Crossover and accessibility in dynamic semantics *

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Abstract

Chierchia (2020) develops a theory of Weak Crossover effects couched in terms of dynamic semantics, the central idea being that discourse referents are *only* introduced in thematic positions. Intrasentential anaphora proceeds via standard dynamic mechanisms, thus grounding WCO in an independently motivated notion of *accessibility*. While conceptually appealing, I show that Chierchia's account is hamstrung by a central design feature of first-generation dynamic theories — the *external staticity* of negation and related operators. This gives rise to the (I argue, erroneous) prediction that any expression that can outscope negation for the purposes of intersentential anaphora can obviate Weak Crossover. To address this issue, I suggest that we need a dynamic semantics in which operators such as negation *selectively* destroy *in*determinate discourse referents, but leave determinate discourse referents unscathed. I develop a version of monadic dynamic semantics (Charlow 2014, 2019a) which accomplishes just this, by tweaking the semantics of indefinites. The resulting system is independently interesting, addressing as it does some otherwise recalcitrant problems for first-generation dynamic theories, stemming from the problematic status of double negation.

1 Introduction

The goals of this paper are two-fold: first, I'll demonstrate that a recent account of Weak Crossover (WCO) put forth by Chierchia (2020) is hamstrung by some central design features of Dynamic Semantics (DS) (Heim 1982, Groenendijk & Stokhof 1991), surrounding the treatment of negation, and other externally static operators. Concretely, Chierchia predicts that: *if* the quantificational scope of a Quantificational Phrase (QP) feeds its *dynamic* scope, then the QP should give rise to WCO obviation. The main empirical contribution of this paper is a demonstration that this can't

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be correct: there are many instances in which quantificational scope feeds dynamic scope, but WCO is still in effect. I'll take this as an initial motivation for rethinking the underlying dynamic plumbing.

Independently, it's been known for some time (Groenendijk & Stokhof 1991, Krahmer & Muskens 1995, Gotham 2019), that DS's treatment of negation leaves something to be desired. In effect, DS departs *too far* from a classical setting, invalidating Double Negation Elimination (DNE). This property of DS will be shown to have problematic consequences in a number of domains, such as Partee's famous *bathroom* sentences. Taking Charlow's (2014, 2019a) monadic DS as a starting point, I'll show that it's possible to address these problems while making DS *more* classical. Concretely, I'll argue that, once we have a dynamic system in which we keep track of both verifying *and* falsifying assignments, we can simply adopt a classical semantics for negation while maintaining all of the typical advantages of DS. All we need to do is tweak the semantics of indefinites, by *collapsing* any outputted falsifying assignments into a single assignment.

The resulting semantics will be shown to have some very useful properties. One consequence will be that negation and related operators are only *selectively* externally static — indeterminate Discourse Referents (DRS), such as those introduced by indefinites, are rendered inaccessible, but determinate DRS, such as those introduced by definites, are left unscathed. I'll show that this fixes the problem originally identified for Chierchia's account of WCO.

More generally, the resulting semantics will validate DNE. As we'll see, this will help us account for some of some recalcitrant problems of first-generation dynamic theories involving accessibility, double negation, and disjunction. A conceptually appealing aspect of this proposal is that it rids DS of a stipulation regarding the semantics of negation, while achieving superior empirical coverage.

2 The dynamics of crossover

WCO (Postal 1971, and subsequent work) has long been a thorn in the side of theories of the syntax-semantics interface, demonstrating poorly understood restrictions on the interaction between scope and binding. An instantiation of the WCO paradigm is given in (1). In order to understand the problem, consider that any theory of scope needs to explain why QPS interpreted in places other than where they're pronounced, in order to account for, e.g., inverse scope readings. A popular approach to this question in the generative literature is Quantifier Raising (QR) (May 1977). If such an operation is independently necessary, then why can't scoping the QP feed binding in (1b)?[1]

(1) a. Every boy^1 loves his$_1$ mother.

b. * His$_1$ mother loves every boy^1.

[1]Of course, there are also many other approaches to scope-taking which *don't* involve movement, such as continuation semantics (Barker 2002, Barker & Shan 2014) — the reasoning outlined here applies equally to such theories.

It's tempting to conclude that scope simply can't feed binding, but the empirical landscape is more nuanced that this, as has long been recognized (see, e.g., Ruys 2000). For example, QPs can bind out subjects (2a), and into right-adjoined VP modified (2b).

(2) a. Every boy^1 's mother loves him$_1$.

b. Mattie petted every kitten^1 against it$_1$'s will.

Chierchia's account side-steps this issue by adopting an account on which anaphora proceeds from left-to-right, and potentially to non-c-commanding positions — Dynamic Semantics (DS). The central move is to claim that the binder is always the *thematic* position of the QP, but, by dint of how anaphora is licensed in DS, the thematic position of the binder must be to the left of the pronoun.

In order to understand how Chierchia's theory works in detail, and ultimately in order to understand its pitfalls, it will be useful to first go through some basic design features of DS.

2.1 Dynamic semantics: the basics

DS (Heim 1982, Groenendijk & Stokhof 1991, a.o.) purports to account for linear asymmetries in the domain of anaphora, as exemplified by the following contrast.[2]

(3) a. A woman^1 walked in. She$_1$ sat down.

b. * She$_1$ sat down. A woman^1 walked in.

Assuming that the indefinite takes scope within its containing sentence, first-generation dynamic theories account for the contrast in (3) by building an Input/Output (I/O) asymmetry into semantic composition. In first generation dynamic theories, such as Groenendijk & Stokhof's Dynamic Predicate Logic (DPL), the asymmetry is precompiled into the entry for the sentential sequencing operator. In order to see how this works, it will be useful to start laying out some details.[3] In DS, sentences denote *dynamic propositions* — functions from assignments to sets of assignments. We'll abbreviate the type of a dynamic proposition as T (4).

(4) Dynamic propositional type (def.)[4]
$\mathsf{T} \coloneqq g \to \{\, g \,\}$

Indefinites induce, in the dynamic parlance, *random assignment* — that is to say that the sentence takes an input assignment g and outputs a set of modified assignments. This is shown in

[2] Throughout this paper, we use superscript indices for the binder, and matching subscript indices for the expression(s) it binds.

[3] The presentation here is rather idiosyncratic, but more-or-less equivalent to DPL.

[4] We write $\{\, a \,\}$ for the type of a set of values of type a.

(5).[5]

(5) Indefinites in DS
$[\![\text{A woman}^1 \text{ walked in}]\!] = \lambda g \,.\, \{\, g^{[1\to x]} \mid \mathsf{woman}\ x \wedge \mathsf{walked\text{-}in}\ x \,\}$ $\quad \mathsf{T}$

We can illustrate the effect of random assignment by considering the application of (5) to $g_\varnothing$.[6].

(6) Context: *The women are Shirley, Britta, and Annie.*
$[\![\text{a woman}^1 \text{ walked in}]\!]\ g_\varnothing = \{\, [1 \to \mathsf{shirley}], [1 \to \mathsf{britta}], [1 \to \mathsf{annie}] \,\}$ $\quad \{\, g \,\}$

Pronouns, on the other hand, induce a simple form of environment-sensitivity, as shown in (7). The consequences of feeding a sentence with a pronoun (a) an assignment which passes the test, and (b) an assignment which fails the test are shown in the two respective examples in (8).

(7) Pronouns in DS
$$[\![\text{She}_1 \text{ sat down}]\!] = \lambda g \,.\, \begin{cases} \{\, g \,\} & \mathsf{sat\text{-}down}\ g_1 \\ \varnothing & \text{otherwise} \end{cases} \qquad \mathsf{T}$$

(8) Context: *only Shirley sat down*

a. $[\![\text{She}_1 \text{ sat down}]\!]\ [1 \to \mathsf{shirley}] = \{\, [1 \to \mathsf{shirley}] \,\}$ $\quad \{\, g \,\}$

b. $[\![\text{She}_1 \text{ sat down}]\!]\ [1 \to \mathsf{annie}] = \varnothing$ $\quad \{\, g \,\}$

Note that a direct consequence of the definitions given here is that a sentence with a pronoun is never defined for the initial assignment (Heim's 1991 *familiarity* condition). In order for pronouns to co-vary with indefinites in previous sentences, it is standard to define a dynamic sequencing operator ($;$), which threads dynamic propositions together by feeding the outputs of the first sentence into the second, pointwise, and gathering up the results.

(9) Dynamic sequencing (def.)
$$p \,;\, q \coloneqq \lambda g \,.\, \bigcup_{g' \in p\, g} \{\, g'' \mid g'' \in q\, g' \,\} \qquad ; : \mathsf{T} \to \mathsf{T} \to \mathsf{T}$$

It follows from the definition of dynamic sequencing that the input of the second sentence can be sensitive to the output of the first, but not vice versa. We can see how this works by sketching the result of interpreting (3a) via dynamic sequencing, and applying the result to the initial assignment.

[5]Technically speaking, we'll assume that assignments are *partial* — that is to say, given a stock of variables $\mathbb{N}$, an assignment is a function whose domain is a subset of $\mathbb{N}$, and who codomain is the set of individuals. This means that a given assignment g may be *un*defined for a variable n.

We write $g^{[n\to x]}$ is the assignment that is just like g, other than mapping n to g — we assume that it is defined iff g_n is *undefined* (Heim's 1991 *novelty condition*).

[6]In order to understand the contribution of a dynamic proposition, it can be useful to apply it to the unique initial assignment $g_\varnothing$, whose domain is the empty set. The initial assignment represents an idealized state in which no DRS have been introduced.

(10) Context: *Three women walked in and sat down: Shirley, Britta and Annie*

$$
\begin{aligned}
[\![(3a)]\!] &= ([\![\text{A woman}^1 \text{ walked in}]\!] \,;\, [\![\text{she}_1 \text{ sat down}]\!])\; g_\varnothing \qquad \{g\}\\
&= \bigcup_{g' \in [\![\text{a woman}^1 \text{ walked in}]\!]\; g_\varnothing} [\![\text{she}_1 \text{ sat down}]\!]\; g'\\
&= [\![\text{she}_1 \text{ sat down}]\!]\; [1 \to \text{shirley}]\\
&\quad \cup [\![\text{she}_1 \text{ sat down}]\!]\; [1 \to \text{britta}]\\
&\quad \cup [\![\text{she}_1 \text{ sat down}]\!]\; [1 \to \text{annie}]\\
&= \{ [1 \to \text{shirley}], [1 \to \text{britta}], [1 \to \text{annie}] \}
\end{aligned}
$$

In the dynamic parlance we say that, by dint of how dynamic sequencing is defined, in a sentence of the form $p\,;q$, p is *accessible* to q, but not vice-versa. Chierchia (2020) accounts for WCO by generalizing this notion of accessibility to sentence-internal positions. Another variant of the WCO paradigm given below. The generalization, at first blush, is that a pronoun may only be bound by an expression X, if the thematic position of X is to the left of the pronoun.

