Presuppositions, implicatures, and contextual equivalence

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Abstract  Classical versions of Maximize Presupposition! (MP) posit a competition between contextually equivalent sentences that differ in regard of the logical strength of their presuppositions. Yet recent work has unveiled novel MP-like cases which fall beyond the empirical scope of MP precisely because the relevant competitors are not contextually equivalent to their base sentence. To account for these cases, Spector & Sudo (2017) propose a novel principle, the Presupposed Ignorance Principle (PIP), whose formulation parallels that of MP but leaves out the former condition on contextual equivalence. This amendment of MP allows the PIP to capture the novel cases while preserving the classical ones. In this paper, we show that the PIP, however, overgenerates and undergenerates in a variety of other examples, and we argue that the main culprit is precisely dropping the condition on contextual equivalence. We discuss two directions moving forward. The first is the Logical Integrity approach by Anvari 2019, 2018, which, among other things, replaces contextual equivalence with contextual entailment. As we discuss, this approach cannot account for our problematic cases either, and thus doesn’t fare much better than Spector & Sudo (2017) in that respect. The second is the implicature-based approach by Marty & Romoli 2019, which combines insights from Magri 2009, Marty 2017, 2019b and Meyer 2013 and which subsume the conditions on contextual equivalence and presupposition satisfaction from MP under the broader notion of relevance. This approach can account for the cases by Spector & Sudo (2017) as well as for some of our novel cases, yet not all of them. We conclude that the issue of how to properly restrict the competition for MP-like cases, accounting for the classical cases, those of Spector & Sudo (2017) and the novel ones we present here, remains an important challenge for all accounts in the literature.

Keywords: Maximize Presupposition!, Presupposed Ignorance Principle, Logical Integrity, exhaustivity, presuppositions, implicatures, contextual equivalence, oddness

* For very helpful discussion and feedback, we would like to thank Amir Anvari, Patrick Elliott, Matt Mandelkern, Uli Sauerland, Benjamin Spector, Yasu Sudo and an anonymous reviewer for NALS. This work was supported by the Leverhulme trust grant RPG-2018-425.
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1 Introduction

The minimal pairs in (1)-(3) exemplify the long-standing observation originating in Heim 1991 that the utterance of a sentence \( \phi \) is infelicitous at a context \( c \) if \( \phi \) has a presuppositionally stronger competitor \( \psi \) whose presupposition is already common knowledge at \( c \), i.e. mutually accepted by the interlocutors in \( c \), and makes the same contribution as \( \phi \) at \( c \).

(1) Context: it is common knowledge that there is a unique sun.
   a. #A sun is shining.
   b. The sun is shining.

(2) Context: it is common knowledge that John has two parents.
   a. #All of John’s parents left.
   b. Both of John’s parents left.

(3) Context: it is common knowledge that Paris is in France.
   a. #John believes that Paris is in France.
   b. John knows that Paris is in France.

Heim (1991) proposed to derive such contrasts from a general principle of language use, which has come to be known as Maximize Presupposition! (MP henceforth), formulated as in (4) and exhorting speakers to make their conversational contributions by ‘presupposing as much as possible.’

(4) Maximize Presupposition
A sentence \( \phi \) is infelicitous in context \( c \), where \( c \) is the context to which \( \phi \) is to be added, if there is an alternative \( \psi \) to \( \phi \) such that:
   a. \( \psi \)’s presupposition asymmetrically entails \( \phi \)’s presupposition, and
   b. \( \psi \)’s presupposition is satisfied in \( c \), and
   c. \( \psi \) and \( \phi \) are contextually equivalent in \( c \).

Since Heim’s natural language formulation of this principle, many researchers have contributed to describe and refine the formal aspects of the competition at work in (1)-(3) together with the contextual conditions on which this competition effectively leads to infelicity effects. MP has been successfully applied to a variety of phenomena and the classical picture above extended and implemented in different versions (a.o., Percus 2006, 2010, Sauerland 2008, Chemla 2008, Schlenker 2012a, Katzir & Singh 2013, Rouillard & Schwarz 2017, Magri 2009, Marty 2017, Anvari 2019).

A recent line of work has investigated minimally different cases like (5) and (6) (a.o., Sharvit & Gajewski 2008, Gajewski & Sharvit 2012, Spector & Sudo 2017):
Given the similarities between (5)-(6) and the classical cases in (1)-(3), it is tempting to try and subsume the effects in (5)-(6) under MP as well. However, as Sharvit & Gajewski (2008) and Spector & Sudo (2017) discuss, cases like (5) and (6) are beyond the scope of application of MP: although the competing (b)-sentences carry stronger presuppositions which are met in context, they are not contextually equivalent to the (a)-sentences and therefore the third condition in (4-c) is not met.

To account for these cases, Spector & Sudo (2017) (S&S henceforth) extends the classical MP approach above, by dropping the contextual equivalence requirement and by proposing a system based on two distinct forms of scalar strengthening, which operate independently but interact with one another. First, they adopt a regular theory of scalar implicatures that operates at the assertion level and allows the presuppositions of the negated alternatives to project. Second, in place of MP, they propose the pragmatic principle in (7), a generalised version of MP, which they call the *Presupposed Ignorance Principle* (PIP henceforth).

\[
\textbf{(7) Presupposed Ignorance Principle (from Spector & Sudo 2017)}
\]

A sentence $\phi$ is infelicitous in context $c$, where $c$ is the context to which $\phi$ is to be added, if there is an alternative $\psi$ to $\phi$ such that:

a. $\psi$’s presupposition asymmetrically entails $\phi$’s presupposition, and

b. $\psi$’s presupposition is satisfied in $c$.

In short, the formulation of the PIP parallels that of MP up to one critical stage: the PIP leaves out the MP-requirement in (4-c) that the presuppositional competitors to a given sentence be contextually equivalent to that sentence, allowing in principle more presuppositional competitors than MP. This minimal amendment allows S&S to capture the cases in (5)-(6), where contextual equivalence does not obtain, while preserving the classical ones in (1)-(3). Finally, S&S argues that the interaction between the PIP and the computation of scalar alternatives can account for contrasts like the one between (5-a) and its positive counterpart in (8).

\[
\textbf{(8) Context: it is common knowledge that all of the students passed.}
\]

John is aware that some of the students passed.

Just like (5-a), (8) has a presuppositionally stronger alternative whose presupposition is met in context, namely *John is aware that all of the students passed*. However, as we discuss
in detail below, the meaning of (8), unlike that of (3), can be strengthened by computing the scalar implicature associated with this alternative, that is by negating this alternative and subsequently by letting its presupposition project up. As a result, (8) together with its scalar implicature presupposes in fact that all of the students passed, and therefore the application of the PIP becomes vacuous in this case (i.e., there is no alternative with a stronger presupposition), hence the felicity of (8).

After presenting S&S’s proposal in more detail in Section 2, we show in Section 3 that, despite its immediate successes, this proposal overgenerates in a variety of other cases, involving (1) environments giving rise to existential presuppositions, (2) cardinal partitives, and (3) the restrictor of definites, universal quantifiers, which-phrases, and the antecedent of conditionals. We argue that the culprit is precisely dropping the condition on contextual equivalence when moving from MP to the PIP. In addition, we show that S&S’s system also undergenerates with certain variants of their case in (6). We discuss two directions moving forward. The first is the Logical Integrity approach by Anvari 2019, 2018, which, among other things, replaces contextual equivalence with contextual entailment. As we discuss, this approach cannot account for our problematic cases or minimal variants thereof, and thus doesn’t fare much better than Spector & Sudo (2017) in that respect. The second is the implicature-based approach stemming from Magri 2009, Marty (2017, 2019b) and Marty & Romoli 2019, which reintroduces the notion of contextual equivalence in some form through the broader notion of relevance. This approach can account for the case by Spector & Sudo (2017) in (5) and the case of cardinal partitives we present below, but the other two cases remain problematic for this approach as well. Finally, following Marty & Romoli 2019, we discuss how the epistemic grammatical layer from Meyer 2013 can be integrated with this approach to account for the other case by Spector & Sudo (2017) in (6) and its variants. All in all, the issue of how to properly restrict the competition for MP-like phenomena, accounting for the classical cases, those of Spector & Sudo (2017) and the novel ones we present here, remains an important challenge for all accounts in the literature.

2 Background

2.1 Spector and Sudo’s (2017) proposal

Motivated by cases like (5) and (6), Spector & Sudo (2017) propose a modification of the classical MP picture by putting forward a system based on the following three ingredients: (i) the Presupposed Ignorance Principle (PIP) in (7), which operates at the presuppositional level, (ii) a mechanism for computing implicatures at the assertion level, and (iii) the interaction between the mechanisms in (i) and (ii).

Consider first the presuppositional level. The PIP essentially requires that among a set of alternative sentences, one should use the one(s) with the strongest presupposition(s) satisfied in context, regardless of whether it makes the same contribution in the context as the other alternatives (i.e., regardless of whether the considered alternatives are contextually
At the assertion level, scalar strengthening proceeds in standard ways. For concreteness, S&S assume that scalar implicatures are computed by applying a covert exhaustivity operator, notated by ‘exh.’ Adopting Fox’s (2007) notion of Innocent Exclusion (IE), (9-a), this operator can be defined as in (9-b), where ε is any sentence and alt(ε) the set of formal alternatives to ε. In short, applying exh to a sentence ε outputs the conjunction of ε and the negation of all of ε’s alternatives that are innocently excludable, i.e., those alternatives to ε that can be consistently negated together without contradicting ε or entailing the truth of other alternatives.

\[
(9) \quad \text{IE}(\epsilon, S) := \cap \left\{ S' \subseteq S \text{ and } S' \text{ is a maximal subset of } S \text{ such that } \{ \neg \psi : \psi \in S \} \cup \{ \epsilon \} \text{ is consistent} \right\}
\]

\[
(9) \quad [\text{exh } \epsilon](w) = [\epsilon](w) \land \forall \psi \in \text{IE}(\epsilon, \text{alt}(\epsilon)) [\neg [\psi](w)]
\]

In addition, S&S refines the scalar strengthening mechanism above to account for its interaction with presuppositions. Specifically, adopting a trivalent semantics for presuppositions, S&S proposes to adjust the bivalent definition of exh in (9) to a trivalent setting so as to let exh pass up the presuppositions of the alternatives it excludes, just like negation passes up the presuppositions of the sentence it negates. Excludable alternatives are thus negated in a strong sense: the negation of an alternative ψ with presupposition p is true if and only if p is true and ψ is false. In short, S&S’s adjustments are twofold: first, the notion of Innocent Exclusion in (9-a) is redefined by making use of strong negation and, second, exh is defined so as to behave as a ‘presupposition hole’ with respect to the presupposition of the alternatives. In other words, [exh ε](w) is undefined if any of its alternative is undefined. The novel definition of exh from Spector & Sudo (2017: 63) is given in (10).

\[
(10) \quad [\text{exh } \epsilon](w) = \begin{cases} 
\# & \text{ iff } [\epsilon](w) = \# \text{ or for some } \psi \in \text{IE}(\epsilon, \text{alt}(\epsilon)), [\psi](w) = \# \\
1 & \text{ iff } [\epsilon](w) = 1 \text{ and for all } \psi \in \text{IE}(\epsilon, \text{alt}(\epsilon)), [\psi](w) = 0 \\
0 & \text{ iff } [\epsilon](w) = 0 \text{ or for some } \psi \in \text{IE}(\epsilon, \text{alt}(\epsilon)), [\psi](w) = 1 \\
& \text{ and for no } \psi \in \text{IE}(\epsilon, \text{alt}(\epsilon)), [\psi](w) = \#
\end{cases}
\]

The PIP can be now more explicitly formulated as in (11), by taking into account the potential role of exh. That is, a sentence ε is infelicitous if it has an alternative ψ, the presupposition of which is satisfied in the context and asymmetrically entails the presupposition of the strengthened meaning of ε.

\[
(11) \quad \text{Presupposed Ignorance Principle}
\]

A sentence ε is infelicitous in context c, where c is the context to which ε is to be added, if there is an alternative ψ to ε such that:

a. ψ’s presupposition asymmetrically entails exh(ε)’s presupposition, and
b. $\psi$’s presupposition is satisfied in c.

