w$ given any such $M'$.

In what follows, I will argue that the FC-disjunctive answers ($\text{dou}_C[\Diamond \phi_{c_2} \lor \Diamond \phi_{c_3}]$ and $\text{dou}_C[\Diamond (\phi_{c_1} \lor \phi_{c_2})]$) are semantically deviant. Once these answers are removed, RelExh predicts the local uniqueness effects for disjunctive-MA-interpretations in the same way as it does for MS-interpretations.

Cross-linguistically, the licensing of a universal FC-item is subject to modal obviation. For example, English FC any can only grammatically appear in a pre-verbal position in the presence of an existential modal, as seen in (85). Modal obviation is also seen in the licensing of pre-verbal FC-disjunctions in Mandarin: as exemplified in (86), when the existential modal keyi ‘can’ is dropped or replaced with a universal modal bixu ‘must’, dou cannot be grammatically associated with a pre-verbal disjunction.

\[ ^{21} \text{Xiang 2020c also explores another solution which assumes syntactic reconstruction and local uniqueness. This solution will require the higher-order wh-trace to take narrow scope.} \]
As for the Exclusivity (RelExcl) presupposition is schematized as follows, where \( \phi \)

The only difference is that, in (90a) where the disjunctive-MA interpretations of 

not derive a reading that requires 

dou is associated with a disjunction or an existential quantifier and functions as a universal FC-licenser.

modal obviation effects for cases where 

dou combines with a disjunction, for every \( \phi \) that is a disjunct of this disjunction, every minimal set of accessible worlds that verifies the truth of \( \phi \) also verifies the truth of the exhaustification of \( \phi \). This presupposition is schematized as follows, where \( C' \) denotes a subset of \([S]\)\(^M\).

(87) Relativized Exclusivity

\[
[[\text{dou}_C(S)]^M] / [[dou_C(S)]^M] \text{ is defined in } w \text{ only if }
\forall \phi, \exists \phi' \in \text{Sub}([S]^M, C) \wedge [[\phi']^M \subseteq [S]^M
\rightarrow \forall M' \in \min\{M' \mid M'_w \subseteq M_w \wedge [[\phi']^M(w) = 1\}) \rightarrow O_C([[\phi']^M]) (w) = 1\}
\]

(For any sentence \( \phi \) that denotes a sub-alternative of the prejacent \( S \) logically stronger than \([S]^M\), the exhaustification of \( \phi \) is true in \( w \) relative to any \( M' \) such that \( M' \) is a modal base that introduces a minimal set of accessible worlds that verifies the truth of \( \phi \) in \( w \).)

Let me first show how the RelExcl presupposition explains the modal obviation effects in universal FC-declaratives. In a non-modal disjunction, the choice of modal base does not matter. RelExcl simply means that \( \phi_j \wedge \phi_m, O_C\phi_j, \) and \( O_C\phi_m \) are simultaneously true, which is clearly contradictory.

(88) * dou [John or Mary teach Intro Chinese]

\[
\phi_j \vee \phi_m \not\text{dou} \phi_j \wedge \phi_m
\]

As for the \( \Box \)-disjunction, there is no modal base relative to which the universal FC-inference \( \Box \phi_j \wedge \Box \phi_m \) and RelExcl presupposition are simultaneously true: \( \Box \phi_j \wedge \Box \phi_m \) is true in \( w \) relative to \( M \) if and only if both John and Mary teach in all worlds in \( M_w \), \( O\Box \phi_j \) is true relative \( M' \) if and only if only if only John teaches in all worlds in \( M'_w \), and clearly \( M'_w \not\subseteq M_w \) (unless \( M'_w = \emptyset \)).

22 Modal obviation effects only arise when \textit{dou} functions as a trigger of universal FC inferences. To avoid over-predicting modal obviation effects for cases where \textit{dou} functions as a distributive or an even-like operator, I add the restriction \([\phi]^M \subseteq [S]\) (viz., \( \phi \) is logically stronger than the prejacent \( S \)); sub-alternatives are logically stronger than the prejacent only in cases where \textit{dou} is associated with a disjunction or an existential quantifier and functions as a universal FC-licenser.

23 This condition is very similar to Dayal’s (2013) Viability Constraint, which says that every exhausted alternative is true relative to a subset of the accessible worlds. The following formulates this constraint in parallel to (87):

\[
[[\text{dou}_C(S)]^M] \text{ is defined in } w \text{ only if }
\forall \phi, \exists \phi' \in \text{Sub}([S]^M, C) \wedge [[\phi']^M \subseteq [S]^M \rightarrow \exists M' \in \min\{M' \mid M'_w \subseteq M_w \wedge O_C([[\phi']^M]) (w) = 1\}
\]

(For any sentence \( \phi \) that denotes a sub-alternative of \( S \) that is logically stronger than \([S]^M \), there is a modal base \( M' \) such that \( M'_w \) is a subset of the worlds accessible to \( w \) and the exhaustification of \( \phi \) is true in \( w \) relative to \( M' \).)