(11) a. Which girl[1] __ loves her$_1$ father?

b. * Which girl[1] does her$_1$ father love __?

Chierchia's account of WCO is founded on the assumption that semantic composition is dynamic *all the way down*, and therefore exhibits left-to-right asymmetries sentence internally. Concretely, in Chierchia's system, *theme* positions will be accessible to *agent* positions, but not vice versa. Where Chierchia departs significantly from DS is in the conjecture that DRS may be introduced *only* at thematic positions; really, what is doing the binding in (11) is the trace position, and coupled with a sentence-internal notion of accessibility, WCO follows straightforwardly.

In DS, there are certain *externally static* expressions, such as negation, which are known to induce inaccessibility. As we'll see, Chierchia's account will face a major obstacle once we examine in detail how such expressions interact with the basic theory of crossover described above. Before laying out the issue, we'll first turn to a consideration of Chierchia's system in more detail.

2.2 Chierchia's dynamic event semantics

Chierchia assumes a version of DS roughly equivalent to the one outlined in the previous section. In order to get the notion of accessibility to do some work in the sentence-internal domain, Chierchia adopts a view of semantic composition that is fundamentally *conjunctive* — *neo-Davidsonian event semantics* (Castañeda 1967). According to neo-Davidsonian event semantics, verbs uniformly denote (monadic) predicates of events, participants are related to events via *thematic functions*,[7] and sentences are existential statements about events. This is illustrated for an non-argument-taking predicate in (12a) and a predicate with more complex argument structure in (12b).

[7] The literature sometimes diverges as to whether thematic roles are modelled as relations or functions. This distinction will be immaterial for our purposes.

(12) a. $[\![\text{it rained}]\!] = \exists e[\mathsf{rain}\ e]$

b. $[\![\text{Shirley gave Jeff the book}]\!] = \exists e \begin{bmatrix} \mathsf{agent}\ e = \mathsf{shirley} \\ \wedge\ \mathsf{theme}\ e = \mathsf{the\text{-}book} \\ \wedge\ \mathsf{goal}\ e = \mathsf{jeff} \end{bmatrix}$

Following Chierchia, we can translate this basic setup into a dynamic setting fairly straightforwardly. In order to do so, we'll first define a helper function *dynamic lift* which shifts a truth value into the corresponding dynamic *test*.[8]

(13) Dynamic lift (def.)

$$t^{\uparrow} := \lambda g\,.\begin{cases}\{g\} & t \\ \varnothing & \text{otherwise}\end{cases} \qquad \uparrow : \mathsf{t} \to \mathsf{T}$$

With dynamic lift in hand, predicates of events can be shifted into functions from individuals to dynamic sentential meanings using the following recipe:[9]

(16) Dynamically lifting a verbal predicate

$$[\![\text{rain}]\!]^{\uparrow} := \lambda e\,.\,([\![\text{rain}]\!]\ e)^{\uparrow} \qquad \mathsf{v} \to \mathsf{T}$$

$$= \lambda e g\,.\begin{cases}\{g\} & \mathsf{rain}\ e \\ \varnothing & \text{otherwise}\end{cases}$$

In a static setting, thematic functions take individuals and return event predicates. These can be lifted into the current setting using an analogous recipe:[10]

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[8] A test is a dynamic proposition that, given as input assignment g, either returns $\{g\}$ or $\varnothing$.

N.b. that our decomposition differs somewhat from the exact presentation in Chierchia (2020), but the results are equivalent.

[9] This recipe takes advantage of the fact that $((\to)\ \mathsf{v})$ is a *functor* in category theoretic terms, which we can characterize via the following type constructor:

(14) $\texttt{Reader}_{\mathsf{v}}$ (def.)

$\mathsf{R_v\ a} := \mathsf{v} \to \mathsf{a}$

Since $\mathsf{R_v}$ is a functor, we can define a structure-preserving operation that maps a function into the functorial structure introduced by $\mathsf{R_v}$.

(15) $\mathsf{map}\ f := \lambda m e\,.\,f\ (m\ e) \qquad \mathsf{map} : (\mathsf{a} \to \mathsf{b}) \to (\mathsf{v} \to \mathsf{a}) \to (\mathsf{v} \to \mathsf{b})$

Our recipe for converting a verbal predicate into a dynamic verbal predicate, simply involves mapping dynamic lift into the verbal predicate.

[10] Similarly, this recipe takes advantage of the fact that in addition to $((\to)\ \mathsf{v})$, $((\to)\ \mathsf{e})$ is also a functor. A reconstruction of the details is left to the reader.

(17) Dynamically lifting a thematic function
$\mathsf{agent}^{\uparrow} \coloneqq \lambda xe \,.\, (\mathsf{agent}\ \mathsf{x}\ \mathsf{e})^{\uparrow}$ $\qquad \mathsf{e} \to \mathsf{v} \to \mathsf{T}$

$$= \lambda gxe \,.\, \begin{cases} \{g\} & \mathsf{agent}\ e = x \\ \varnothing & \text{otherwise} \end{cases}$$

We also need a composition rule for gluing together dynamic event predicates — *dynamic predicate modification*. This is simply an instantiation of dynamic sequencing, generalized in an analogous way to Partee & Rooth's (1983) generalized conjunction.

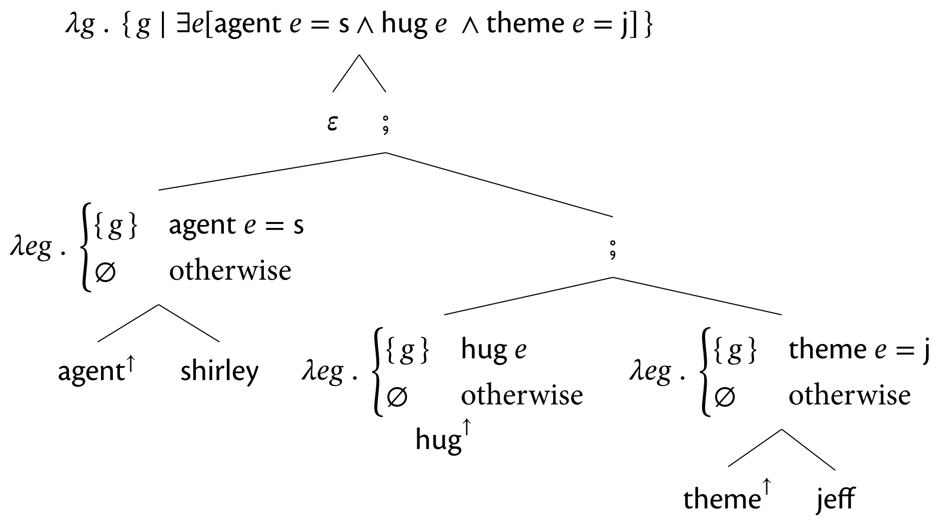
(18) Predicate modification in dynamic event semantics (def.)
$f \,;\, g \coloneqq \lambda e \,.\, f\ e \,;\, g\ e$ $\qquad (\mathsf{v} \to \mathsf{T}) \to (\mathsf{v} \to \mathsf{T}) \to \mathsf{v} \to \mathsf{T}$

Finally, we need a way of closing off the event variable, in order to get back a sentential meaning. In event semantics, this is typically done via an operation of existential closure. A dynamic version of this operation is defined below.

(19) Existential closure in dynamic event semantics
$\varepsilon\ m \coloneqq \lambda g \,.\, \bigcup_{e \in D_{\mathsf{v}}} m\ e\ g$ $\qquad \varepsilon : (\mathsf{v} \to \mathsf{T}) \to \mathsf{T}$

Before we discuss expressions with dynamic import — indefinites, pronouns, etc. — let's first consider an example derivation, to illustrate how semantic composition proceeds in Chierchia's dynamic event semantics. The verbal predicate is lifted into a *dynamic* predicate, and similarly, lifted thematic functions applied to their arguments return dynamic predicates. The verb and its arguments thus compose via dynamic predicate modification, and the resulting dynamic predicate is existentially closed, returning a dynamic proposition. Since no DRS are introduced here, the result is a test.

Figure (1): "Shirley hugged Jeff"



We've shown how Chierchia systematically lifts neo-Davidsonian event semantics into a dynamic setting. It should already be reasonably clear that, given how the system is set up, accessibility relations between thematic positions automatically fall out which mirror their linear order, i.e., the agent phrase is accessible to the theme phrase, but not vice versa. In order to illustrate the account of WCO it will only be necessary to discuss the semantics of indefinites and pronouns. Departing slightly from Chierchia here, are defined such that they may compose directly with thematic functions.

(20) $\mathsf{pron}_n \coloneqq \lambda r \, . \, \lambda e \, . \, \lambda g \, . \, r \, g_n \, e \, g$ $\qquad (\mathsf{e} \to \mathsf{v} \to \mathsf{T}) \to \mathsf{v} \to \mathsf{T}$

Indefinites on the other hand denote functions from dynamic predicates to dynamic propositions. One important thing to note here is that they are *not* attributed any inherent anaphoric potential. By dint of its type,[11] it is clear that the indefinite is a scope-taker; they may not compose with thematic functions *in-situ*, but rather must be scoped out.