Given the assumptions about $\text{exh}$, the PIP, and their interplay, the system by S&S can account for the infelicity of cases like (5)-(6), which are not covered by the classical MP approach. First, when (8) is strengthened by $\text{exh}$, the presupposition of the negated all-alternative projects to the whole sentence. In other words, exhaustification strengthens the meaning of $\text{exh}$’s prejacent in two related ways: (i) by negating the assertion of its all-alternative, and subsequently (ii) by passing up the (stronger) presupposition of that alternative. As a result, the application of the PIP becomes vacuous, which accounts for the felicity of (8) in the relevant context.

(8)  
Context: it is common knowledge that all of the students passed.  
\begin{align*}  
\text{exh } [\phi \ \text{John is aware that some of the students passed}] \\
\text{IE}(\phi, \text{ALT}(\phi)) & = \{ \text{John is aware that all of the students passed} \} \\
\end{align*}

\begin{itemize}
  \item a. Asr: John believes that some of the students passed, but he doesn’t believe that all of the students passed
  \item b. Prs: all of the students passed
\end{itemize}

By contrast, in negative cases like (5-a), the corresponding all-alternative is not excludable, and so exhaustification is vacuous and the PIP effectively applies, giving rise to a conflicting inference. That is, by application of the PIP, an utterance of (5-a) triggers the presupposed ignorance inference $\neg \text{CK}(\text{all of the students passed})$. This makes the sentence contradictory with common knowledge and this accounts for its infelicity.

(5-a)  
\begin{align*}  
\text{Context: it is common knowledge that all of the students passed.} \\
\#\text{exh } [\phi \ \text{John is unaware that some of the students passed}] \\
\text{IE}(\phi, \text{ALT}(\phi)) & = \{ \} \\
\end{align*}

\begin{itemize}
  \item a. Asr: John doesn’t believe that some of the students passed
  \item b. Prs: some of the students passed
\end{itemize}

(5-a) $\rightarrow \neg \text{CK}(\text{all of the students passed})$ by the PIP

To summarise, Spector & Sudo (2017) propose two forms of scalar strengthening, operating at different levels and interacting with each other. At the assertion level, $\text{exh}$ negates the assertion of certain alternatives and passes up their presuppositions. At the presuppositional level, the PIP derives weaker inferences about what is common knowledge. Importantly, the scope of application of the PIP can be tempered by the effect of exhaustification: scalar strengthening via $\text{exh}$ can sometimes ‘rescue’ a sentence from the infelicity that would otherwise arise from applying the PIP directly to the plain meaning of that sentence, like for instance in the analysis of (8) above. The interaction between both scalar strengthening mechanisms is thus at the heart of S&S’s account of the asymmetry between (5) and (8). In the following subsection, we turn to state a general prediction of this system which will be systematically tested in Section 3.
2.2 A general prediction

As mentioned, a critical aspect of Spector & Sudo’s (2017) system is that a (non-vacuous) application of `exh` can sometimes rescue the infelicity that would otherwise result from the application of the PIP. This gives rise to the general prediction in (12).

(12) **General prediction following from S&S’s system**

Let $S_p$ be any sentence with presupposition $p$, $S'_q$ be an alternative to $S_p$ such that $q$ asymmetrically entails $p$, and $c$ be the utterance context of $S_p$. If $c$ entails $q$ and $\text{exh}(S_p)$ does not entail $q$, then $S_p$ is predicted to be infelicitous by the PIP.

The prediction in (12) holds because, in the absence of a bleeding relation between `exh` and the PIP relative to the presupposition $q$ — i.e., $q$ does not end up presupposed as a result of applying `exh` — if $q$ is satisfied in $c$, then the PIP will generate the conflicting ignorance inference $\neg CK(q)$, from which infelicity should follow.

In the next section, we test this prediction and evaluate the empirical adequacy of the PIP. The result of this investigation is a variety of cases which are challenging for S&S’s system, and specifically for the idea that contextual equivalence can be harmlessly eliminated from the set of conditions restricting the set of presuppositional competitors to a given sentence. The cases to be discussed involve, among others, environments giving rise to existential presuppositions, cardinal partitives, the restrictor of universal quantifiers and the antecedent of conditionals, and a variant of S&S’s presupposed ignorance case.\(^1\)

3 Problems: overgenerating and undergenerating infelicity

In this section, we present in turn four cases which are problematic for the prediction in (12) and challenge S&S’s proposal. In the first three cases, we will see that the PIP overgenerates in predicting infelicity for sentences which are intuitively felicitous, while in the last one it undergenerates by not predicting the infelicity of intuitively infelicitous sentences. But before going on, some methodological considerations are in order.

A simple way of testing the prediction in (12) is considering cases which do not involve exhaustification, given the general properties of the context or the property of the linguistic environment in question. There are at least two ways to do that. First, we can consider the

\(^1\) At least two other challenges for S&S’s proposal have been pointed out in the recent literature. One challenge comes from the interaction between presuppositions and free choice disjunction through a phenomenon called *Presupposed Free Choice* (Marty & Romoli 2019). Another challenge comes from the observation that MP-like effects may arise even though the stronger presupposition of the competing alternative is not satisfied in context, an observation originally from Percus 2010 and recently extended to S&S’s novel cases in Anvari 2018, 2019. For this second challenge, we note that a solution preserving the original formulation of MP has been put forward in Marty 2019a. As far as we can tell, this solution can also be adapted to the PIP so as to account for Anvari’s extended cases. Our formulations of MP in (4) and of the PIP in (7) are in fact already in that spirit.
possible implicatures that a sentence may give rise to and verify that the presupposition of those implicatures do not affect the subsequent application of the PIP. Second, we can rely on linguistic contexts that render the application of EXH vacuous in the first place. This can be done for instance by setting up the surrounding context so as to force implicature suspension, (13), or by embedding the relevant sentence in a downward-entailing (DE) environment (e.g., under negation), (14). Interestingly, conditionals combine both advantages: the antecedent of conditionals is a DE-environment (or at least not-UE) and conditionals give rise to an ignorance inference about their antecedent, as illustrated in (15) (a.o., Gazdar 1979).

(13)  **Suspension Test 1: Speaker’s explicit ignorance**
I don’t know whether all of the students passed, but some of them did.
\[ \neg \text{the speaker doesn’t know whether all of the students passed} \]

(14) **Suspension Test 2: Downward-entailing environments**
It is not the case that some of the students passed.
\[ \neg \text{it is not the case that some but not all the students passed} \]

(15) **Combining Test 1 & 2: Antecedent of conditionals**
If some of the students passed, then John is one of them.
a. \[ \neg \text{the speakers doesn’t know whether some of the students passed (and therefore the speaker doesn’t know whether all of them did)} \]
b. \[ \neg \text{if some but not all the students passed, then John is one of them} \]

We will use both these verification strategies whenever applicable to assess the possible effects of exhaustification and provide additional controls for our test cases.

### 3.1 Case 1: Existential presuppositions

Consider a sentence with a presuppositional predicate like (16). We can paraphrase the presupposition of this sentence as in (16-a) and its asserted content as in (16-b).

(16) Jane didn’t stop smoking.
   a. Prs: Jane used to smoke
   b. Asr: Jane smokes now

Consider now embedding \texttt{stop} in the scope of a quantifier as in (17) and (18). Intuitively, while the presupposition of \texttt{stop} projects universally in the scope of \texttt{all}, it doesn’t do so however in the scope of existential quantifiers like those in (18).²

² See Chemla 2009 for experimental evidence that presuppositions do not project universally in existential quantifiers, and see also Section 8 for discussion.
All of my students stopped smoking.
\[ \sim \text{all of my students used to smoke} \]

Some/At least one/Few/Less than three of my students stopped smoking.
\[ \lnot \text{all of my students used to smoke} \]

In the following, we will use the quantifier and scalar item *some* and assume for simplicity that the presupposition of (18) is an existential presupposition, i.e., *some of my students used to smoke*. We note however that all that is needed for creating the problem below is simply that the presupposition be weaker than universal. Against this background, consider again the sentence in (18), with its presupposition in (18-a) and its asserted content in (18-b). Importantly for our purposes, (18) can be felicitously uttered in a context in which it is known that all of my students used to smoke.

(18)  \hspace{1cm} \text{Context: all of my students used to smoke.}
Some of my students stopped smoking.
\begin{itemize}
  \item a. PRS: some of my students used to smoke
  \item b. ASR: those students don’t smoke now
\end{itemize}

Note, however, that (18) has (19) as a presuppositionally stronger alternative, the presupposition of which is also satisfied in those contexts. If the PIP were to apply on the basis of the competition between (18) and (19), it would incorrectly predict (18) to be infelicitous.

(19)  \hspace{1cm} \text{Context: all of my students used to smoke.}
All of my students stopped smoking.
\begin{itemize}
  \item a. PRS: all of my students used to smoke
  \item b. ASR: all of my students don’t smoke now
\end{itemize}

S&S’s system, however, does not make this unwarranted prediction. Since (19) is also assertively stronger than (18), the meaning of (18) can be first strengthened by computing the implicature associated with (19), the presupposition of which projects then to the whole sentence, as shown in (20). This meaning strengthening operation renders the application of the PIP vacuous since (20) and (19) are now presuppositionally equivalent. As a result, the sentence in (18) is in fact expected on its strengthened meaning to be felicitous in the context above.

(20)  \hspace{1cm} \text{EXH [some of my students stopped smoking]}
\[ \iff \text{some of my students stopped smoking but not all of them did} \]
\begin{itemize}
  \item a. ASR: some of my students stopped smoking, but not all of them did
  \item b. PRS: all of my students used to smoke
\end{itemize}

This result is intuitively correct and, in fact, such cases could even be taken as an argument
for S&S’s proposal regarding the interactions between EXH and the PIP. There is, however, an immediate expectation that follows from the general prediction in (12): in contexts in which (19)’s presupposition is satisfied but the ‘rescuing’ implicature in (20) gets suspended, the PIP should apply in a non-vacuous fashion and therefore infelicity should follow. This expectation can be tested using the suspension tests outlined above.

Consider first a case like (21), where the speaker explicitly asserts that he is ignorant about the *all*-alternative in (19). This short discourse is felicitous and the fact that it is tells us that, in that context, the implicature in (20) is suspended, for otherwise the continuation in (21) would give rise to a contextual contradiction. But precisely, in the absence of this implicature, one would expect the PIP to apply and therefore the second sentence in (21) to be infelicitous, contrary to facts.