The only difference is that, in (90a) where the \( \phi \)-disjunction is parsed without local exhaustification, the Viability Constraint predicts that universal FC is possible if and only if \( O\Box \phi_j \) and \( O\Box \phi_m \) are true. In other words, the Viability Constraint does not derive a reading that requires \( \Box (\phi_j \vee \phi_m) \) to be false; it is therefore too weak to derive universal uniqueness effects in disjunctive-MA interpretations of \( \phi \)-questions.

40
(89)  *dou [John or Mary must teach Intro Chinese]
\[ \square \varphi_j \lor \square \varphi_m \xrightarrow{\text{dou}} \square \varphi_j \land \square \varphi_m \]

As for the \( \diamond \)-disjunction, illustration (90) considers three evaluation worlds (viz., \( w, w', w'' \)) where the FC-inference is true and two ways to parse the prejacent disjunction (viz., with or without local exhaustification under the modal). In (90a) where the sentence is parsed without local exhaustification, RelExcl is satisfied in \( w \): the minimal set of accessible worlds that verifies the truth of \( \diamond \varphi_j / \diamond \varphi_m \), namely \( \{ w_1 \} / \{ w_2 \} \), also verifies the truth of its exhaustification \( O \diamond \varphi_j / O \diamond \varphi_j \). By contrast, RelExcl is not satisfied in \( w' \): \( \{ w_3 \} \) also verifies the truth of the disjunct \( \diamond \varphi_j / \diamond \varphi_m \), but it does not verify the truth of \( O \diamond \varphi_j / O \diamond \varphi_m \). In (90b) where the sentence is parsed with local exhaustification, RelExcl is also satisfied in \( w' \): the violation of exclusivity in \( w_3 \) does not affect RelExcl because \( \{ w_3 \} \) does not verify any locally exhaustified disjunct (\( \diamond O_{C} \varphi_j \) or \( \diamond O_{C} \varphi_m \)).

(90)  dou [John or Mary can teach Intro Chinese]

Let \( \text{teach-IC} = \begin{bmatrix} w_1 \rightarrow \{ j \} \\ w_2 \rightarrow \{ m \} \\ w_3 \rightarrow \{ j, m \} \end{bmatrix} \) and \( M = \begin{bmatrix} w \rightarrow \{ w_1, w_2 \} \\ w' \rightarrow \{ w_1, w_2, w_3 \} \\ w'' \rightarrow \{ w_1, w_3 \} \end{bmatrix} \)

a. \( \diamond \varphi_j \lor \diamond \varphi_m \xrightarrow{\text{dou}} \diamond \varphi_j \land \diamond \varphi_m \)

Given \( M \), RelExcl is satisfied in \( w \), violated in \( w' \) and \( w'' \). More generally:
Universal FC is possible if and only if \( \diamond O_{C} \varphi_j \land \diamond O_{C} \varphi_m \land \neg \diamond (\varphi_j \land \varphi_m) \).

b. \( \diamond O_{C} \varphi_j \lor \diamond O_{C} \varphi_m \xrightarrow{\text{dou}} \diamond O_{C} \varphi_j \land \diamond O_{C} \varphi_m \)

Given \( M \), RelExcl is satisfied in \( w \) and \( w' \), violated in \( w'' \). More generally:
Universal FC is possible if and only if \( \diamond O_{C} \varphi_j \land \diamond O_{C} \varphi_m \).

The RelExcl presupposition also explains two facts in question interpretations. First, it explains why non-modalized questions and \( \square \)-questions do not have a universal FC-disjunctive answer: wide-scope disjunctive answers of the form \( \text{dou}(\varphi \lor \psi) \) or \( \text{dou}[\square \varphi \lor \square \psi] \) violate RelExcl and are undefined. In particular in (91b), in responding to a \( \square \)-question, the elided disjunction can be interpreted as an existential FC item, derived from a narrow-scope reading, but not as a universal FC item.

(91)  a. Which chapter did we assign to the students?

Chapter 1 or chapter 2.  (Ignorance)

b. Which chapter do we have to assign to the students?

Chapter 1 or chapter 2.  (Ignorance; existential FC)

Second, RelExcl restricts the acceptability of a FC-disjunctive answer to a \( \diamond \)-question. When parsed without local exhaustification, a disjunctive answer of the form \( \text{dou}[\diamond \varphi \lor \diamond \psi] \) or \( \text{dou}\diamond(\varphi \lor \psi) \) satisfies RelExcl if and only if \( \diamond O \varphi \) and \( \diamond O \psi \) are both true and \( \diamond (\varphi \land \psi) \) is false. Hence, for the singular \( \diamond \)-question (92a) (repeated from (84)), since \( \diamond (\varphi_c \lor \varphi_c) \) is not false, the disjunctive answers \( \text{dou}\diamond(\varphi_c \lor \varphi_c) \) and \( \text{dou}[\diamond \varphi_c \lor \diamond \varphi_c] \) violate the RelExcl presupposition of \( \text{dou} \) and are undefined. Once these answers are removed from the answer space, the RelExh presupposition predicts a universal local uniqueness effect in the same way as how it does for the MS-interpretation in (80).