(21) $[\![\text{some boy}]\!] \coloneqq \lambda k g \, . \, \bigcup_{\mathsf{boy}\, x} k \, x \, g$ $\qquad (\mathsf{e} \to \mathsf{T}) \to \mathsf{T}$

So just how do DRS get introduced on this set-up, if not by indefinites? The most substantial contribution of Chierchia's paper is the Dynamic Predication Principle (DPP). A direct consequence of the DPP is that only verbs, and (more relevantly for our purposes) thematic functions may introduce DRS.[12]

(22) The Dynamic Predication Principle (DPP)
DRS can only be introduced by predicates. (Chierchia 2020: p. 32)

This is cashed out in the formal system via a superscript operator that applies to a thematic function, introducing a *determinate* DR relative to its individual argument. Note that it's easy to define a version of the superscript operator that may apply to an individual or a quantifier, and therefore it's a crucial claim of the theory that such operators don't exist.

(23) DR introduction (def.)
$r^n \coloneqq \lambda x e g \, . \, r \, x \, e \, g^{[n \to x]}$ $\qquad (\mathsf{e} \to \mathsf{v} \to \mathsf{T}) \to \mathsf{e} \to \mathsf{v} \to \mathsf{T}$

Some terminological clarification will be useful at this point: we say that a dynamic proposition *introduces a* DR *n* if it is undefined for any input defined at *n*, and its outputs are defined

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[11] Any expression of type $(\mathsf{a} \to \mathsf{b}) \to \mathsf{b}$ may take scope at a type b node, leaving behind a trace of type a.

[12] In order to account for, e.g., interactions A-movement and WCO, the generalization in (22) ultimately must be extended in an apparently *ad-hoc* fashion. Since we're not concerned with A-movement here, the original version of the generalization will be sufficient for our purposes.

Elliott (in progress) presents an alternative to Chierchia's theory of WCO which aims to account for the A/A' distinction for the purposes of WCO in a more principled fashion — the idea, in brief, is that whereas A'-movement involves *bona fide* scope and phonological displacement, A-movement is cashed out as null operator movement — A-moved expressions are in fact base-generated in their landing sites. Once married with a dynamic account of WCO, the distinction between A/A' movement falls out automatically; as an added bonus, *improper movement* becomes unrepresentable.

at n. We say that n is a *determinate* DR if every assignment in the output agrees on n (assuming they are defined at n), and n is an *indeterminate* DR if the output assignments *dis*agree at n (again, assuming they are defined at n). In classical DS, indefinites introduce indeterminate DRS by triggering random assignment. In Chierchia's system, x^n is guaranteed to introduce a determinate DR, since output assignments map n to the same individual: x.

We're now in a position to understand Chierchia's theory of WCO. Due to the DPP, only thematic positions may introduce DRS — scope never feeds DR-introduction. Anaphora therefore always respects the accessibility hierarchy initially established in the thematic core of the sentence. The classic dynamic entry for an indefinite is, in essence, decomposed into two components — the base position is responsible for introducing the DR, and at the scope site, the indefinite induces non-determinism by feeding in a set of alternatives. A derivation involving successful binding is given in figure (2), and a WCO derivation is given in figure (3). In the latter instance, binding is unsuccessful due to the accessibility relations set up in the thematic core of the sentence, which may not subsequently be disrupted.

Figure (2): A girl $\_\_^1$ loves her$_1$ mother

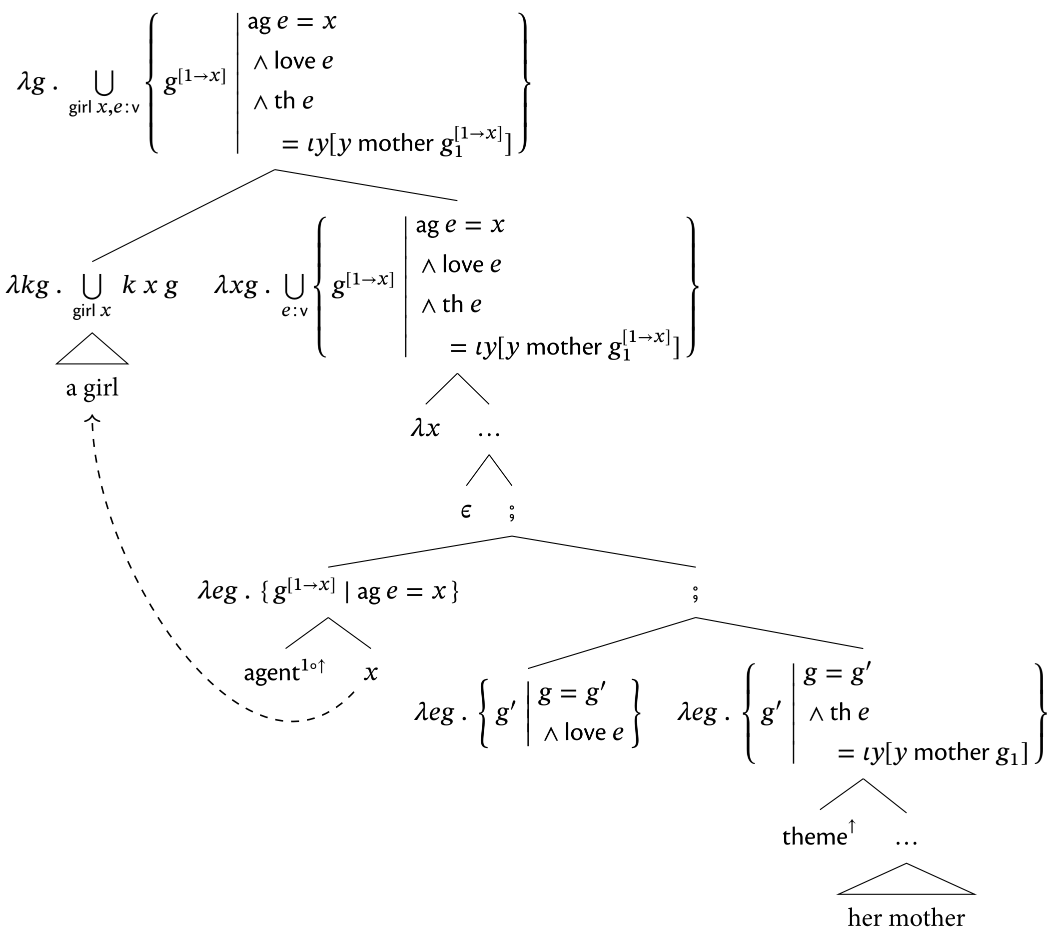
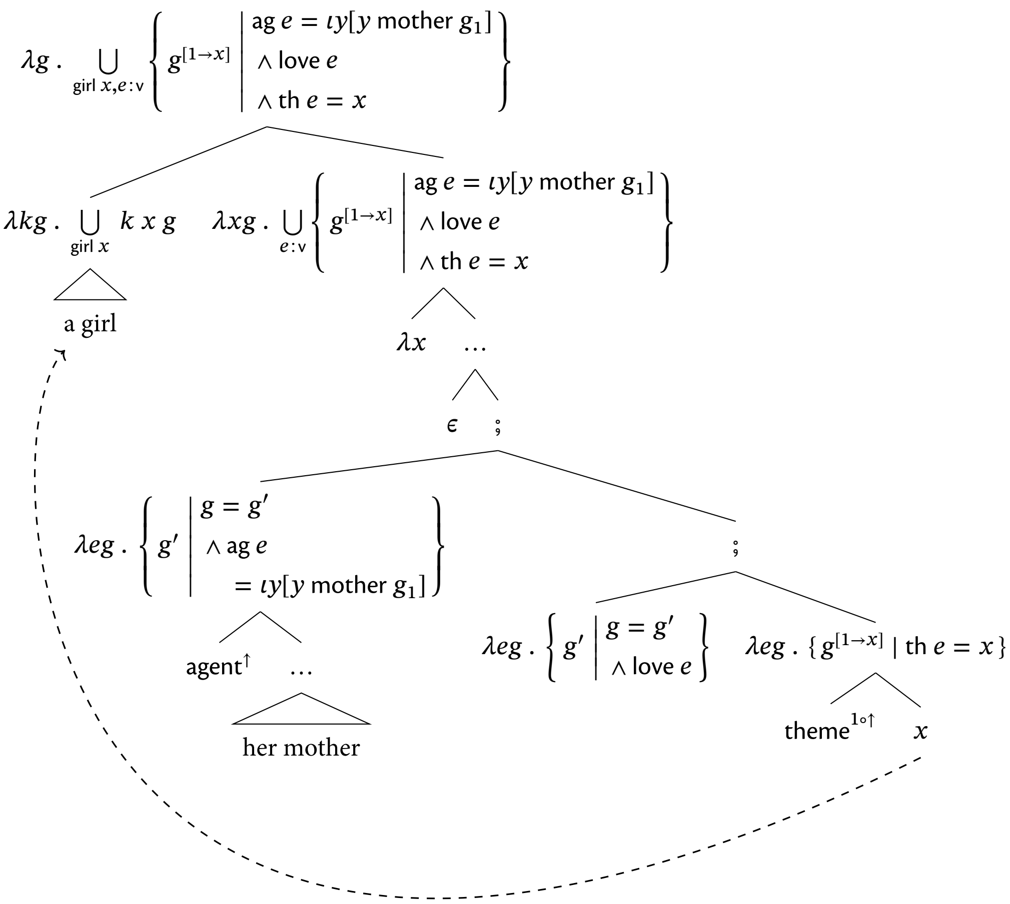


Figure (3): A girl her$_1$ mother loves __[1]



One aspect of Chierchia's account which is crucial, yet somewhat glossed over in the paper, is that, assuming a QR perspective, a QP cannot bind its trace via dynamic mechanisms, as this is incompatible with the DPP. Rather, there must be a syntactically distinct stock of variables responsible for trace-binding.

Putting this conceptual issue to one side, Chierchia's explanation for WCO is rather neat, and arguably worth preserving; taking as it does independently motivated mechanisms for anaphora, and generalizing them to the subsentential domain. As discussed in detail in the paper, this account has the further advantage of straightforwardly accounting for binding into adjuncts, without recourse to shell-like descending structures.

(24) John [[loves every cat[1]] [against it_1 s will]]. Chierchia 2020: 43

As discussed by Chierchia this follows straightforwardly from the system as laid out here, on the assumption that the adjunct is a right-adjoined dynamic event predicate. Since composition proceeds via discourse sequencing, it follows that the *theme* is accessible to adjunct for the purposes of anaphora.

In the next section, we'll introduce the central problem, as we see it, for Chierchia's theory of WCO, which arises by dint of how negation functions in DS.