(21)  
*Context:* all of my students used to smoke.
I don’t know whether all of them stopped. But (I know that) some of them did.
\(\neg\) not all of my students stopped smoking

In response to (21), one could try and make use of a notion of relevance to explain why the stronger *all*-alternative is considered neither by EXH, nor by the PIP in such cases. For instance, one could hypothesize that an alternative \(\psi\) to a sentence \(\phi\) cannot take part to any strengthening operation affecting \(\phi\)’s meaning if the speaker is known to be ignorant about \(\psi\). This explanation, however, does not extend to our second way of testing implicature suspension. Consider for instance the sentences in (22), where (18) and (19) are now embedded in the antecedent of conditionals:

(22)  
*Context:* all of my students used to smoke.
\begin{enumerate}
  \item If some of them stopped smoking, Jane will be happy.
  \item If all of them stopped smoking, Jane will be happy.
\end{enumerate}

Intuitively, (22-a) conveys its plain (i.e., non-strengthened) meaning, compatible with that of (22-b). And indeed, the computation of a *not-all* implicature in this environment is generally disfavored as it would weaken (rather than strengthen) the global meaning of (22-a), by conveying that *Jane will be happy if some but not all my students stopped smoking*. In the absence of an implicature, the PIP should thus apply on the basis of the competition between (22-a) vs. (22-b), predicting (22-a) to be felicitous only if it is not

\[\text{We note that the question of whether the antecedent of conditionals is a DE-environment is a debated one. What is important for us is that this environment is uncontroversially one in which scalar implicatures tend to disappear and out of which presuppositions tend to project. In addition, the problem can be recreated with other, less controversial, DE-environments as in (i).}\]

(i)  
*Context:* all of my students used to smoke.
I doubt that some/many of them stopped smoking.
common knowledge that all the students used to smoke. But this prediction is incorrect since (22-a) is in fact felicitous in the context above.\(^4\)

Note that it could be argued that \textsc{exh} applies in (22-a) nonetheless, precisely because its application is needed to rescue the sentence from infelicity. Yet this explanation faces two serious issues. First, it does not align with speakers’ intuitions about the meaning of (22-a): speakers accept (22-a) as felicitous in the absence of the \textit{not-all} implicature. Second, if we were to assume that \textsc{exh} nonetheless applies in DE-environments like (22-a), we would lose S&S’s explanation for the asymmetry between (8) and (5-a), since \textsc{exh} could be then applied in (5-a) too, preventing the PIP from applying and thus rescuing that sentence from infelicity.\(^5\) Similar data can be reproduced with other presuppositional triggers (e.g., \textit{another, again}, definite descriptions), and other downward entailing contexts.\(^6\)

In sum, one crucial feature of S&S’s system is the interaction between \textsc{exh} and the PIP, where the application of the former takes precedence and may lead to a vacuous application of the latter. The cases we have discussed in this subsection are problematic for this architecture because they are cases where \textsc{exh} does not apply and the conditions of application of the PIP are met, and therefore they are incorrectly predicted to be infelicitous.

### 3.2 Case 2: Cardinal partitives

Consider the sentences in (23), each of which involves the cardinal partitive phrase \textit{three of the lawyers that John hired} associated with the existential presupposition that there are at least three individuals that are lawyers and that John hired.

\(^4\) For completeness, we note that one also needs to make sure that it is possible in context that all of the speaker’s students used to smoke, as it is arguably another presupposition of the \textit{all}-alternative in this environment. This requirement can easily be met by adding for instance the background information that smoking was rampant in the university with lots of the students smoking.

\(^5\) We note that integrating Meyer’s (2013) proposal to S&S’s system, as we discuss later in Section 7, would provide another solution to the case in (21). However, it would not provide a solution for the case in (22-a). Thanks to Benjamin Spector for discussion on this point.

\(^6\) Specifically, we could then analyse (5-a) as in (i), where \textsc{exh} occurs below negation.

\begin{align*}
\text{(i)} & \quad \text{not } [\textsc{exh} \{\phi \text{ John is aware that some of the students passed}\}] \\
& \quad \text{IE}(\phi, \textit{alt} (\phi)) = \{\text{John is aware that all of the students passed}\} \\
& \quad \text{a. Asr: } \neg[\text{John believes that some of the students passed and doesn’t believe that all of them passed}] \\
& \quad \text{b. Prs: all of the students passed}
\end{align*}

\(^7\) There might be discrepancies between presuppositional triggers in regard of projection in quantificational environments. In particular, some triggers have been argued to project universally from the scope of all quantifiers; see Charlow 2009, Romoli 2012, Chemla 2009 for discussion. Once again, our argument above holds only of those triggers for which there is less than universal projection.
(23)  a. Three of the lawyers that John hired just arrived.
     PRS: there are (at least) three lawyers that John hired
b. Will three of the lawyers that John hired come to the trial?
     PRS: there are (at least) three lawyers that John hired
c. If three of the lawyers that John hired have arrived, the trial can begin.
     PRS: there are (at least) three lawyers that John hired

It has long been observed that the use of cardinal partitives is subject to an anti-maximality requirement (a.o., Jackendoff 1977, Hoeksema 1984, Barker 1998, Zamparelli 1998, Sauerland & Yatsushiro 2004, 2017, Marty 2017, 2019b). That is, the sentences in (23) can be felicitously used in a conversation only if it is not common knowledge that John hired exactly three lawyers. This observation can be further exemplified by the contrast in (24) adapted from Marty (2019b: (23)).

(24)  a. John hired exactly three lawyers, and
     #[three of them/three of his lawyers] just arrived.
   b. I can’t remember how many lawyers in total John hired but
      [three of them/three of his lawyers] just arrived.

In (24-a), the speaker makes it common ground that John hired exactly three lawyers, and this prevents the subsequent use of the partitive phrase *three of John’s lawyers* from being felicitous. By contrast, in (24-b), the speaker is ignorant as to whether John hired exactly three or more lawyers, and the use of this same phrase is felicitous.

It has been proposed in Marty (2017, 2019b) that the anti-maximality condition on the use of those partitives follows from the general competition between indefinite phrases and their presuppositionally stronger definite alternatives which has been traditionally subsumed under the scope of MP (cf. Heim 1991). In short, cardinal partitives like *three of the lawyers that John hired* are indefinite phrases headed by a silent indefinite determiner and compete with their definite cardinal variants, e.g. *the three lawyers that John hired*. Thus, a sentence like (23-a) has the sentence in (25) as an alternative. This alternative carries a stronger presupposition but is equivalent, in its assertion part, to (23-a) (i.e., in every context in which the presuppositions of (23-a) and (25) are satisfied, the two sentences are equivalent).

(25)  The three lawyers that John hired just arrived
     a.  PRS: there are exactly three lawyers that John hired
     b.  ASR: those three lawyers just arrived

By MP, an utterance of (23-a) is thus predicted to be felicitous only in contexts in which the presupposition of (25) is not satisfied, that is if it is not common ground that John hired exactly three lawyers. It is worth noting here that MP does not impose any further requirement. In particular, note that the hypothetical alternatives to (23-a) in (26), although similar in structure to (25) and presuppositionally stronger than (23-a), are not contextually
equivalent to (23-a), and therefore they do not qualify as presuppositional competitors to (23-a) in regard of MP.

(26) a. The *four* lawyers that John hired just arrived  
    Prs: there are exactly four lawyers that John hired  

b. The *five* lawyers that John hired just arrived  
    Prs: there are exactly five lawyers that John hired  

c. The *six* lawyers that John hired just arrived  
    Prs: there are exactly six lawyers that John hired  

d. etc…

The situation changes however if one adopts the PIP in place of MP here: if we are to eliminate the condition on contextual equivalence, then all the definite alternatives to (23-a) above are now expected to compete with (23-a). Applying the PIP to (23-a) on the basis of these alternatives amounts to generate, for each of these alternatives, an ignorance inference targeting their presuppositional content (e.g., $\neg CK(\text{exactly three})$ & $\neg CK(\text{exactly four})$ & $\neg CK(\text{exactly five})$, etc.). Summing up these inferences gives rise in the end to the following prediction: (23-a) should be felicitous only if (a) it is common knowledge that John hired at least three lawyers (i.e., the plain presupposition of (23-a)), but (b) it is not common knowledge how many lawyers John exactly hired (i.e., by application of the PIP). This prediction is, however, incorrect as (23-a) is fully felicitous in a context where the number of lawyer hired by John is common knowledge.

(23-a) Context: John hired exactly four lawyers.
    Three of the lawyers that John hired just arrived.

Could it be then the application of the PIP is blocked here by the application of EXH? In the above case, it would be so for instance if there was an innocently excludable alternative to (23-a) against which the meaning of (23-a) could be exhaustified so as to add the presupposition that *John hired exactly four lawyers*, blocking in effect the application of the PIP. As we will now see, however, there is no such an alternative to fulfill this role.

Consider first the formal alternatives to (23-a) in (26). Those definite alternatives are all logically stronger than (23-a) and, taken independently, any of those alternatives can be negated consistently with the plain of meaning (23-a). For instance, negating the *four*-alternative to (23-a) in (26-a) would give rise to the implicature that *the four lawyers that John hired didn’t arrive*, which would then add to (23-a)’s presupposition the presupposition of interest, namely that *John has exactly four lawyers*. However, those alternatives cannot be negated all together consistently with the plain of meaning (23-a) since the presuppositions associated with those alternatives are logically inconsistent with one another (i.e., *John has exactly four lawyers, John has exactly five lawyers, John has exactly six lawyers*, etc.). As a result, the definite alternatives to (23-a) in (26) are not innocently excludable and thus cannot be used to prompt scalar reasoning and block the application of the PIP.
Next, we note that sentences like (23-a) also have indefinite cardinal alternatives such as (27), which are also logically stronger than them, both at the presupposition and at the assertion level:

(27) Four of the lawyers that John hired just arrived.
    a. Prs: there are (at least) four lawyers that John hired
    b. Asr: those four lawyers just arrived

Yet adding those alternatives to the picture does not solve the issue at hand. First, while exhaustifying the plain meaning of (23-a) against the alternative in (27) would strengthen (23-a)’s presupposition, as shown in (28), the representation resulting from this strengthening would still be presuppositionally weaker than the definite four-alternative in (26-a), and therefore would not prevent the PIP from applying.