(92)  (Context: We can assign either chapter 1, or chapter 2, or both, to the students.)

Which chapter can we assign to the students?  (Universal local uniqueness)

\( \neg \rightarrow \) We are only allowed to assign exactly one chapter to the students.
This analysis also applies to (93) (cf. (81)). When parsed with local exhaustification, the disjunctive answers $\text{dou} \Diamond \Box_c \phi_1 \lor \phi_2$ and $\text{dou} \left( \Diamond \Box_c \phi_1 \lor \Diamond \Box_c \phi_2 \right)$ are true if and only if $\Diamond \Box_c \phi_1$ and $\Diamond \Box_c \phi_2$ are true, regardless of whether $\Diamond (\phi \land \psi)$ is false or true. Hence, RelExh predicts a seemingly non-universal local uniqueness effect in the same way as it does for the MS-interpretations of (81).

(93) Which SINGLE chapter can we assign to the students? (‘Existential’ local uniqueness)

\[
\Rightarrow \text{We are allowed to assign exactly one chapter to the students.}
\]

To sum up, singular $\Diamond$-questions admit FC-disjunctive answers derived based on Boolean disjunctions, the existence of which unpleasantly can make RelExh be satisfied even in contexts that violate local uniqueness. In responding to this problem, I argue that the FC-triggering operator $\text{dou}/\text{dou}$ carries a RelExcl presupposition. According to RelExcl, a disjunction can have a universal FC reading only if any modal base that verifies the truth of a disjunct of this disjunction also verifies the truth of the exhaustification of this disjunct. This presupposition explains the modal obviation effects in licensing universal FC, and moreover, it allows to extend the RelExh analysis of local uniqueness effects to MA-interpretations of $\Diamond$-questions.

4.6. Interim summary

In this section, I have proposed a novel ‘Relativized Exhaustivity (RelExh)’ presupposition to solve the dilemma between uniqueness and MS in questions. This presupposition requires Dayal’s presupposition to be satisfied relative to any modal base that introduces a minimal set of accessible worlds that verifies the truth of a true answer.

The RelExh presupposition has many advantages: it allows for MS-interpretations, carries forward the merits of Dayal’s presupposition in deriving uniqueness effects, and overcomes the overgeneration problem with Fox’s answerhood. Moreover, in contrast to the non-relativized Dayal’s presupposition, the RelExh presupposition can account for the local uniqueness effects in $\Box$-questions and $\Diamond$-questions. Predictions of Dayal’s presupposition and RelExh are summarized as follows:

<table>
<thead>
<tr>
<th>Modal-type</th>
<th>Reading-type</th>
<th>Dayal’s</th>
<th>RelExh</th>
</tr>
</thead>
<tbody>
<tr>
<td>No modal</td>
<td>± uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\Box$-modal</td>
<td>- uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>global uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>local uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\Diamond$-modal</td>
<td>- uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>MS</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>- uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>MA</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>global uniq.</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>local uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>MS</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>local uniq.</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 1: Predictions of Dayal’s presupposition and the RelExh presupposition

In addition to RelExh, I have also proposed a ‘Relativized Exclusivity (RelExcl)’ condition for the derivations of universal FC inferences and disjunctive-MA readings. RelExcl explains the modal obviation effects in licensing universal FC items, and moreover, it solves the challenges to the RelExh analysis in accounting for the local uniqueness effects in $\Diamond$-questions with a disjunctive-MA reading.
5. Conclusions

This paper made three contributions to the study of MS readings and uniqueness effects in questions. First, observing that MS answers are subject to a ‘mention-one-requirement’ which cannot be explained by pragmatic factors, I argued that MS readings are primarily licensed by grammatical factors, such as the presence of an existential modal. Further, given that this modal expression has to be interpreted within the question nucleus, I argued that the MS/MA-ambiguity in ◊-questions should be analyzed in terms of structure-ambiguities within the question nucleus.

Second, taking insights from Fox 2013, I derived MS readings and MA readings of ◊-questions with a single non-exhaustive answerhood and attributed the MS/MA-contrast to structural variations within the question nucleus. I argued that MA readings arise if one of the following conditions is met, and MS readings arise otherwise: (i) the higher-order wh-trace takes scope above the existential modal, and (ii) a FC-triggering operator dou (≈ the Mandarin FC-triggering particle dou) appears above the existential modal and is associated with the higher-order wh-trace. In particular, condition (i) yields readings calling for a conjunctive-MA answer, and condition (ii) yields a reading calling for a disjunctive-MA answer.

However, allowing non-exhaustive answers to be complete would cause a concerning conflict with ‘Dayal’s presupposition’, which says that a question must have a true exhaustive answer. This condition is crucial in accounting for the uniqueness effects in questions. Hence, last and the most, I proposed that question interpretations can violate Dayal’s presupposition but are mandatorily subject to a ‘Relativized Exhaustivity’ condition. This condition solves the dilemma between uniqueness and MS, overcomes the over-generation problem of Fox’s answerhood in predicting MS-readings, and is especially advantageous in deriving local uniqueness effects in modalized questions.

Acknowledgements [To be added …]

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