3 An accessibility issue

The problem for Chierchia's account is a simple one — it can be observed that certain operators, such as negation, render their arguments inaccessible for the purposes of anaphora. This is illustrated in (25) — we use a Negative Polarity Item (NPI) to ensure that the expression takes scope below negation.

(25) *It's not true that any man[1] walked in. He_1 sat down. $\neg > \exists$

If an indefinite takes scope *above* negation, however, anaphora is possible, even if its surface position is below negation. This is illustrated in (26) — in fact, anaphora *disambiguates* in favor of the wide scope reading. This would seem to suggest that the indefinite may introduce a DR at its scope site, rather than at its thematic position, in conflict with the DPP.

(26) It's not true that a man[1] walked in. He_1 sat down. $\exists > \neg$

This however subverts Chierchia's explanation for WCO — there are two possible responses, if the indefinite is permitted to introduce a DR at its scope site, then we correctly account for the interaction with negation, but predict that such expressions should always obviate WCO. Chierchia suggests something along these lines for specific indefinites, but we'll argue that it can't be correct in the general case. Alternatively, one could maintain the DPP, in which case the interaction with negation remains puzzling. We'll ultimately argue that a version of Chierchia account of WCO can be salvaged, but this requires us to rethink the basic dynamic foundations upon which the system is built.

In order to understand exactly why this issue arises, we'll consider the status of negation in DS in somewhat more detail.

3.1 Negation as a destructive operation

In order to account for the paradigm in (25) and (26), the standard move in DS is to adopt an *externally static* entry for negation — i.e., to define negation in such a way that it is guaranteed to return a test.[13] The definition in (53) accomplishes this. Even if a DR is introduced in m, not m can only ever return $\{g\}$ or $\varnothing$, for a given input g.

(27) Externally static negation (def.)

$$\text{not } m \coloneqq \lambda g \,.\, \begin{cases} \{g\} & m\,g = \varnothing \\ \varnothing & \text{otherwise} \end{cases} \qquad \mathsf{T} \to \mathsf{T}$$

If we consider the Logical Form (LF) for a sentence involving an indefinite taking wide quantificational scope over negation, as in (28), we can see exactly how the problem will arise — the thematic function is responsible for introducing the DR, and since the thematic function is within the scope of negation, it in effect doesn't matter where the indefinite takes scope with respect to negation — the DR is always going to get wiped out.

(28) Some man λx [not [x^1 walked in]]
$[\![\text{some man}]\!]\ (\lambda x g \,.\, \{ g' \mid g' = g \land \{ g^{[1 \to x]} \mid \text{walked-in } x \} = \varnothing \})$

Since negation is fundamentally a *destructive* operation in DS, we can't tweak the entry for the indefinite in order to "resurrect" the DR introduced by the thematic function. The only option is to assume that indefinites can introduce DRS at their scope sites. We can modify the denotation of an indefinite as follows in order to cash this out (this is simply the standard semantics for indefinites in first-generation dynamic theories such as DPL):

(29) $$[\![\text{some man}^1]\!] \coloneqq \lambda k g \,.\, \bigcup_{\text{man } x} k\, x\, g^{[1 \to x]} \qquad (\mathsf{e} \to \mathsf{T}) \to \mathsf{T}$$

Naturally, this gives rise to the prediction that, if an indefinite can outscope negation, it obviates WCO. This prediction has some initial plausibility since, as pointed out by Chierchia (2020: 49), specific indefinites, which generally take wide scope over negation, seem to ameliorate WCO, as illustrated by the examples below (judgments due to Chierchia 2020):

(30) His$_1$ father hates { a boy^1 I know | a friend^1 of mine | a certain boy^1 }

This response will however not work in the general case. Much like indefinites, *wh-expressions* are externally dynamic, as illustrated by the sentence in (31):

(31) I know [which boy^1 Mary invited to the party and why he$_1$ was invited].

Furthermore, scoping a *wh*-expression over negation feeds anaphora, as illustrated by example (32). In order to account for this example, we'd need to pursue the same strategy as for indefinites, and allow *wh*-expressions to introduce DRS. It's fairly clear however that this makes extremely bad predictions for WCO, essentially predicting that *wh*-expressions should never give rise to WCO effects.

[13] Negation is not the only expression which is taken to be externally static — non-existential quantifiers, also, are typically taken to be externally static. We'll come back to those later.

(32) I know which boy^1 Mary didn't invite __ to the party, and why he$_1$ was excluded.

The problem here does not exclusively effect the account of WCO for expressions such as indefinites and *wh*-expressions. For other scope takers too, just so long as the thematic position is in the scope of negation, any DRS introduced will be rendered inaccessible.[14] We can construct an example in which this predicts the impossibility of a universal binding into an adjunct, just so long as the adjunct takes scope above negation. (33) has a salient reading which can be paraphrased as follows: *Each puppy is such that, after it peed on the carpet, John decided not to keep it.*. This involves negation in an embedded clause, with the adverbial attaching high. The universal, in turn, takes scope over both the embedding verb and the adverbial.

(33) John [decided [not to keep each of these puppies^1] [after it$_1$ peed on the carpet]].
$\forall > \text{decide} > \neg$

Surprisingly, Chierchia predicts that anaphora should in fact be impossible here. This is because a DR is introduced in the position corresponding to the *theme* of *keep*, rather than at the scope site of the universal. This DR should be wiped out by embedded negation prior to attachment of the adverbial clause.

Note that this problem isn't specific to negation — we can create examples illustrating the same point using other externally static operators, such as (non-indefinite) determiners. Like negation, determiners fail to destroy DRS introduced in a lower thematic position. Rather, the *dynamic* scope of the indefinite is at least as high as its quantificational scope.

(34) Most girls danced with a boy^1. He$_1$ was pleased. $\exists > \text{most}$

4 Rethinking external staticity

In this section, we'll suggest a way to fix these bad predictions while retaining Chierchia's basic account of WCO by tweaking the semantics of negation. The idea will be as follows: negation, and similar operators, are externally static with respect to *indeterminate* DRS, but externally dynamic with respect to *determinate* DRS.[15] To illustrate, consider the meaning of the sentence "A girl^1 likes Jeff^2" in DS, where both Determiner Phrases (DPS) introduce DRS (n.b., we're abstracting away from events and other complications here). If there are three girls who like Jeff, namely *Shirley*, *Britta*, and *Annie*, then the dynamic proposition expressed by the sentence is as follows:

[14]Thanks to Filipe Hisao Kobayashi (p.c.) for drawing my attention to examples such as this.

[15]In antecedent unpublished work, Simon Charlow conjectures an entry for dynamic negation that collapses nondeterminism in its argument. The proposal advanced in this paper is inspired by Charlow's, but differs in the following respects: (i) our entry for negation is simply classical negation lifted into a dynamic setting, (ii) collapse of non-deterministic information is instead built into the semantics of *indefinites*, via the positive closure operator. I'm grateful to Simon Charlow (p.c.) for sharing his work with me.

(35) Context: *Three girls like Jeff*
$[\![\text{A girl}^1 \text{ likes Jeff}^2]\!] = \lambda g \,.\, \{\, g^{[1\to\text{shirley},2\to\text{jeff}]}, g^{[1\to\text{britta},2\to\text{jeff}]}, g^{[1\to\text{annie},2\to\text{jeff}]} \,\}$

Since the standard entry for negation in DS is destructive — applying it in this instance would eliminate all DRS, including 2. Rather, we want negation to eliminate all *indeterminate* DRS (here 1), and but retain determinate DRS (here 2). We want the effect of applying negation to be as follows:

(36) $\lambda g \,.\, \{\, g^{[2\to\text{jeff}]} \,\}$

It's not clear how to achieve this in a first-generation DS. The reason for this is as follows — in classical DS, we only distinguish between assignments that are in the output set, and those that are not. If there are no girls that like Jeff, then "A girl^1 likes Jeff^2" will simply return the empty set, for any input assignment g. There is no obvious way to get back the information associated with the DR 2. In order to address this issue, we will need to adopt a more expressive dynamic system — the monadic DS of Charlow (2014, 2019a).

4.1 From classical DS to monadic DS

We'll now shift gears and introduce a system with somewhat more expressive power than classical DS — this is because classical DS only distinguishes between assignments that are in the output set, and those that are not. We will rather need a three-way distinction between assignments that are paired with *true*, in the output set, those that are paired with *false*, and those absent from the output set. Charlow's (2014) monadic DS delivers exactly this design feature.[16]

In monadic DS, rather than defining the type of a dynamic proposition directly, we provide a recipe for mapping an "ordinary" type to its dynamic counterpart. Technically speaking, this makes use of the `State.Set` monad, but the formal niceties will be largely unimportant for our purposes. What will be important here is that, in monadic DS, we keep track of ordinary semantics values, while simultaneously engaging in the familiar acrobatics of DS.

(37) The `State.Set` type constructor (def.)
$\mathsf{S\ a} \coloneqq \mathsf{g} \to \{\, (\mathsf{a}, \mathsf{g}) \,\}$

The type of a dynamic proposition is just what we get if we apply the recipe in (37) to type t — the type of a truth value. In monadic DS, therefore, a dynamic proposition maps an input assignment to a set of truth-value-assignment pairs. This is illustrated in (38).

(38) $\mathsf{T} \coloneqq \mathsf{g} \to \{\, \mathsf{g} \,\}$ Type of a dynamic proposition in classical DS
$\mathsf{S\ t} \coloneqq \mathsf{g} \to \{\, (\mathsf{t}, \mathsf{g}) \,\}$ Type of a dynamic proposition in monadic DS

[16] Although this is relatively orthogonal to the central point here, n.b. that the expressive power afforded by the monadic approach is independently motivated, since it provides a compelling account of exceptional scope phenomena. Unlike other theories of exceptional scope based on pseudo-scope mechanisms such as *choice functions*, Charlow's theory accounts for the *binder roof constraint* (Brasoveanu & Farkas 2011).

In monadic DS, the counterpart of a *test* returns either $\{(\top, g)\}$ or $\{(\bot, g)\}$ for a given input g. This is illustrated for a simple sentence in (39). If the conditions are satisfied (namely, if *John left*), the test returns the (singleton set containing) *true-tagged* input assignment. If the conditions aren't satisfied, in returns the false-tagged input assignment — a classical test, by way of contrast, would simply return the empty set in the latter instance.