(28) \[\text{exh } \phi \text{ three of the lawyers that John hired just arrived} \]
\[\text{alt}(\phi) = \{\text{four of the lawyers that John hired just arrived}\} \]
    a. Asr: three but not four of the lawyers that John hired just arrived
    b. Prs: there are (at least) four lawyers that John hired

\[\Rightarrow \text{there are exactly four lawyers that John hired}\]

Second, while we illustrated the point above with four for simplicity, sentences like (23-a) have in fact infinitely many indefinite alternatives of that sort, all of which are innocently excludable (e.g., five of the lawyers that John hired just arrived, six of the lawyers that John hired just arrived, etc.). Thus, the reasoning in (28) should apply in principle to any numeral \(n\) larger than three, in which case we would have \text{exh} negating all alternatives of the form \(n\text{ of the lawyers John hired arrived, for any } n\text{ larger than three. Adding the presupposition of each of those negated alternatives, i.e. } \text{John hired at least}\ 4, 5, 6, 7, \ldots \text{ lawyers}, \text{ is now going to entail that John hired infinitely many lawyers; this entailment is certainly not an inference that people draw upon hearing sentences like (23-a).}\)

(29) \[\text{exh } \phi \text{ three of the lawyers that John hired just arrived} \]
\[\text{IE}(\phi, \text{alt}(\phi)) = \{ n \text{ of the lawyers that John hired just arrived} \ | \ n > 3 \} \]
    a. Asr: three but not \(n\) of the lawyers that John hired just arrived, for any \(n > 3\)
    b. Prs: there are (at least \(n\)) lawyers that John hired, for any \(n > 3\)

\[\Rightarrow \text{there are infinitely many lawyers that John hired}\]

In light of our discussion, one may still wonder whether a solution for this problem in S&S’s system would not be to stipulate that numerals may only compete with their immediate ‘neighbours’. On this view, the alternatives for ‘three’ would just be just ‘two’ and ‘four’. Another way to formulate that idea would be to state that the closer a numeral is to
the one used in EXH’s prejacent, the more likely it is to contribute to alternatives.\(^8\) That assumption, prima facie, would indeed permit one to account for instance for the case in (23-a) by singling out the potential role of the definite four-alternative in (26-a): if we are to preserve only the ‘two’ and ‘four’ alternatives to ‘three’, then the four-alternative in (26-a) would become innocently excludable and its exclusion would block the PIP successfully.

As it stands, however, the above stipulation would face serious issues beyond the mere absence of an independent motivation. First, it cannot account for variants of (26-a) such as (30). In order for the PIP to be blocked by scalar reasoning, the domain of EXH would need here to include the definite alternative for ‘eleven’, which is quite distant from ‘three’, and crucially to include only that definite alternative since, if other alternatives of the same sort were in EXH’s domain (e.g., all alternatives up to ‘eleven’), then that alternative would no longer be innocently excludable.

(30)  
Context: John hired exactly eleven lawyers.  
Three of the lawyers that John hired just arrived.

Second, the problem for Spector & Sudo’s (2017) would re-emerge in DE-environments. As before, embedding (23-a) in a DE-environment as in (23-c) does not change the picture: a sentence like (23-c) can be felicitously used if it is common knowledge that John hired exactly \(n\) lawyers, for any \(n > 3\). Crucially, note that, as expected, the natural reading of (23-c) does not have an embedded implicature, i.e. (23-c) does not suggest that if three but not four/five/six/etc. of the lawyers that John hired arrived, the trial can begin.

(23-c)  
Context: John hired exactly four lawyers.  
If three of the lawyers that John hired have arrived, the trial can begin.

We now turn to a third set of overgenerating cases involving the restrictor of universal quantifiers and which-phrases, and the antecedent of conditionals.

### 3.3 Case 3: Restrictors

A common assumption in the literature is that universally quantified sentences like (31), or definite descriptions like (32), require that their restrictor be non-empty. That is (31) and (32) presuppose that there is at least one individual who is a student and came to class.\(^9\)

(31) All of the students who came to class understood the puzzle.  
Prs: some students came to class

(32) The students who came to class understood the puzzle.  
Prs: some students came to class

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8 We thank Benjamin Spector for pointing out to us this possibility.
9 See Heim & Kratzer (1998: Chapter 6) and references therein.
Consider now the variant in (33), containing a disjunction in the restrictor of all/the.\footnote{Spector & Sudo (2017: Section 6.1) discuss similar cases as an open problem for their account and point to a proposal by Sudo (2016), involving analogous cases with implicatures, as a possible solution. It is unclear to us whether that proposal can extend to our cases here. Thanks to Benjamin Spector and Yasu Sudo for discussion on this point.}

(33) \textit{Context: some students met with either Danny or Irene, others with both.}\newline All/The students who met with Danny or Irene understood the puzzle.\newline Prs: some students met with Danny or Irene

(33) has the alternative in (34), which is assertively weaker but presuppositionally stronger than (33). Thus, according to the PIP, (33) should be felicitous only in contexts where the presupposition of (34) is not satisfied, that is if it is not common knowledge that some student(s) met with both Danny and Irene. Yet this prediction is not borne out: as evidenced above, (33) is in fact felicitous in such contexts. Crucially, note that exhaustification cannot help in this case either: EXH is vacuous if applied globally (i.e., at root level), and it would lead to an intuitively wrong meaning if applied locally (i.e., in the restrictor of all/the).

(34) \textit{Context: some students met with either Danny or Irene, others with both.}\newline All/The students who met with Danny and Irene understood the puzzle.\newline Prs: some students met with Danny and Irene

Similar data can be reproduced with the antecedent of conditionals. Thus for instance, (35-a) has the alternative in (35-b) which, in a way similar as above, is assertively weaker but presuppositionally stronger than (35-a). Contra the predictions of the PIP, however, (35-a) can be felicitously uttered in a context where (35-b)’s stronger presupposition is met.

(35) \textit{Context: it is possible that Jane met with both Danny and Irene.}\newline a. If Jane met with Danny or Irene, she understood the puzzle.\newline Prs: it is possible that Jane met with Danny or Irene\newline b. If Jane met with Danny and Irene, she understood the puzzle.\newline Prs: it is possible that Jane met with Danny and Irene

And the very same problem arises with the restrictor of which-phrases:

(36) \textit{Context: some students met with either Danny or Irene, others with both.}\newline a. Mary knows which students met with Danny or Irene.\newline Prs: some students met with Danny or Irene\newline b. Mary knows which students met Danny and Irene.\newline Prs: some students met with Danny and Irene

One could hope to address the issues above by restricting the scope of application of the
PIP, for instance by restricting its application to alternatives whose assertive content is not entailed by the assertion of the base sentence, as suggested in (37).

(37) **Presupposed Ignorance Principle (modified)**

A sentence $\phi$ is infelicitous in context $c$, where $c$ is the context to which $\phi$ is to be added, if there is an alternative $\psi$ to $\phi$ such that:

a. $\psi$’s presupposition asymmetrically entails $\phi$’s presupposition, and
b. $\psi$’s presupposition is satisfied in $c$, and
c. $\psi$’s assertion is not entailed by $\phi$’s assertion.

The additional restriction in (37-c) would take care of the universal quantifier cases above and, under certain assumptions about the semantics of conditionals and embedded questions, it might extend to those cases as well. However, the restrictor of the definite determiner *the* is a non-monotonic context and so, in contrast to the other cases, the presuppositionally stronger alternative is not entailed; in those cases, the same issue would thus arise even on the modified version of the PIP in (37). Similar data can be reproduced with other non-monotonic contexts like the restrictor of *most*, as exemplified in (38). Here again, the problem is that the presuppositionally stronger alternative to (38-a) in (38-b) is not entailed by (38-a), and so the issue re-emerges.

(38) **Context: some students met with either Danny or Irene, others with both.**

a. Most students who met with Danny or Irene understood the puzzle.  
*PRS: some student met with Danny or Irene*

b. Most students who met with Danny and Irene understood the puzzle.  
*PRS: some student met with Danny and Irene*

3.4 **Case 4: Speaker-oriented ignorance**

Our fourth and last set of cases shows that, when it comes to presupposed ignorance, S&S’s proposal also faces undergeneration issues. As a starting point, consider the example in (6-a), which corresponds to the second case motivating S&S’s departure from contextual equivalence in favor of the PIP.

(6-a) **Context: it is common knowledge that Mary will go to Yale.**  
#John, too, will go to Yale or Harvard.

a. *Asr: John will go to Yale or Harvard*

b. *PRS: someone other than John will go to Yale or Harvard*

As S&S show, the infelicity of (6-a) is left unexplained by MP while it is accounted for by the PIP. The reason for that is that, unlike MP, the PIP allows (6-b) to be a presuppositional competitor to (6-a). Hence, (6-a) is expected on S&S’s proposal to give rise through the PIP to the presupposed ignorance inference $\neg \text{CK(someone salient other than John (i.e. Mary) will go to Yale)}$. 

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This inference contradicts common knowledge, since the interlocutors know that Mary will go to Yale, accounting for the infelicity of (6-a).

(6-b) John, too, will go to Yale.
   a. ASR: John will go to Yale
   b. PRS: someone other than John will go to Yale

However, Marty & Romoli (2019) observe that the PIP fails to account for minimally different versions of (6-a), which also involve disjunctive presuppositions. To illustrate, consider first the example in (39), which offers a different instance of the same problem:

(39) Context: the interlocutors know that Olivia only took Logic
   #Noah is unaware that Olivia took Logic or Algebra.
   (cf. Noah is unaware that Olivia took Logic)

The observation here is simply that (39) cannot be felicitously uttered in a context in which it is common knowledge among the interlocutors that Olivia took Logic (or, alternatively, that Olivia took Algebra). It is easy to see that the PIP can account for this observation through the exact same reasoning as before: an utterance of (39) is infelicitous in the suggested context because, by application of the PIP, it gives rise to two inferences, \(\neg\text{CK}(\text{Olivia took Logic})\) and \(\neg\text{CK}(\text{Olivia took Algebra})\), one of which contradicts common knowledge. Yet as Marty & Romoli (2019) argue, the explanatory challenge surrounding presupposed ignorance is more general. Specifically, they observe that similar infelicity effects reproduce in cases like (40) even though neither of the embedded disjuncts are common knowledge in the context. Thus for instance, (40-a) is infelicitous even if it is not commonly known whether the speaker has children.

(40) a. #Noah is unaware that I have two or more children.
    b. #Sue didn’t realize that my wife is from France or Italy.
    c. #Mary was sorry that Sue had lunch with Noah or me yesterday.

The infelicity effects of those examples are in fact similar to those previously observed for their non-embedded, non-presuppositional variants in (41) (a.o., Gazdar 1979, Fox 2007, Singh 2008, 2010, Fox & Katzir 2011).

(41) a. #I have two or more children.
    b. #My wife is from France or Italy.
    c. #Sue had lunch with Noah or me yesterday.

Taken at face value, all those examples appear to be infelicitous because they give rise to speaker-oriented ignorance inferences that stand in contradiction with common knowledge. That is, a sentence like (40-a), just like its unembedded variant in (41-a), sounds odd because it conveys that the speaker herself is ignorant as to how many children she actually has, and
this piece of information conflicts with the common assumption that people are normally knowledgeable about such personal facts.\textsuperscript{11} The problem, however, is that the PIP is not designed to mandatorily generate such speaker-oriented ignorance inferences. In fact, the PIP only generates for (40-a) the inference that the exact number of children that the speaker has is not common ground; the issue is that this inference is and remains compatible with common knowledge as long as this information is not mutually shared by the interlocutors (e.g., if this information is not known to the speaker’s addressee). In sum, Spector & Sudo’s (2017) proposal only offers a partial solution to the empirical challenge of presupposed ignorance: it accounts for the infelicity effects in (6-a) but leaves those of the variants in (40) unexplained.\textsuperscript{12}

3.5 Intermediate summary

We have presented in this section several problematic cases for S&S’s system. In the first three cases, we observed that the PIP overgenerates in predicting infelicity for felicitous sentences, with no clear way for exhaustification to save the day by bleeding its application. In the fourth case, we showed that the PIP undergenerates by not capturing the infelicity of infelicitous sentences carrying disjunctive presuppositions. This conclusion leaves us at this point with the following dilemma. On the one hand, none of the overgeneration cases that we discussed (Cases 1-3) are problematic for MP for none of them involves a competition between contextually equivalent alternatives; but the scope of application of MP is too restrictive to capture S&S’s cases and their variants above, including Case 4. On the other hand, the PIP generalises MP by dropping the condition on contextual equivalence and successfully covers the novel cases observed by S&S; but its broader scope of application now leads to systematic overgeneration issues in the Cases 1-3 above while, at the same time, undergenerating in Case 4. In the next section, we turn to discussing two directions moving forward, based on two recent proposals in the literature.