(39) $$[\![\text{John left}]\!] = \lambda g \,.\, \{(\text{j left}, g)\} =: \lambda g \,.\, \begin{cases} \{(\top, g)\} & \text{John left} \\ \{(\bot, g)\} & \text{otherwise} \end{cases} \qquad \text{S t}$$

We can retrieve the classical dynamic proposition from a monadic proposition by simply filtering out all the false tagged assignments — for illustrative purposes, we define a helper function $\mathcal{C}$ to do just this in (40). We retrieve the classical test from the monadic test in (41).

(40) Converting from a monadic DS proposition to a classical DS proposition:
$$\mathcal{C}\ p = \lambda g \,.\, \{g \mid (g, \top) \in (p\ g)\} \qquad \text{S t} \to \text{T}$$

(41) $$\mathcal{C}\ (\lambda g \,.\, \{(\text{j left}, g)\}) = \lambda g \,.\, \begin{cases} \{g\} & \text{John left} \\ \varnothing & \text{otherwise} \end{cases} \qquad \text{T}$$

In monadic DS, just as in classical DS, a sentence with an indefinite, such as "A boy left", induces an indeterminate output. Unlike in classical DS, we keep track of both the *verifiers* (i.e., the boys that left) and the *falsifiers* (i.e., the boys that didn't leave). This is illustrated in (42): true-tagged assignments map 1 to a boy who left, and false-tagged assignments map 1 to a boy who didn't leave. In classical DS, on the other hand, we only retain assignments corresponding to verifiers. Again, applying $\mathcal{C}$ to the meaning in (42) returns the corresponding meaning in classical DS. This is shown in (43).

(42) $$\begin{aligned} [\![\text{a boy}^1\ \text{left}]\!] &= \lambda g \,.\, \{(\text{left}\ x, g^{[1\to x]}) \mid \text{boy}\ x\} \qquad \text{S t} \\ &=: \lambda g \,.\, \bigcup_{\text{boy}\ x} \begin{cases} \{(\top, g^{[1\to x]})\} & \text{left}\ x \\ \{(\bot, g^{[1\to x]})\} & \text{otherwise} \end{cases} \end{aligned}$$

(43) $$\mathcal{C}\ (\lambda g \,.\, \{(\text{left}\ x, g^{[1\to x]}) \mid \text{boy}\ x\}) = \lambda g \,.\, \{g^{[1\to x]} \mid \text{boy}\ x \wedge \text{left}\ x\} \qquad \text{T}$$

Just as an classical DS, in monadic DS sentences with pronouns induce input sensitivity. We leave it as an exercise to the reader to retrieve the corresponding classical dynamic proposition.

(44) $$[\![\text{he}_1\ \text{was upset}]\!] = \lambda g \,.\, \{(\text{upset}\ g_1, g)\} \qquad \text{S t}$$

Dynamic sequencing behaves just as in classical DS, only in additional to passing outputs from the first conjunct into the second, it conjoins the contained truth values.[17]

[17] In a monadic setting, it isn't in fact necessary to define sequencing directly – it arises through the interaction of classical conjunction and exceptional scope mechanisms. See Charlow (2019a) for details. We do so here only be illustrative purposes.

(45) $m \mathbin{;} n \coloneqq \lambda g \,.\, \bigcup_{(t,g')\in m\,g} \{\, (t \wedge u, g'') \mid (u,g'') \in n\,g' \,\}$ $\qquad$ $\mathsf{S\,t} \to \mathsf{S\,t} \to \mathsf{S\,t}$

Let's now consider a concrete example of cross-sentential anaphora in monadic DS, to get a feel for how things work. Note that we keep track of both the *verifiers* (here: the women who walked in and sat down), and the *falsifiers* (here: the woman who didn't walk in but sat down).

(46) Context: *Only Britta and Shirley walked in, but Britta, Shirley, and Annie all sat down.*

$(⟦\text{a woman}^1 \text{ walked in}⟧ \mathbin{;} ⟦\text{she}_1 \text{ sat down}⟧)\, g_\varnothing$

$$= \bigcup_{(g',t)\in⟦\text{a woman}^1 \text{ walked in}⟧\, g_\varnothing} \{\, (t \wedge u, g'') \mid (u,g'') \in ⟦\text{she}_1 \text{ sat down}⟧\ g' \,\}$$

$$= \{\, (\top \wedge u, g'') \mid (u,g'') \in ⟦\text{she}_1 \text{ sat down}⟧\ [1 \to \mathsf{s}] \,\}$$
$$\cup \{\, (\top \wedge u, g'') \mid (u,g'') \in ⟦\text{she}_1 \text{ sat down}⟧\ [1 \to \mathsf{b}] \,\}$$
$$\cup \{\, (\bot \wedge u, g'') \mid (u,g'') \in ⟦\text{she}_1 \text{ sat down}⟧\ [1 \to \mathsf{a}] \,\}$$
$$= \{\, (\top, [1 \to \mathsf{s}]), (\top, [1 \to \mathsf{b}]), (\bot, [1 \to \mathsf{a}]) \,\}$$

4.2 Refining the semantics of indefinites

Now that we have a better understanding of monadic DS at the sentential level, we will need to rethink our assumptions concerning the meanings of sentences with indefinites. First, let's zoom in to the subsentential level, and consider the standard semantics for indefinites in monadic DS, after Charlow (2014, 2019a).[18]

(49) Indefinites in monadic DS (first attempt)

$⟦\text{a boy}^1⟧ = \lambda k g \,.\, \bigcup_{\text{boy}\, x} k\, x\, g^{[1\to x]}$ $\qquad$ $(\mathsf{e} \to \mathsf{S\,t}) \to \mathsf{S\,t}$

Indefinites compose with a scope and return a function from an input assignment, to a (potentially) heterogeneous set of true- and false-tagged assignments. This is illustrated in (50).

(50) Context: *Of the boys, only Troy and Abed left*

$⟦\text{a boy}^1 \text{ left}⟧ = ⟦\text{a boy}^1⟧\ (\lambda x g \,.\, \{\, (\mathsf{left}\, x, g) \,\})$ $\qquad$ $\mathsf{S\,t}$

$= \lambda g \,.\, \{\, (\top, g^{1\to \mathsf{troy}}, (\top, g^{[1\to \mathsf{abed}]}), (\bot, g^{[1\to \mathsf{jeff}]})) \,\}$

[18] As discussed in detail by Charlow, the meaning in (49) can be decomposed into the basic meaning for the indefinite, given in (47), and monadic bind ($\star$), defined in (48):

(47) $\lambda g \,.\, \{\, (x, g^{[1\to x]}) \mid \mathsf{boy}\, x \,\}$ $\qquad$ $\mathsf{S\,e}$

(48) $m^\star \coloneqq \lambda k g \,.\, \bigcup_{(x,g')\in m\, g} k\, x\, g'$ $\qquad$ $(\mathsf{a} \to \mathsf{S\,b}) \to \mathsf{S\,b}$

We abstract away from this here, but this decomposition plays a *crucial* role in Charlow's monadic DS, since the availability of bind directly accounts for the exceptional scoping abilities of indefinites.

Recall that we ultimately want negation to be externally dynamic, at least selectively. In monadic DS, there's a very straightforward way of achieving this by simply lifting propositional negation into a dynamic setting. The definition is given in (53) — externally dynamic negation simply flips the contained truth-values in the output set.[19]

(53) Externally dynamic negation (def.)
$\text{not}\ m := \lambda g\ .\ \{\,(\neg\, t, g') \mid (t, g') \in m\ g\,\}$ $\qquad \mathsf{S\,t} \to \mathsf{S\,t}$

Perhaps we can simply adopt the semantics for negation in (53) and be done with it. Unfortunately, this will make bad predictions in tandem with the semantics for indefinites given in (49). This is because the entry for negation in (53) is externally dynamic with respect to *all* discourse referents. In effect, it simply allows the dynamic scope of an indefinite to pass straight through it. This is illustrated below:

(54) $\text{not}\ [\![\text{someone}^1\ \text{left}]\!] = \text{not}\,(\lambda g\ .\ \{\,(\text{left}\ x, g^{[1\to x]}) \mid \text{person}\ x\,\})$
$= \lambda g\ .\ \{\,(\neg\,(\text{left}\ x), g^{[1\to x]}) \mid \text{person}\ x\,\}$

This won't do at all — as we've seen, if an indefinite takes scope below negation, negation roofs its potential to bind. In order to avoid this result, the strategy we'll pursue here is one of tweaking the semantics of indefinites, while maintaining that negation is simply externally dynamic. Intuitively, what is going wrong here is that a sentence with an indefinite may give rise to a heterogeneous output set, consisting of both true-tagged and false-tagged assignments. Rather, we need to ensure that a sentence with an indefinite only *ever* outputs either (a) a set of true-tagged assignments, or (b) a set of false-tagged assignments.

4.2.1 Positive & negative extension

Towards this aim, we'll first define a helper function: the *positive extension* operator filters out the false-tagged assignments (55a). It is of course straightforward to define the negative counterpart, which we do in (55b) (this will be useful when we come to define discussing disjunction).

[19] Why might we want t define negation in this way? There's actually a strong conceptual motivation for doing so. It turns out that the entry in (53) is exactly what we get if we *map* plain old propositional negation into a dynamic setting — since S is a functor, the recipe for achieving this mapping can be defined straightforwardly for any function:

(51) $\text{map}\ f := \lambda m\, \lambda g\ .\ \{\,(f\ x, g') \mid (x, g') \in m\ g\,\}$ $\qquad \text{map} : (\mathsf{a} \to \mathsf{b}) \to \mathsf{S\,a} \to \mathsf{S\,b}$

(52) $\text{map}\ (\neg) =: (53)$

A criticism often leveled at DS is that it precompiles the desired behavior directly into the meanings of the logical connectives (see, e.g., Schlenker 2009 for discussion). Here however, all we've done is lifted classical negation into a dynamic setting via a totally general and logically well-behaved operation.