4 Moving forward

We now turn to two alternative approaches to MP, which broaden its scope of application while maintaining relatively strict conditions on competing alternatives. The first is the Logical Integrity approach by Anvari 2019, 2018 which, among other things, replaces

\textsuperscript{11} Of course, this common assumption need not hold in all contexts. For instance, in amnesia-like contexts, it can be assumed instead that the speaker has forgotten such personal information. And indeed, in such contexts, all of the examples above become felicitous.

\textsuperscript{12} As Marty & Romoli (2019) discuss, the present issue would disappear if some additional mechanism could be postulated so as to force the outcome of the PIP to be narrowed down from common knowledge to the speaker’s epistemic state, for then the conflicting speaker-oriented inferences that we described would obtain. We refer the readers to Marty & Romoli (2019) for a critical discussion of this and related analytical options and why it appears non-trivial to integrate them with S&S’s system in order to account for the effects above.
contextual equivalence with contextual entailment. The second is the implicature-based approach stemming from Magri 2009 and further developed in Marty 2017, 2019b, which, among other things, extends the application of EXH to presuppositional competitors and subsumes the condition on contextual equivalence under the broader notion of relevance. As we explain below, that condition is further extended to the presuppositional level in the version in Marty 2019b and Marty & Romoli 2019. Both these approaches aim at capturing a broad class of unacceptable sentences, starting with the classical MP-cases. For space reasons, we will compare their empirical coverage in the next sections only with regard to the seven cases discussed in this paper: classical MP cases like (42), the PIP-motivating examples from S&S in (43) and (44) (which we call 'the asymmetry’ and 'presupposed common ignorance', respectively, for convenience) and our four new cases in (45)-(48).

(42) **Classical MP cases**

*Context: John has two parents.*

a. #All of John’s parents left.
b. Both of John’s parents left.

(43) **The asymmetry**

*Context: all of the students passed.*

a. #John is unaware that some of the students passed.
b. John is aware that some of the students passed.

(44) **Presupposed common ignorance**

*Context: Mary will go to Yale.*

#John, too, will go to Yale or Harvard.

(45) **Presupposed speaker’s ignorance**

*Context: it is not known whether the speaker has children*

#Noah is unaware that I have two or more children.

(46) **Existential Presuppositions**

*Context: all of my students used to smoke.*

a. Some of my students stopped smoking.
b. If some of my students stopped smoking, Jane will be happy.

(47) **Cardinal partitives**

*Context: John hired exactly four lawyers.*

a. Three of the lawyers that John hired just arrived.
b. If three of the lawyers that John hired arrived, the trial can begin.

(48) **Restrictors**

*Context: some students met with both Danny and Irene; it is possible that Jane is one
of them.

a. All/Most/The students who met with Danny or Irene understood the puzzle.
b. If Jane met with Danny or Irene, she understood the puzzle.

The remainder of this paper is structured as follows. Section 5 focuses on the Logical Integrity approach. We will see that this approach captures the classical MP cases as well as the asymmetry with factives; however, it leaves the presupposed ignorance cases in (44)-(45) unexplained and encounters the same overgeneration issues as the PIP for the cases in (46)-(48). Section 6 turns to the implicature-based approach. By reintegrating the notion of contextual equivalence in some form, this approach can deal with one of our novel cases (the case of cardinal partitives in (47)), in addition to the classical MP ones and the asymmetry with factives; however, the cases in (46) and (48) still remain problematic for this approach, which is also insufficient by itself to account for the presupposed ignorance cases in (44)-(45). In Section 7, we show however that this latter limitation can be overcome: once we consider the grammatical approach to ignorance implicatures from Meyer 2013, the implicature-based approach offers a satisfying solution to the presupposed ignorance challenge (see also Marty & Romoli 2019). As we discuss, while Meyer’s (2013) proposal is also compatible with the PIP and LI, the resulting systems are unable to address the presupposed ignorance challenge in its full generality.

5 Moving to contextual entailment

5.1 The proposal in brief

Logical Integrity (LI henceforth) is a generalisation which aims at capturing the unacceptability of a variety of sentences, part of which were previously captured by MP or the PIP. The gist of this generalisation is that a sentence $\phi$ is deemed infelicitous if it has an alternative $\psi$ that is logically non-weaker, yet contextually entailed by $\phi$. In other words, LI forces the logical relation between a sentence and its alternatives to be preserved once contextual information is considered, hence the name of ‘logical integrity’. We will consider the formulation of this principle in (49) and assume that a sentence is infelicitous if any part of it violates (49).\footnote{This is not the final version of the principle, which is associated with an additional ‘projection principle’ for local applications, but it is enough for our purposes; see Anvari 2019 for discussion.}

(49) **Logical Integrity** (adapted from Anvari 2019: (5))

A sentence $\phi$ must not be uttered in context $c$ if it has an alternative $\psi$ such that (i) $\phi$ contextually entails $\psi$ in $c$, but (ii) $\phi$ does not logically entail $\psi$. 

\begin{itemize}
  \item \textbf{Logical Integrity} (adapted from Anvari 2019: (5))
  \item A sentence $\phi$ must not be uttered in context $c$ if it has an alternative $\psi$ such that (i) $\phi$ contextually entails $\psi$ in $c$, but (ii) $\phi$ does not logically entail $\psi$.
\end{itemize}
5.2 Good predictions

As Anvari (2019) shows, LI can account for the classical MP cases and the asymmetry with factives. To illustrate, consider first the sentence in (42-a). The both-alternative in (42-b) is not logically entailed by (42-a) because (42-b) can be undefined when (42-a) is true. In the given context, however, if (42-a) is true, then so is (42-b). In sum, we have contextual but not logical entailment, and consequently (42-a) violates LI in this context. Next, consider the contrast in (43): the all-alternative to (43-a) in (50) is not logically entailed by (43-a); but when we add the assumed contextual information that all of the students passed, (50) becomes contextually entailed by (43-a). Thereby (43-a) is correctly predicted to be infelicitous by LI.

(50) John is unaware that all of the students passed.

On the other hand, (43-b) has the all-alternative in (51) which is neither logically nor contextually entailed by (43-b) in the given context and so, unlike (43-a), (43-b) does not violate LI. Hence, the contrast in (43) is nicely captured by this approach.

(51) John is aware that all of the students passed.

5.3 Presupposed ignorance unexplained

As Anvari (2019) discusses, LI doesn’t account however for the presupposed ignorance case in (44): since the formal alternatives to (44) in (52) are neither logically nor contextually entailed by (44), the infelicity of (44) is left unexplained by LI.

(52) a. John too will go to Yale.
   b. John too will go to Harvard.

For similar reasons, LI doesn’t account for our variant of (44) in (45): in a context in which it is not known whether the speaker has a son or a daughter, the formal alternatives to (45) in (53) are neither logically nor contextually entailed by (45). The LI approach cannot therefore account for the infelicity effects in (44)-(45) either.

(53) a. Noah is unaware that I have a son.
   b. Noah is unaware that I have a daughter.

5.4 Overgeneration problems

Turning to our other cases, the LI approach encounters there the same overgeneration issues as the PIP. Consider first the case of existential presuppositions, as in (46). This approach has no problem with the example in (46-a) since the all-alternative to (46-a) in (54)
is not logically entailed by (46-a) and this absence of entailment is preserved contextually when we add the contextual information that all of my students used to smoke.

(54) All of my students stopped smoking.

However, if we move to the minimal variant of (46-a) in (46-b), the situation changes: the all-alternative to (46-b) in (55) is not logically entailed by (46-b), yet if we add the contextual information above, contextual entailment this time obtains. Therefore, LI incorrectly predicts (46-b) to be infelicitous.14

(55) If all of my students stopped smoking, Jane will be happy.

The same observations hold for the case of cardinal partitives in (47). The felicity of (47-a) is unproblematic for LI since the alternative the four lawyers that John hired just arrived is neither logically, nor contextually entailed by (47-a) in the given context. Yet the variant of (47-a) in (47-b) recreates for LI the same problem as above: (47-b) does not logically entail the definite four-alternative in (56), but it does contextually entail it in contexts in which it is common knowledge that John hired exactly four lawyers. Like (46-b), (47-b) is therefore predicted to be infelicitous by LI, contra speakers’ intuitions.

(56) If the four lawyers that John hired arrived, the trial can begin.

For similar reasons, LI makes incorrect predictions for the last case in (48). This is because the sentences in (48) do not logically entail their and-alternatives in (57) (i.e., these alternatives can be undefined given the compatibility presupposition of the antecedent). Nonetheless, in the given context, both these sentences contextually entail their and-alternatives. Therefore, they are predicted to be infelicitous by LI and, once again, this prediction is incorrect.

(57) a. All/Most/The students who met with Danny and Irene understood the puzzle.
   b. If Jane met with Danny and Irene, she understood the puzzle.

In sum, LI aims at capturing a broad class of unacceptable sentences, starting with the classical MP cases and the asymmetry with factives. Although we cannot make full justice to the richness of the proposal, which goes beyond the cases we focused on here, we conclude that, as far as the novel cases above are concerned, LI does not make much progress with respect to S&S’s proposal in facing similar overgeneration issues.

14 This argument holds if one assumes a semantics of conditionals which makes their antecedents downward monotonic. But the same argument can be reproduced with other, less controversial DE-environments (see footnote 3 above).
6 Moving to relevance

In this section, we outline the implicature-based approach to MP effects stemming from Magri (2009) and Marty (2017, 2019b). We will refer to this approach as the M&M system. This system integrates back contextual equivalence within a notion of relevance, which allows it to account for the classical MP-cases in the same way as the original MP approach, and further applies that notion of relevance at the presuppositional level. The resulting approach accounts for the asymmetry case in (43) as well as for the case of cardinal partitives in (47). Yet the existential presuppositions and restrictor cases remain problematic for this approach as well.

6.1 The proposal in brief

On the theory of implicatures developed in Magri 2009, 2011, 2013, the infelicity of a sentence like (58) is hypothesized to result from the mandatory computation of a mismatching implicature, that is an obligatory implicature which contradicts common knowledge.

(58) #Some Italians come from a beautiful country.

a. Parse: \( \text{EXH}_R [\phi \text{ some Italians come from a beautiful country}] \)

b. Alternative: \( [\psi \text{ all Italians come from a beautiful country}] \)

c. Relevance: \( \psi \iff \phi \) and so \( \psi \in R \)

d. Obligatory implicature: #Not all Italians come from a beautiful country

The gist of Magri’s theory is that a sentence like (58) must be parsed with an exhaustive operator, (58-a). Just like any other quantifier, the domain of this operator is taken to be restricted by a contextually assigned relevance predicate \( R \); the exhaustivity operator with its restriction is written ‘\( \text{EXH}_R \)’. The denotation of \( R \) is assigned by the context of \( \text{EXH}_R \)’s prejacent and thus varies across contexts, accounting for the context-dependency of implicatures, i.e., for the possibility to suspend an implicature in certain contexts but also for the impossibility to do so in others.\(^{15}\) In particular, since relevance is assumed to be closed under contextual equivalence, if the prejacent \( \phi \) of \( \text{EXH}_R \) in (58-a) is relevant, then so is its \( \text{all}-\text{alternative} \psi \) in (58-b) since \( \psi \) and \( \phi \) are contextually equivalent relative to \( \phi \)’s context (corresponding here to the global context \( c \)). As a result, the implicature associated with \( \psi \) becomes mandatory in this case, resulting in a representation that contradicts common knowledge (i.e., \( \psi \cap \text{EXH}_R (\phi) = c \cap (\phi \land \neg \psi) = \emptyset \)). In addition to relevance considerations, the domain of quantification of \( \text{EXH} \) is also regulated by general economy considerations (a.o., Fox & Spector 2009, Magri 2011, Spector & Sudo 2017, Fox & Spector 2018): since the computation of an implicature must lead to meaning strengthening, an alternative that can

\(^{15}\) Suitable algorithms for computing the domain restriction \( R \) as well as evaluating contextual equivalence at embedded levels are proposed in Schlenker (2012b: Sections 4.1 & 4.2.1) and Marty (2017: Appendix A) based on the original proposal in Singh (2011).
be pruned from the domain of EXH must effectively be pruned if the implicature associated with that alternative would weaken or leave unaffected the global meaning of the sentence. Building on Magri’s insights, Marty (2017) proposes to extend Magri’s theory to presuppositional effects.\(^{16}\) In essence, Marty (2017) argues that, when computing the implicatures of a sentence \(\phi\), speakers entertain two sets of alternatives that are mutually exclusive and distinguished on the basis of Strawson-entailment: (i) a set of presuppositional alternatives, comprising the formal alternatives to \(\phi\) that can only be undefined when \(\phi\) is true (i.e., those alternatives that are Strawson-entailed), \(^{(59-a)}\), and (ii) a set of assertive alternatives, comprising the formal alternatives to \(\phi\) that can be false when \(\phi\) is true (i.e., those alternatives that are not Strawson-entailed), \(^{(60-a)}\).\(^{17}\) As Marty (2017) discusses, on this proposal, we need exclusion to be performed innocently on both sets of alternatives. For presuppositional alternatives, we adopt the procedure of innocent exclusion (IE\(_{prs}\)) proposed in Marty (2017), \(^{(59-b)}\), which applies Fox’s (2007) original notion to the presuppositional domain. For assertive alternatives, we propose that innocent exclusion (IE\(_{asr}\)) is computed as shown in \(^{(60-b)}\), by taking all maximal sets of assertive alternatives that can be negated consistently with the prejacent and the negation of the presupposition of all IE\(_{prs}\) alternatives. This second definition slightly departs from Marty (2017) in that it assumes that IE\(_{asr}\) is computed on the basis of IE\(_{prs}\) rather than independently.

\(^{(59)}\) **Excludable and Innocently Excludable presuppositional alternatives**

\(^{a}\) \(E^{prs}(\phi) = \{ \psi : \psi \in ALT(\phi) \land \phi \neq \psi \land psp(\psi) \equiv \psi \} \)

\(^{b}\) \(IE^{prs}(\phi, S) := \bigcap \left\{ S' \mid S' \subseteq S \land S' \text{ is a maximal subset of } S \text{ such that } \{ \neg psp(\psi) : \psi \in S' \} \cup \{ \phi \} \text{ is consistent} \right\} \)

\(^{(60)}\) **Excludable and Innocently Excludable assertive alternatives**

\(^{a}\) \(E^{asr}(\phi) = \{ \psi : \psi \in ALT(\phi) \land \phi . psp(\psi) \neq \psi \} \)

\(^{b}\) \(IE^{asr}(\phi, S) := \bigcap \left\{ S'' \mid S'' \subseteq S \land S'' \text{ is a maximal subset of } S \text{ such that } \{ \neg \psi : \psi \in S \} \cup \{ \phi \} \cup \{ \neg psp(\chi) : \chi \in IE^{prs}(\phi, E^{prs}(\phi)) \} \text{ is consistent} \right\} \)

\(^{16}\) Marty (2017) implements this theory in a uni-dimensional approach to meaning, using partial semantics for presuppositions. The core theory is then integrated into a dynamic theory of interpretation, using a notion of local contexts along the lines of Karttunen (1974) and Heim (1983).

\(^{17}\) As Spector & Sudo (2017) discuss, once we move to a trivalent or partial semantics, there are various ways to define the notions of entailment and consistency. We assume here that entailment is defined as in (i) (i.e., ‘strict entailment’) and that consistency is defined as in (ii).

(i) \(\phi\) entails \(\psi\) iff, for any world at which \(\phi\) is true, \(\psi\) is also true at that world.

(ii) A set of propositions \(S\) is consistent iff all of its members can be true at the same world.

Finally, we assume that the negation used in the definitions of this subsection is the so-called ‘strong’ negation (see Spector & Sudo 2017 for discussion). That is, \(\neg \phi\) is defined only if \(\phi\) is defined (i.e., \(psp(\phi)\) is true) and, where defined, \(\neg \phi\) is true iff \(\phi\) is false.
Following this characterization of assertive and presuppositional alternatives, Marty (2017) proposes that the exhaustivity operator be defined as in (61).

(61) **Exhaustivity operator for assertive and presuppositional alternatives**

a. \([\exh R \phi]\) is defined at a world \(w\) only if
   
   (i) \(\psp(\phi)\) is true in \(w\), and
   
   (ii) for all \(\psi\) s.t. \(\psi \in \IEasr(\phi, Easr(\phi))\) and \(\psi \in R\), \(\psp(\psi)\) is true in \(w\), and
   
   (iii) for all \(\psi\) s.t. \(\psi \in \IEprs(\phi, Epres(\phi))\) and \(\psp(\psi) \in R\), \(\neg \psp(\psi)\) is true in \(w\)

b. Where defined, \([\exh R \phi]\) is true in \(w\) iff

   (i) \(\phi\) is true in \(w\), and

   (ii) for all \(\psi\) s.t. \(\psi \in \IEasr(\phi, Easr(\phi))\) and \(\psi \in R\), \(\neg \psi\) is true in \(w\)

In this framework, applying \(\exh\) to a sentence \(\phi\) can strengthen \(\phi\)’s presupposition in one of two ways. First, in a way similar to Spector & Sudo’s (2017) proposal, presupposition strengthening can happen indirectly upon projection of the presupposition of \(\IEasr\) alternatives: \(\exh R\) passes up to the whole sentence the presuppositions of the relevant \(\IEasr\) alternatives to its prejacent that it negates, (61-a-ii). Second, presupposition strengthening can happen as a direct result of an implicature: \(\exh\) also passes up to the whole sentence the negation of the relevant presuppositions of the \(\IEprs\) alternatives to its prejacent, (61-a-iii).

Finally, M&M’s system inherits from Magri’s original system the idea that \(\exh\)’s domain is modulated both by relevance and economy considerations. In particular, with respect to relevance, if the prejacent \(\phi\) of \(\exh\) is relevant, then any *assertive alternative* to \(\phi\) that is contextually equivalent to \(\phi\) is also relevant (Magri 2009, 2011). Conversely, if \(\phi\) is *not* relevant to begin with, then \(\exh(\phi)\) is infelicitous. This logic is extended in Marty 2019b and Marty & Romoli 2019 to presuppositional alternatives by assuming that, for those alternatives, speakers assess relevance by considering the relevance of their presuppositional contribution. For our immediate purposes, it is enough to observe that, given the way the notion of relevance is usually defined in the literature, if a proposition is entailed by the context, then that proposition trivially counts as relevant. As a result, if

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18 In works like Magri (2011) and Marty (2017), it is assumed that an occurrence of \(\exh\) must be present at every propositional node. For our purposes, however, we only need to assume that \(\exh\) is mandatory at matrix level, since we will not be concerned for now with infelicity effects arising from the computation of conflicting embedded implicatures. We refer the reader to Marty (2017) for discussion and refinements of this assumption in the case of presupposed implicatures.

19 We refer the reader to Marty (2019b: 4.3-4.4) for an explanation and a discussion of the pragmatic rationale underlying the formulation of this extended requirement.

20 We assume that the notion of relevance is modelled using questions under discussion (see Roberts 2004, Beaver & Clark 2009 among many others). Assume that the question under discussion is associated with a partition of the context set, which corresponds to the set of complete answers to this question (a.o., Heim 1994, Groenendijk et al. 1984, Groenendijk & Stokhof 1984, Bennett 1979). The notion of relevance can then be defined as in (i), where \(Q\) is the partition associated with the question under discussion: a proposition is relevant if and only if it does not distinguish between two worlds within a cell of \(Q\).
the prejacent $\phi$ of EXH is assertable at a context $c$ (i.e., if $c$ entails $\phi$’s presupposition), then $\phi$’s presupposition is relevant in $c$ and so are the presuppositions of $\phi$’s presuppositional alternative that are satisfied in $c$. Conversely, if $\phi$ is not assertable to begin with, e.g., $\phi$’s presupposition isn’t met prior to utterance and fails to be accommodated, then EXH($\phi$) is infelicitous. Crucially, if an assertive alternative or the presupposition of a presuppositional alternative is deemed relevant in that sense, it cannot be pruned from $R$ and thus from the domain of EXH; consequently, any assertive or presupposed implicature associated with such an alternative is predicted in M&M’s system to be mandatory.

### 6.2 Good predictions

First, M&M’s system readily accounts for the classical MP effects. To illustrate, consider the sentence in (42-a), which is parsed with an occurrence of the exhaustivity operator at matrix level, as shown below. The presuppositional both-alternative to EXH’s prejacent is innocently excludable and, since its presupposition is satisfied in the suggested context, its presupposition counts as relevant and thus must be excluded. In a way similar to what we saw in (58), the mandatory computation of this implicature results in a contextually contradictory representation, from which infelicity follows.\(^21\)

(42-a)  
**Context:** *John has only two parents.*

#All of John’s parents left

(i) Let $Q$ be a partition of the context set. A proposition $p$ is relevant given $Q$ iff for any cell $q \in Q$ and any two worlds $w, w' \in q$, $p(w) = p(w')$.

It follows from (i) that, if a proposition $p$ is entailed by the context set (e.g., if $p$ is a presupposition satisfied in the context) then, for any partition $Q$ of that context set, $p$ cannot distinguish between any world within any cell of $Q$, and therefore $p$ is relevant.

21 Anvari (2019) argues that, contrary to MP, M&M’s system generates certain inferences that are stronger than attested. For instance, in M&M’s system, a sentence like (i) has the potential to give rise to a global presuppositional implicature, i.e. *not every professor has exactly two students*, but also to a local presuppositional implicature which, upon projection under every, gives rise to the stronger presupposition that *every professor has more than two students*. Yet as Anvari (2019) observes, speakers generally accept sentences like (i) in contexts where it is known that some professors have exactly two students, i.e. in contexts where the local implicature is false.