(55) Positive and negative extension (def)

a. $p^{+} := \{ (\top, g') \mid (\top, g') \in p \}$ $\qquad + : \{ \mathsf{g}, \mathsf{t} \} \to \{ \mathsf{g}, \mathsf{t} \}$

b. $p^{-} := \{ (\top, g') \mid (\top, g') \in p \}$ $\qquad - : \{ \mathsf{g}, \mathsf{t} \} \to \{ \mathsf{g}, \mathsf{t} \}$

4.2.2 Positive & negative collapse

We'll now define another auxiliary operation in terms of the helper function in (55a): the *positive collapse* operation †. The intuition behind the positive collapse operation is as follows — it takes a dynamic proposition, which could, in principle, output a heterogenous set of true- and false-tagged assignments. If there are any verifiers, it simply filters out the falsifiers, but if there *aren't* any verifiers it collapses the modified assignments associated with the falsifiers into a single output assignment, thereby eliminating any indeterminacy from the output.

(56) Positive collapse operation (def.)

$$m^{\dagger} := \lambda g \,.\, \begin{cases} (m\ g)^{+} & (m\ g)^{+} \neq \varnothing \\ \{ (\bot, \bigcap \{ g \mid (g, \bot) \in (m\ g) \}) \} & \text{otherwise} \end{cases} \qquad \mathsf{S\,t} \to \mathsf{S\,t}$$

What does it mean to take the *intersection* of a set of assignments? We can take assignment functions to be equivalent to their graphs, i.e., sets of variable-value pairs. This is illustrated below for an assignment mapping the variables $\{ 1, 2, 3 \}$ to boys.

$$\begin{bmatrix} 1 \to \mathsf{jeff} \\ 2 \to \mathsf{troy} \\ 3 \to \mathsf{abed} \end{bmatrix} =: \begin{Bmatrix} (1, \mathsf{jeff}), \\ (2, \mathsf{troy}), \\ (3, \mathsf{abed}) \end{Bmatrix}$$

Intersecting two assignments therefore amounts to set intersection of their graphs.[20] This will have the desired effect of preserving determinate DRs, while eliminating indeterminate DRs. Suppose we have a set of assignments which agree on 1 and 2, but differ on 3 — Intersecting their graphs returns an assignment that is undefined at 3, but which preserves information about the determinate DRs.

$$\bigcap \left\{ \begin{Bmatrix} (1, \mathsf{troy}) \\ (2, \mathsf{abed}) \\ (3, \mathsf{shirley}) \end{Bmatrix}, \begin{Bmatrix} (1, \mathsf{troy}) \\ (2, \mathsf{abed}) \\ (3, \mathsf{britta}) \end{Bmatrix}, \begin{Bmatrix} (1, \mathsf{troy}) \\ (2, \mathsf{abed}) \\ (3, \mathsf{annie}) \end{Bmatrix} \right\} = \begin{Bmatrix} (1, \mathsf{troy}) \\ (2, \mathsf{abed}) \end{Bmatrix}$$

Just as in the previous section, it is straightfoward to define a negative collapse operation ‡ — the negative counterpart of positive collapse. This simply returns the false-tagged assignments, if there are any; if not, it collapses the true-tagged assignments via intersection, thereby eliminating any indeterminacy.

[20] I'm grateful to Yasu Sudo (p.c.) for suggesting this simple formulation in terms of graph intersection.

(57) Negative collapse (def.)

$$m^{\ddagger} := \begin{cases} (m\ g)^{-} & (m\ g)^{-} := \varnothing \\ \{(\top, \bigcap\{g \mid (g, \top) \in (m\ g)\})\} & \text{otherwise} \end{cases} \qquad \mathsf{S\,t} \to \mathsf{S\,t}$$

4.2.3 The revised entry for indefinites

We can now revise our semantics for indefinites in terms of (a) Charlow's semantics for indefinites, and (b) the positive collapse operator. The idea behind the new entry will be as follows: if the output set contains a true-tagged assignment, return its positive extension, and otherwise, return the (singleton set of the) *grand intersection* of the false-tagged assignments, and tag it false. The reader may wonder at this point why we don't just return the false-tagged assignments. This will in fact be crucial for explaining why only *determinate* DRS may survive, as we'll see shortly.[21]

(58) Indefinites in monadic DS (second attempt)

$$[\![\text{a}^{n}\ \text{boy}]\!] := \lambda k\ .\ \left(\lambda g\ .\ \bigcup_{\mathsf{boy}\ x} k\ x\ g^{[1 \to x]}\right)^{\dagger}$$

4.3 Negation and crossover

We've done a lot of work in order to define indefinites in a way which seems rather ad hoc, but as we'll see, this new definition will have number of advantages. Originally, the problem with Chierchia's account of WCO arose due to the assumption that negation is fundamentally a *destructive* operation, taken from first-generation dynamic theories. In our new, monadic setting, we can simply make use of externally dynamic operation, as defined in (53), and repeated below.

(59) Externally dynamic negation (def.)

$$\mathsf{not}\ m := \lambda g\ .\ \{(\neg\, t, g') \mid (t, g') \in m\ g\} \qquad \mathsf{S\,t} \to \mathsf{S\,t}$$

With our refined semantics for indefinites, this simplistic treatment of negation will pay dividends. On the one hand, due to the way that indefinites have been (re-)defined, we will retain the basic results of classical DS — namely, negation will still render any DRS introduced by an indefinite inaccessible[22] On the other hand, determinate DRS will survive. As we'll see later, an added bonus of this system will be an account of double-negation — a recalcitrant problem for classical DS.

[21] One potentially worrying consequence of this move is that the decomposition mentioned in footnote 18 no longer goes through. It's possible that one could think of the positive collapse operation not as something built into lexical entry for the indefinite article, but rather as a covert operation inserted at LF, subject to certain constraints. We leave this question to future research.

[22] This statement has an important caveat – if the indefinite introduces a *determinate* DR, it will survive.

4.3.1 Negation roofs the dynamic scope of indefinites

First off, let's make sure that we get the basic static truth-conditions correct. We can retrieve a classical notion of truth relative to assignment from a dynamic proposition by checking whether the output contains at least one true-tagged assignment.[23] Now, consider the meaning of "A boy^1 left", where Troy and Abed left, but Jeff didn't. Since the positive extension is non-empty, this sentence will *only* output true-tagged assignments, given the entry for indefinites in (58):

(61) $[\![\text{A boy}^1 \text{ left}]\!] = \lambda g \,.\, \{ (\top, g^{[1\to\mathsf{troy}]}), (\top, g^{[1\to\mathsf{abed}]}) \}$ $\quad$ S t

Negating the sentence will simply flip the contained truth values, so the sentence will come out false, as desired.

(62) Context: *Of the boys, only Troy and Abed left.*
$\mathsf{not}\,([\![\text{a boy}^1 \text{ left}]\!]) = \lambda g \,.\, \{ (\bot, g^{[1\to\mathsf{troy}]}), (\bot, g^{[1\to\mathsf{abed}]}) \}$ $\quad$ S t

Now, consider what happens if the facts are such that *no boy left*. First, let's consider the output set induced by Charlow's original semantics for the indefinite, without the positive collapse operator.

(63) Context: *No boys left*
$\lambda g \,.\, \{ (\mathsf{left}\ x, g^{[1\to x]}) \mid \mathsf{boy}\ x \} \coloneqq \lambda g \,.\, \{ (\bot, g^{[1\to \mathsf{j}]}), (\bot, g^{[1\to \mathsf{t}]}), (\bot, g^{[1\to \mathsf{a}]}) \}$

In order to compute the meaning of the sentence "A boy^1 left", we first filter out the true-tagged assignments — since there are none, we just get back the same meaning. Based on how the indefinite is defined in our new setting, the output of "A boy^1 left" should be the grand intersection of the false-tagged assignments. Since the output assignments disagree only in 1, the output simply contains g (in other words, we end up with a false-tagged test).

(64) Context: *No boys left.*
$[\![\text{a boy}^1 \text{ left}]\!] \coloneqq \lambda g \,.\, (\bot, g)$

Applying negation simply flips the contained truth-value — the result is a true-tagged test. We therefore retain a core result of DS, that negation roofs the dynamic scope of indefinites.

(65) Context: *No boys left.*
$\mathsf{not}\ [\![\text{a boy}^1 \text{ left}]\!] = \lambda g \,.\, (\top, g)$

[23] We can define this formally as an operation that existentially closes the output set.

(60) $\mathsf{clo}\ m \coloneqq \lambda g \,.\, \exists g' [(\top, g') \in m\ g]$ $\quad$ $\mathsf{clo} : \mathsf{S\,t} \to \mathsf{g} \to \mathsf{t}$

4.3.2 The dynamic translucency of negation

This is all well and good, but so far we haven't done anything above and beyond classical DS. The interesting part comes when alongside an indefinite, we also have some determinate DR introduced. Consider the sentence "A boy[1] likes Annie[2]" in a context where no boy in fact likes Annie. Again, let's start out with the meaning predicted by Charlow's semantics, and just take the false-tagged assignments:

(66) $\lambda g\,.\,\{\,(\bot, g^{[1\to\text{jeff},2\to\text{annie}]}), (\bot, g^{[1\to\text{troy},2\to\text{annie}]}), (\bot, g^{[1\to\text{abed},2\to\text{annie}]})\,\}$

To compute the meaning of "A boy[1] likes Annie[2]" in this context, recall that we first check whether the positive extension is empty. Since it is, we take the grand intersection of the false-tagged assignments. Since all the output assignments map 2 to Annie, but disagree on 1, the unique such assignment is one that is just like g, but maps 2 to Annie.

(67) Context: *No boys like Annie*
$[\![\text{A boy}^1 \text{ likes Annie}^2]\!] = \lambda g\,.\,\{\,(\bot, g^{[2\to a]})\,\}$

Negating the resulting meaning simply flips the resulting meaning, and the result is no longer a test! We correctly predict that determinate DRS can survive negation. We can think of negation as being a dynamically *translucent* operator — it allows only determinate DRS to survive — namely, only those that all output assignments agree upon.

(68) Context: *No boys like Annie*
not $[\![\text{A boy}^1 \text{ likes Annie}^2]\!] = \lambda g\,.\,\{\,(\top, g^{[2\to a]})\,\}$

4.3.3 Applying the results to crossover

On the dynamic regime adopted here, we can now resolve the problem for Chierchia's account of WCO. For the purposes of illustration, in the previous section we attributed DR-introduction to indefinites directly. Since we want to maintain Chierchia's account of WCO, the revised entry for indefinites is identical to the definition in (58), only it doesn't introduce a discourse reference (this, as on Chierchia's account, will be the preserve of the thematic position).