(i) Every professor invited all his students.

<table>
<thead>
<tr>
<th>Parse: EXH$<em>{\pi}$[every professor $\lambda x$ EXH$</em>{\pi}$[x invited all $x$’s students]]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Global: $\neg$[for every professor $x$, $x$ has exactly two students]</td>
</tr>
<tr>
<td>b. Local: for every professor $x$, $\neg$[x has exactly two students]</td>
</tr>
</tbody>
</table>

This observation is discussed in Marty (2017: Chapter 2) and it is shown to be unproblematic for M&M’s system: as long as the presupposition of the both-alternative is not satisfied in its local context, the local implicature in (i-b) can be suspended, independently of the global one, simply by pruning the both-alternative from the domain of the embedded EXH, hence ensuring felicity in the kind of contexts considered by Anvari (2019).
a. Parse: \( \text{exh}_{R} [\phi \text{ all of John’s parents left}] \)
b. \( \text{IE}^{\text{prs}}(\phi, \text{IE}^{\text{prs}}(\phi)) = \{ [\psi \text{ both of John’s parents left}] \} \)
c. Relevance: \( c \subseteq \text{psp}(\psi) \) and so \( \text{psp}(\psi) \in \mathcal{R} \)
d. Obligatory implicature: #John doesn’t have exactly two parents

Second, M&M’s system offers a simple solution to the issue raised by cardinal partitives: of all the definite alternatives to indefinite partitives of the form ‘the \( n \) of the NPs’, only the one of the form ‘the \( n \) NPs’ qualifies as a presuppositional alternative by the definition in (59). All other definite alternatives qualify instead as assertive alternatives and are not innocently excludable since negating them all upon exhaustification would project presuppositions that are mutually inconsistent (e.g., John has exactly 4 and exactly 5 and exactly 6 etc. lawyers). Consequently, M&M’s system, in a way similar to MP, correctly predicts (47-a) and (47-b) to be infelicitous only in those contexts in which John is known to have hired exactly 3 lawyers, i.e., in contexts in which the presupposition of their definite three-alternative is satisfied.

(47-a) \( \text{exh}_{R} [\phi \text{ three of the lawyers that John hired just arrived}] \)
a. \( \{ \text{the three lawyers that John hired just arrived} \} \subseteq \text{IE}(\phi, \text{IE}^{\text{prs}}(\phi)) \)
b. \( \{ \text{the four/five/etc. lawyers that John hired just arrived} \} \nsubseteq \text{IE}(\phi, \text{IE}^{\text{prs}}(\phi)) \)
c. \( \{ \text{the four/five/etc. lawyers that John hired just arrived} \} \nsubseteq \text{IE}(\phi, \text{IE}^{\text{asr}}(\phi)) \)

Finally, the M&M approach accounts for the asymmetry with factives from Spector & Sudo (2017). Consider again the contrast in (43), repeated below for convenience:

(43) Context: all of the students passed.

a. #John is unaware that some of the students passed.
b. John is aware that some of the students passed.

In M&M’s system, this contrast lies in the different status of the target all-alternatives in both cases. In (43-a), the target all-alternative is a presuppositional alternative to exh’s prejacent: upon exhaustification, the negation of its (stronger) presupposition can be added to the plain presupposition of exh’s prejacent. In the present case, since the presupposition of the presuppositional all-alternative to exh’s prejacent is satisfied, this strengthening is mandatory and results in a contextual contradiction, hence the infelicity of (43-a).

(43-a) #John is unaware that some of the students passed.
a. Parse: \( \text{exh}_{R} [\phi \text{ John is unaware that some of the students passed}] \)
b. \( \text{IE}^{\text{prs}}: [\psi \text{ John is unaware that all of the students passed}] \)
c. Relevance: \( c \subseteq \text{psp}(\psi) \) and so \( \text{psp}(\psi) \in \mathcal{R} \)
d. Obligatory implicature: #Not all the students passed

In (43-b), by contrast, the corresponding all-alternative is an assertive alternative to exh’s
prejacent: upon exhaustification, its presupposition and the negation of its (stronger) assertion can be added to the plain meaning of EXH’s prejacent. Note that, in the absence of contextual equivalence, this strengthening process is predicted yet to remain optional.

(43-b) John is aware that some of the students passed.
   a. Parse: EXHₗ [φ John is aware that some of the students passed]
   b. IEₗφ: [ψ John is aware that all of the students passed]
   c. Relevance: ψ ↛ φ and so pruning ψ from 𝓁 is possible
   d. Possible implicature: John is not aware that all the students passed

M&M’s account of the contrast in (43) is therefore similar to Spector & Sudo’s (2017). In particular, both accounts predict the sentence in (43-a) to be odd due to the mandatory generation of some conflicting inference (attributed to the working of the PIP in one case, and to the working of EXH in the other), while no such conflict need to arise in (43-b).

In closing, we note that the two accounts make however different predictions regarding two issues surrounding the felicity conditions of (43-b). The first issue has to do with focus sensitivity: S&S argue that, in a context in which it is known that all of the students smoke, the scalar term some needs to be stressed in order for (43-b) to be felicitous. That is, while (62-a) is felicitous such contexts, (62-b) isn’t:

(62) Context: it is common knowledge that all of the students smoke
   a. John is aware that some of the students smoke.
   b. #John is aware that some of the students smoke

On S&S’s approach, this contrast follows if one assumes that prosodic prominence on the scalar term strongly correlates with the presence of EXH. In (62-a), since some is stressed, scalar strengthening happens and the sentence is predicted to be felicitous. In (62-b), by contrast, some isn’t stressed and, in the absence of scalar strengthening, the sentence is predicted to be infelicitous through the PIP. On M&M’s approach, on the other hand, the implicature is predicted to be optional in this case, given the absence of contextual equivalence with the target assertive alternatives. This approach can thus account for the fact that stress on some, signaling the active work of EXH, is the most natural choice in the given context. It can also account for why (62-b) is felicitous in a context in which it is not known that all of the students smoke. However, it does not readily account for the infelicity of (62-b). The contrast in (62) seems therefore to favour S&S’s approach.

The second issue also pertains to the obligatoriness vs. optionality of the implicature in (43-b). This time, however, it is the M&M approach which appears to make the right prediction. In particular, Spector & Sudo’s (2017) account predicts that, in order for (43-b) to be felicitous in the context at hand, an implicature must be computed to avoid the PIP from generating a contextual contradiction. By contrast, M&M’s account predicts (43-b) to be felicitous in that same context independently from such a strengthening process. With this in mind, consider the example in (63):
As before, the context at hand is one in which it is common ground that *all of the students smoke*. However, the speaker is now explicitly stating that he is ignorant as to whether John is aware that all the students smoke, and this information subsequently leads one to suspend the implicature previously associated with (43-b). Crucially, we observe here that the suspension of that implicature leaves the felicity of (43-b) unaffected. This is directly in line with M&M’s predictions but problematic at first sight for Spector & Sudo (2017): in the absence of the target implicature, the PIP should apply just like in (43-a), and therefore the discourse in (63) should be perceived as infelicitous, contra speakers’ intuitions.\(^{22}\)

In sum, the two issues above have to do with the obligatoriness vs. optionality of the implicature associated with sentences like (43-b), in a context in which the presupposition of the alternative is or is not satisfied. The first issue suggests a strong correlation between the presence of focus, the generation of the target implicature and the felicity of (43-b), while the second reveals that (43-b) can be felicitous also in the absence of that implicature. S&S’s approach easily accounts for the former but not the latter issue, while M&M’s approach easily accounts for the latter but not the former.

### 6.3 Presupposed ignorance unexplained

Without further assumptions, the M&M approach cannot account for the presupposed ignorance cases in (44)-(45). To illustrate, consider the example in (44), which is parsed on this approach as shown in (64): of all the excludable assertive alternatives to (44), only the two conjunctive alternatives qualify as innocently excludable alternatives. Neither of those assertive alternatives is contextually equivalent to their base disjunctive sentence

\(^{22}\) We note right away that integrating Meyer’s (2013) proposal to S&S’s system, as we discuss below in Section 7, can provide a solution to this problem. Specifically, if we integrate Meyer’s (2013) Matrix \(K\) Axiom into S&S’s system (see Section 7 for details), the second sentence in (63) (i.e., (43-b)) could be parsed as shown in (i-a) and \(\text{exh’s prejacent would then have the formal alternative in}

\begin{enumerate}
  \item a. \(\text{exh}\ [\phi \ K_f [\text{John is aware that some of them smoke}]]\)
  \item b. \(\psi \ K_f [\text{John is aware that all of them smoke}]\)
\end{enumerate}

Crucially, on this enriched theory, the unwarranted prediction we pointed out can be circumvented by computing for (i-a) the implicature associated with (i-b), i.e., the speaker doesn’t know whether John is aware that all of them smoke. In passing up to the whole sentence its presupposition, that implicature can block the PIP from applying effectively, solving the issue at hand. In fact, this case could be taken as independent evidence that S&S’s system need to be supplemented with a theory along the lines of Meyer’s (2013) to counteract some otherwise unwarranted effects of the PIP.
and, more to the point, neither of the implicatures associated with those alternatives has
the potential to give rise to a contextual contradiction.

(64)  \[ \text{EXH}_R [\phi \text{ John, too, will go to Yale or Harvard}] \]
a.  \[ E^{asr}(\phi) = \{ [\text{John will go to Yale}], [\text{John will go to Harvard}], [\text{John will go to}
Yale and Harvard], [\text{John, too, will go to Yale}], [\text{John, too, will go to Harvard}],
[\text{John, too, will go to Yale and Harvard}] \} \]
b.  \[ IE^{asr}(\phi, E^{asr}(\phi)) = \{ [\text{John will go to Yale and Harvard}], [\text{John, too, will go}
to Yale and Harvard}] \}

Similar observations hold of the speaker-oriented variant in (45), as illustrated in (65): of
the three excludable presuppositional alternatives to (45), only the (presupposition of the)
conjunctive one is innocently excludable. The resulting presupposed implicature (i.e., that
I don’t have both a son and a daughter) is not in conflict with common knowledge and
cannot, therefore, account for the infelicity of (45).

(65)  \[ \text{EXH}_R [\phi \text{ Noah is unaware that I have a son or a daughter}] \]
a.  \[ E^{prs}(\phi) = \{ [\text{Noah is unaware that I have a son}], [\text{Noah is unaware that I have}
a daughter], [\text{Noah is unaware that I have a son and a daughter}] \} \]
b.  \[ IE^{prs}(\phi, E^{prs}(\phi)) = \{ [\text{Noah is unaware that I have a son and a daughter}] \} \]

The presupposed ignorance cases constitute for now a challenge for M&M’s system, which
is not equipped to operate on scalar alternatives involving independent disjuncts. We will
see in Section 7 that this limitation can yet be overcome by combining M&M’s system with
Meyer’s (2013) grammatical approach to ignorance implicatures.

6.4 Remaining overgeneration problems

While M&M’s system can account for the case of cardinal partitives, it encounters similar
issues as the PIP and LI for the other two cases. Consider first the case of existential
presuppositions, repeated from above:

(46)  \[ \text{Context: all of my students used to smoke.} \]
a.  Some of my students stopped smoking.
b.  If some of my students stopped smoking, Jane will be happy.