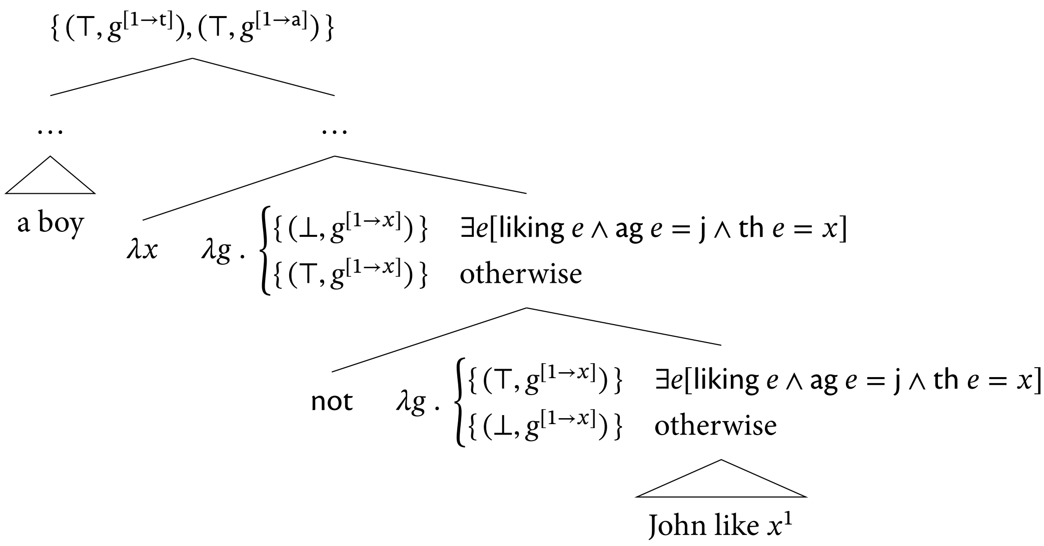
(69) Indefinites in monadic DS (final ver.)

$$[\![\text{a}^n \text{ boy}]\!] \coloneqq \lambda k\,.\left(\lambda g\,.\bigcup_{\text{boy}\,x} k\,x\,g\right)^{\dagger}$$

Recall that examples where indefinites take scope over negation were problematic for Chierchia's account. The insight here is that, if the indefinite takes wide scope, at the point in the derivation at which negation composes, the DR introduced by the thematic position is *determinate*, and therefore survives. We can therefore retain Chierchia's account of WCO just so long as we couch it in terms of the refined dynamic system outlined previously. The task of lifting

Chierchia's dynamic event semantics is a largely mechanical one, and therefore the details are elided.[24]

Figure (4): A boy John doesn't like __[1]
(context: *John likes Troy and Abed but not Jeff*)



5 Bathrooms and double-negation

5.1 Addressing the problem of double negation

An independent motivation for the variation on monadic DS outlined here is a principled account of two otherwise recalcitrant problems for first generation dynamic theories — double negation and (relatedly) so-called *bathroom sentences* (see, e.g., Krahmer & Muskens 1995, Gotham 2019 for discussion). We'll begin with a discussion of double negation, since our solution to this problem will be rather simple, and it will underlie our subsequent account of bathroom sentences. As is well known, classical DS with equipped with externally static negation cannot handle anaphora in discourses such as the following:[25]

(71) John doesn't own no car[1]; it_1's in the garage.

[24]More concretely, We can simply replace instances of *dynamic lift* with a more general operation *monadic return*, defined in (70).

(70) Monadic return (def.)
$a^{\eta} \coloneqq \lambda g\,.\,\{(a, g)\}$ $\qquad$ $a \to S\,a$

[25]We assume here that, at LF negative indefinites are decomposed into a negative and an existential component.

Perhaps unexpectedly, an independent motivation for the semantics of indefinites and negation outlined here is that it provides a straightforward account of (71). This is because, according to the revised semantics for dynamic negation adopted here, all it does is flip the contained truth-values in the output set. Recall furthermore that, indefinites return the set of true-tagged semantics, if non-empty. This means that "John owns a car[1]" returns the following, if the facts are such that John owns exactly one car – the Ferrari (f):

(72) Context: *John owns a ferrari, and no other cars*
$[\![\text{John owns a car}^1]\!] \coloneqq \lambda g \,.\, \{ (\top, g^{[1 \to f]}) \}$

If we apply externally static negation to this meaning *twice*, all it will be will flip the contained truth value twice, giving back...an identical meaning:

(73) $\textsf{not}\,(\textsf{not}\,(\lambda g \,.\, \{ (\top, g^{[1 \to f]}) \})) = \textsf{not}\,(\lambda g \,.\, \{ (\bot, g^{[1 \to f]}) \}) = \lambda g \,.\, \{ (\top, g^{[1 \to f]}) \}$

5.2 Dynamic sequencing and positive collapse

In light of the discussion of the double negation, the attentive reader will have noticed that we still need to tweak the standard monadic entry for dynamic sequencing/conjunction in order to avoid some bad predictions. As it stands, we predict that anaphora should succeed in sentences like (74). The reason why this is an issue is as follows: the standard monadic entry for dynamic sequencing feeds each assignment that the first conjunct outputs, be it true-tagged, or false-tagged into the second conjunct. It follows that a false-tagged assignment outputted by the first conjunct can license anaphora in the second, and the result of the entire conjunction can be flipped by negation, resulting in a true sentence.

(74) It's false [that there's no[1] bathroom, and it_1's upstairs].

The solution we adopt here is straightforward: conjunction only feeds outputs in the *positive collapse* of the first conjunct into the second.

(75) Revised dynamic sequencing (def.)
$m \,;\, n \coloneqq \lambda g \,.\, \bigcup_{(t,g') \in m^{\dagger}\, g} \{ (t \wedge u, g'') \mid (u, g'') \in n\, g' \}$ $\qquad \textsf{S t} \to \textsf{S t} \to \textsf{S t}$

This ensures that, in the problematic example (74), as long as there is more than one bathroom, DR 1 is eliminated by the positive collapse of the first conjunct, and anaphora will fail.

5.3 Bathroom sentences

The problem of double negation also infects classical DS in another guise — namely, the problem of so-called "bathroom sentences" (after Partee), as in (76a). In a disjunctive sentence, an indefinite under negation in the first disjunct apparently licenses anaphora in the second. Based on independent evidence from presupposition projection in sentences such as (77), much work in classical DS has converged on the idea that in a sentence of the form "ϕ or ψ", ψ is interpreted in the context of the negation of ϕ. It is tempting to try to extend this explanation to (76a), in which case we could paraphrase (76a) as in (76b). In a classical dynamic theory this is difficult to cash out however since, as we've seen, double negation elimination isn't valid.

(76) a. Either there isn't a bathroom^1, or it$_1$'s upstairs.

b. Either there isn't a bathroom, or there isn't not a bathroom, and it's upstairs.

(77) Either John has never smoked, or he stopped smoking recently.

Now that we have a dynamic system in which double negation *is* valid however, we can cash out the idea implicit in the paraphrase in (76b). The most straightforward way of doing so is to build the desired behavior directly into the lexical entry for propositional disjunction. An attempt at this is given in (78).[26]. On our revised semantics, disjunction involves taking the union of two output sets — computing the first output set is straightforward: we simply take the outputs of the first disjunct relative to the input assignment. Computing the second output set is more complex: if the first disjunct outputs *any* false-tagged assignments, then we gather up the results of feeding them pointwise into the second disjunct; otherwise, we gather up the results of feeding the intersection of the true-tagged assignments into the second disjunct. Note the parallel with our entry for indefinites.

(78) Disjunction in monadic dynamics (def.)

$$m \text{ or } n \coloneqq \lambda g\,.\, m\,g \cup \bigcup_{g' \in m^{\ddagger}\,g} n\,g'$$

Why this fancy meaning? Well, first off we don't want to allow anaphora from the first disjunct into the second, in the general case as illustrated by the oddness of (79). The semantics for disjunction suggested in (78) blocks this. Let's go through the reasoning informally: given our semantics for indefinites, if there are verifiers of the first disjunct, it will return a set of true-tagged modified assignments, i.e., the negative extension of the first disjunct will be empty. In this case, we feed the the intersection of the true tagged assignments into the second disjunct (due to the *negative collapse* operation). As long as there is more than one bathroom, intersecting the assignments wipes out any indeterminate *bathroom* DR, and anaphora will thereby fail. If there are no verifiers, then the first disjunct outputs the intersection of the false-tagged assignments, eliminating any information about the bathroom DR; feeding this into the second disjunct again fails to resolve anaphora.

(79) #Either there is a bathroom or it's upstairs.

Note there is a prediction that antecedents which introduce *determinate* DRS in the first disjunct will still be accessible from the second. This is because intersecting assignments leaves

[26]Note that this is similar to the non-deterministic semantics for disjunction proposed in Charlow 2014: §4.6.2 (see also Groenendijk & Stokhof's 1991 *program disjunction*). Like Charlow, we'll assume that disjunction *induces indeterminacy* by introducing a set of alternatives. Charlow's entry is internally static however, and therefore doesn't say (or aim to say) anything about bathroom sentences.

determinate DRS unscathed, as we've already seen. At first blush, this seems like a good result, based on sentences such as (80). It's not immediately clear what is responsible for the "bound" reading in (80) however; since the antecedent is a definite, this could merely be accidental coreference.

(80) Either John[1] is downstairs, or he_1's upstairs.

We can attempt to diagnose the possibility of binding from the first disjunct into the second by constructing an elliptical sentence with a sloppy reading, as in (81). (81) has a reading (the one we're interested) in, which entails that *Sally thinks that Sally's brother is either downstairs or upstairs*. As is well known, in order for the parallelism condition on ellipsis to be satisfied under the sloppy reading, the ellipsis antecedent must involve *binding* (Williams 1977 and subsequent work). The fact that the sloppy reading is available here suggests that *Mary* can bind into *her brother*, which in turn binds the pronoun in the second disjunct.[27]

(81) Mary[1] thinks that either [her_1 brother][2] is upstairs, or he_2's outside,
and Sally does Δ too.

Returning to the analysis of the bathroom sentence (repeated below in (82)), let's see how the revised entry for disjunction in (78) accounts for the possibility of anaphora. We would like a (76a) to be true just in case there is no bathroom, or there is a bathroom and it's upstairs. We go through a concrete illustration of the first scenario in (83). Since there are no verifiers, the first disjunct simply returns the singleton of the pair *true* and the initial assignment, as shown in (83a). Since the first disjunct has an empty negative extension, we intersect the assignments in the positive extension, as in (83b); these are fed pointwise into the second disjunct. All this results in here is the initial assignment. Feeding this into the second disjunct is simply undefined. Assuming existential projection, the disjunctive sentence ends up true, and introduces no discourse referents, as desired, as in (83d).

(82) Either there isn't a bathroom[1], or it_1's upstairs.

(83) Context: *There is no bathroom*
  a. $[\![\text{there isn't a bathroom}^1]\!]\ g_\varnothing = \{(\top, g_\varnothing)\}$
  b. $\bigcap\{g \mid g \in \{(\top, g_\varnothing)\}\} = g_\varnothing$
  c. $[\![\text{it}_1\text{'s upstairs}]\!]\ g_\varnothing = \#$
  d. $([\![\text{there isn't a bathroom}^1]\!]\ \text{or}\ [\![\text{it}_1\text{'s upstairs}]\!])\ g_\varnothing = \{(\top, g_\varnothing)\}$

Moving on to the second scenario — there is a bathroom, and it's upstairs. First, we compute the output of the first disjunct relative to the initial assignment; since there is one verifier, this

[27] A note of caution here — given that the kind of monadic grammar we've adopted has a very general mechanism for facilitating *exceptional scope* (monadic bind: see Charlow 2014, 2019b,a for details), there's a concern that, in the antecedent, "her brother" takes exceptional scope out of the disjunctive sentence, and binds into the second disjunct from its scope site. We leave it to future work to disentangle the various factors potentially at play here.

will be a singleton set containing a false-tagged assignment mapping 1 to the bathroom, as in (84a). To compute the meaning of the disjunctive sentence, we take the union of (a) the first disjunct (84a), and the union of the results we get from feeding the negative extension of (84b) into the second disjunct; since we have a single false-tagged assignment mapping 1 to the bathroom, anaphora is successful.

(84) Context: *There is exactly one bathroom (b) and it's upstairs.*

a. $[\![\text{there isn't a bathroom}^1]\!]\, g_\varnothing = \{\, (\bot, [1 \to \mathsf{b}]) \,\}$

b. $[\![\text{it}_1\text{'s upstairs}]\!]\, [1 \to \mathsf{b}] = \{\, (\top, [1 \to \mathsf{b}]) \,\}$

c. $([\![\text{there isn't a bathroom}^1]\!] \text{ or } [\![\text{it}_1\text{'s upstairs}]\!])\, g_\varnothing$

$= \{\, (g, t) \mid (g, t) \in (\{\, (\bot, [1 \to \mathsf{b}]) \,\} \cup \{\, (\top, [1 \to \mathsf{b}]) \,\}) \,\}$

$= \{\, (\bot, [1 \to \mathsf{b}]), (\top, [1 \to \mathsf{b}]) \,\}$

Note that, even if it's false that the bathroom is upstairs, the disjunctive sentence still makes available a false-tagged modified assignment, since there is a verifier. This makes the (good) prediction, that anaphora should be successful in the following:

(85) Neither [is there no bathroom^1, nor is it$_1$ is upstairs]. In fact, it$_1$'s downstairs.

6 Extensions

6.1 Uniqueness

One potential worry for this account is that, as discussed by Gotham (2019), anaphora from under double negation seems to give rise to a uniqueness inference.

(86) a. John owns a shirt^1. It$_1$'s hanging up. The rest are in the closet.

b. John doesn't own no shirt^1. It$_1$'s hanging up. ??The rest are in the closet.

I don't have much to add here, other than to note that there seem to be exceptions to uniqueness, involving maximal reference, as illustrated in (87). Indeed, Gotham does not have a principled explanation for the uniqueness effect, and it seems reasonable to conclude that this phenomenon is still poorly understood.

(87) John doesn't own no shirt^1. They're in the closet.

See also Krahmer & Muskens (1995), who arguе that bathroom sentences have *universal* readings. I don't take a principled stance on this issue here.

6.2 Related work

The fragment outlined in the latter part of this paper is *distributive* and *non-eliminative*, just like, e.g., Groenendijk & Stokhof's (1991) DPL. One may wonder whether either of these logical properties are crucial for validating DNE within a genuinely dynamic setting.

In related work, Elliott (2020) develops a dynamic semantics with non-distributive updates, which uses similar techniques to the fragment outlined here in order to validate DNE. Briefly, sentential updates are analyzed as functions from an input state — a *proposition-assignment* pair — to a set of truth-value, world, assignment triples; the output of an update thereby distinguish between validating world-assignment pairs, and falsifying world-assignment pairs. This allows for a semantics of indefinites which mirrors the strategy adopted here, as illustrated in (90) — indefinites only introduce indeterminate DRS in worlds in which there is a verifier. As shown in detail by Elliott, this gives rise to an update semantics in which negation can be classical, and DNE is validated. Just as in the current setting, negation merely serves to flip the polarity of the outputs.

(88) Update type (def.)
$\mathsf{u} \coloneqq (\{\, \mathsf{s} \,\}, \mathsf{g}) \to \{\, (\mathsf{t}, \mathsf{s}, \mathsf{g}) \,\}$

(89) $[\![\text{Jeff left}]\!] = \lambda(c, g) \,.\, \{\, (\mathsf{left}_w\ j, w, g) \mid w \in c \,\}$ $\quad \mathsf{u}$

(90) $$[\![\text{someone}^1 \text{ left}]\!] = \lambda(c, g) \,.\, \{\, (\top, w, g^{[1 \to x]}) \mid \mathsf{left}_w\ x, x \in \mathsf{dom}, w \in c \,\} \cup \{\, (\bot, w, g) \mid \neg\exists x[\mathsf{left}_w\ x], w \in c \,\} \qquad \mathsf{u}$$

Updates induced by sentences with epistemic modals end up being non-distributive, due to the adopting of the test semantics based on Veltman 1996 and Groenendijk, Stokhof & Veltman 1996. This is illustrated below.

(91) $$[\![\text{someone}^1 \text{ might have left}]\!] \coloneqq \lambda(c, g) \,.\, \begin{cases} \{\, (\top, w, g) \mid w \in c \,\} & \exists(\top, *, *) \in (90)\,(c, g) \\ \{\, (\bot, w, g) \mid w \in c \,\} & \text{otherwise} \end{cases} \qquad \mathsf{u}$$

Elliott also develops an anaphoric account of modal subordination, showing how an update semantics which validates DNE addresses some pitfalls of existing dynamic frameworks which incorporate both anaphora and modality.

It seems, therefore that a dynamic semantics can be either distributive or non-distributive, and validate DNE. What seems to be *crucial* is a distinction between, informally, *true* vs. *false* information in the output. We leave to future work the task of making this intuition more logically precise.[28]

[28] In recent work, Mandelkern (2020c, 2020b) is also concerned with developing a fragment which achieves the basic results of DS while validating DNE. In contrast to the strategy pursued here, Mandelkern's semantics is static, and gives rise to eliminative updates. Mandelkern's core insight is that a sentence with an indefinite such as "someone[1] left" comes with a disjunctive "witness presupposition", which is satisfied if (a) g_1 left, or (b) nobody left. This is somewhat reminiscent to the semantics for indefinites proposed in the current setting. I leave a detailed comparison between these approaches to future work.

7 Conclusion

An important contribution of monadic DS which we've glossed over in this paper is that it provides a way to generalize function-argument application to values with dynamic side-effects, as in (92). Application in monadic DS passes output assignments from the first value m into the second value n pointwise, and does Function Application (FA) of the contained ordinary-semantic values.

(92) Application in monadic DS

$$m \circledast n := \lambda g\,.\bigcup_{(f,g')\in m\,g} \{\,(f\ x, g'') \mid (x, g'') \in n\ g'\,\} \qquad \mathsf{S}\,(\mathsf{a} \to \mathsf{b}) \to \mathsf{S}\,\mathsf{a} \to \mathsf{S}\,\mathsf{b}$$

As discussed at the beginning of this paper, one of Chierchia's motivations for dynamicizing event semantics was to allow for a system in which the machinery of dynamic semantics does some work intrasententially. Adopting monadic DS wholesale arguably renders this move redundant. We've refrained from the more substantial enterprise of recasting Chierchia's ideas in an event-free monadic setting, but this would be helpful in order to understand the core properties of this approach to WCO.

Another issue with Chierchia's theory which we haven't addressed here is its reliance on a syntactic distinction between pronouns and *traces*. Arguably, this is an artifact of its adoption of the QR approach to scope. In ongoing work (Elliott in progress), I attempt to recast the basic ideas of Chierchia's theory in a dynamic semantics where quantificational scope is modeled via *continuations* (Barker 2002, Barker & Shan 2014).

In this paper our aim has been rather more conservative — we've argued that Chierchia's theory, as it stands, makes bad predictions regarding interactions with externally static operators. This motivated a significant rethinking of the underlying dynamic plumbing — ultimately, we argued for the expressive power of monadic DS, where we systematically keep track of both verifiers and falsifiers. The core the account lay in the revised semantics for indefinites which, unlike in standard monadic DS, either return true-tagged assignments *only*, or the grand-intersection of the false-tagged assignments. This revised semantics allowed us to simply lift propositional negation into our dynamic setting, resulting in an operator with all the desired properties — namely, it roofs dynamic scope with respect to indeterminate DRS, but allows determinate DR to survive unscathed. This move, as an added bonus, provided us with a straightforward account of double negation.

The properties of the resulting system deserve a more thorough exploration than we have had space for here. Namely, it remains to be seen whether there is any independent motivation for allowing operations such as negation to leave *determinate* DRS unscathed. Despite the fact that the analysis was motivated by fairly narrow concerns — namely, a technical problem with Chierchia's theory of WCO — we believe that the resulting discussion has broader lessons for any theory of WCO couched in DS, and is suggestive of the properties we ultimately want the dynamic semantics characterizing natural language to have.

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