Like the other two approaches, M&M’s system can readily account for the felicity of (46-a):
the all-alternative to (46-a) in (66) is an assertive alternative to (46-a) and that alternative
is not contextually equivalent to (46-a). As a result, the meaning (46-a) may but need not
be exhaustified on the basis of (66): (46-a) is correctly predicted to be felicitous either way.

(66)  All of my students stopped smoking.
However, the minimal variant of (46-a) in (46-b) is problematic for this approach as well. To understand why, consider the *all*-alternative to (46-b) in (67). First, we verify that (67) is logically non-weaker, yet Strawson-entailed by (46-b); therefore, it counts in M&M’s system as a presuppositional alternative to (46-b). Second, the presupposition of (67) (i.e., *all of my students used to smoke*) and that of (46-b) (i.e., *some of my students used to smoke*) are satisfied in the suggested context; therefore, the computation of the presupposed implicature associated with (67) is predicted to be mandatory. Since the resulting implicature (i.e., *not all of my students used to smoke*) conflicts with the context, M&M’s system predicts (46-b) to be infelicitous, which is incorrect.

(67) If all of my students stopped smoking, Jane will be happy.

A similar problem arises when we move to the case of restrictors:

(48) *Context: some students met with both Danny and Irene; it is possible that Jane is one of them.*

a. All/Most/The students who met with Danny or Irene understood the puzzle.
b. If Jane met with Danny or Irene, she understood the puzzle.

Consider the alternatives to (48-a) and (48-b) in (68-a) and (68-b), respectively. In M&M’s system, those alternatives count as presuppositional alternatives and, moreover, their presuppositions are satisfied in the context at hand. As a result, the presupposed implicatures associated with those alternatives are predicted to be obligatory and, since those implicatures conflict with the contextual assumptions, both (48-a) and (48-b) are incorrectly predicted to be infelicitous.

(68) a. All/Most/The students who met with Danny and Irene understood the puzzle.
b. If Jane met with Danny and Irene, she understood the puzzle.

To summarise, M&M’s approach can account for the original MP cases, for the factive asymmetry from Spector & Sudo (2017) as well as for the case of cardinal partitives. However, it leaves presupposed ignorance unaccounted for at this point and it faces the same problems as the PIP and LI with existential presuppositions and restrictors. While the first issue can be remedied by enriching M&M’s approach, as we shall now see, the latter will remain a problem.

23 To be precise, the presupposition of (46-b) should also include that it is possible that all of my students stopped smoking, which would also have to be satisfied in the context. We leave that presupposition aside since it is orthogonal to the argument we are making here.
7 Back to presupposed ignorance

In this section, we go back to the various challenges posed by presupposed ignorance by investigating in more detail the common and speaker-oriented cases discussed above, as well as further variants. We show that a unified solution to all of those cases can be given by integrating Meyer’s (2013) grammatical view on ignorance implicatures to the M&M approach. As we discuss, while Meyer’s (2013) proposal is also compatible with the PIP and LI, the resulting systems, as they stand, are unable to account for all the cases of presupposed ignorance.

7.1 The challenge in more detail

Consider first the case in (69), which illustrates what we have referred to as presupposed common ignorance (e.g., (44)):

(69) Presupposed common ignorance
    Context: Mary speaks French.
    #John, too, speaks French or Japanese.

As we discussed, of all the approaches we considered, only the approach by S&S can account for the infelicity of such sentences. However, just like the other approaches, it fails to account for the speaker-oriented variants of (69) in (70):

(70) Presupposed speaker’s ignorance
    Context: It is not known whether the speaker is married.
    #Noah is unaware that my wife is French or Japanese.

In this subsection, we add the observation that S&S approach also runs into problems with minimally different versions of (69). Consider for instance the following two variants:\footnote{24}

(71) a. Variant 1: non-split conjunctive antecedent
    Context: Mary speaks French and Japanese.
    #John, too, speaks French or Japanese.

24 Uli Sauerland pointed out to us another interesting variant of (69):

(i) Mary, Bill and Sue each speaks French or Japanese. John, too, speaks French or Japanese.

The first sentence is understood as conveying that at least one of Mary, Bill and Sue speaks French and that at least another one of them speaks Japanese. The local context of the second sentence is thus one in which, in a way similar to (71-b), it is common knowledge that some individual other than John speaks French and that some other individual, also distinct from John, speaks Japanese. Thus, the PIP predicts here that the second sentence should be infelicitous because the alternatives 
\[\text{John too speaks French and John too speaks Japanese}\]
both have stronger presuppositions that are met in context. If this analysis of (i) is on the right track, then (i) is also an issue for S&S approach.
b. **Variant 2: split conjunctive antecedent**

*Context:* Mary speaks (only) French and Bill speaks (only) Japanese.

John, too, speaks French or Japanese.

As S&S themselves discuss, their approach does not capture the infelicity effect in (71-a) in that it predicts (71-a) to be infelicitous under certain parses, but not others. Specifically, if (71-a) is parsed without any occurrence of EXH, it is predicted to be infelicitous by the PIP because the alternatives (i) *John, too, speaks French and Japanese*, (ii) *John, too, speaks (only) French* and (iii) *John, too, speaks (only) Japanese* all have stronger additive presuppositions that are met in context. Similarly, if (71-a) is parsed with an occurrence of EXH taking scope below *too*, as in (72), the predicted presupposition is that there is a salient *x* other than John such that *x* speaks French or Japanese but not both. And this is false in the suggested context since the only salient *x* is Mary and, by assumption, she does speak both languages.

(72) $\text{too [EXH [John speaks French or Japanese]]}$

The problem is that there is also a possible parse for (71-a), namely (73), under which the sentence is predicted to implicate that John speaks one language or the other but not both, while presupposing that there is some salient *x* other than John which speaks both (i.e., the presupposition of the negated alternative *John too speaks French and Japanese*). Since this presupposition is also satisfied in the suggested context, (71-a) is incorrectly predicted to be felicitous under this parse (see Spector & Sudo 2017: Section 6.4).

(73) $\text{EXH [too [John speaks French or Japanese]]}$

The variant in (71-b) raises another non-trivial issue for S&S approach for that approach incorrectly predicts (71-b) to be infelicitous, on all relevant parses. First, if (71-b) is parsed without any occurrence of EXH, it is predicted by the PIP to be infelicitous for the same reasons as above. Next, if we assume the parse in (73), then the predicted presupposition is as before that someone other than John speaks both languages; this time, however, this presupposition is false in the given context since neither Mary, nor Bill speaks both languages. Finally, if we assume the parse in (72), the predicted presupposition is that somebody other than John speaks French or Japanese but not both. This presupposition is satisfied in the context, no matter whether this somebody is understood to be Mary or Bill. However, regardless of the choice of the antecedent, (71-b) has on this parse alternatives whose presuppositions are logically stronger and satisfied in the context. Thus for instance, the alternative in (74) presupposes that some salient individual other than John has the property of speaking only French, a property that Mary is known to have in the given context. Therefore, the PIP shall apply in this case, incorrectly predicting the sentence in (71-b) to be infelicitous.²⁵

²⁵ Benjamin Spector suggested to us that (71-b) may not be problematic for S&S if one assumes that the plural individual *Mary and Bill* is a possible anaphoric antecedent for *too*. In that case, the
In sum, S&S’s approach accounts for the basic case in (69) but it fails to extend to its speaker-oriented variants and to account for its variants in (71-a) and (71-b), by incorrectly predicting the former to be felicitous and the latter to be infelicitous (at least under certain assumptions; see fn. 25 for discussion). In light of those data, we propose in the following an alternative account of (69) and of its variants that extends a proposal by Meyer (2013) and integrates it with the M&M approach.

### 7.2 An exhaustivity-based solution

#### 7.2.1 A grammatical epistemic layer

One common way to account for ignorance inferences is to conceive them as implicatures derived from additional Gricean principles (a.o., Gazdar 1979, Sauerland 2004, Fox 2007, 2016). Meyer (2013) proposes instead that ignorance inferences are derived in the grammar through the interaction of the exhaustivity operator with another covert operator representing the speaker’s beliefs (see also Meyer 2014, Buccola & Haida 2018). At the core of Meyer’s proposal is the assumption — called the Matrix $K$ Axiom — that assertively used sentences contain a covert doxastic operator $K$ which is adjoined at the matrix level at LF (cf. Chierchia 2006, Alonso-Ovalle & Menéndez-Benito 2010). Much like the attitude verb believe, the Matrix $K$ operator universally quantifies over the speaker’s doxastic alternatives, as shown in (75). The subscript $x$ refers to the doxastic source, i.e. the individual whose beliefs $K$ is quantifying over. In the cases that we will be concerned with, $x$ will always be the speaker, hence the notation $K_s$.

**(75)**

\[
[[K_x \phi]] = \lambda w. \forall w' \in Dox(x)(w)[[[\phi]](w')]
\]

Meyer shows that the Matrix $K$ Axiom, together with the possible adjunction of EXH at any propositional node (i.e., below and above $K$), derives speaker-oriented ignorance inferences. To illustrate, consider the simple disjunctive sentence in (76):

**(76)**

\[
K_s [\phi] \text{ iff } \phi
\]

The presupposition of the second sentence would become that the plurality Mary and Bill speaks French or Japanese and that presupposition is satisfied in the context. Moreover, the presupposition of the alternatives in (i-a) and (i-b) would now be that the plurality Mary and Bill speak French and that the plurality Mary and Bill speak Japanese, respectively. Since those presuppositions are not satisfied, the PIP would not apply and, consequently, the sentence would be predicted to be felicitous.

(i) a. John too speaks French.

In sum, under the assumption that a context like (71-b) is sufficient to make the plurality of the two mentioned individual salient, the split conjunctive antecedent case is not problematic for S&S.

26 Here and throughout this paper, we will use $K$ in the meta-language to abbreviate the denotation of the Matrix $K$ operator, i.e., we adopt the following convention: $[[K_x \phi]] \text{ iff } K_x \phi$. 
John speaks French or Japanese.

a. Exclusivity implicature:
   \[ \sim \text{the speaker believes that John doesn’t speak French and Japanese} \]

b. Ignorance inferences:
   \[ \sim \text{the speaker is ignorant whether John speaks French} \]
   \[ \sim \text{the speaker is ignorant whether John speaks Japanese} \]

Sentences like (76) are typically understood as conveying that (a) the speaker believes that John doesn’t speak both French or Japanese, and (b) the speaker is ignorant about whether John speaks French and about whether John speaks Japanese. The inference in (a) corresponds the genuine exclusivity implicature arising from the basic competition between disjunction and conjunction. The inferences in (b) are called ignorance inferences and generally derived on the basis of the competition between the whole disjunction and its independent disjuncts: a disjunction \( \phi \lor \psi \) generates speaker-oriented ignorance inferences about \( \phi \) and about \( \psi \) (e.g., Gazdar 1979). As Meyer shows, the pattern of inferences in (76) can be derived on her proposal with the parse given in (77).<ref>

\[
[\text{exh}_1 [K_s [\text{exh}_2 [\text{John speaks French or Japanese}]]]]
\]

a. Exhaustification below \( K \)
   
   (i) \( I^{exs} = \{ [F and J] \} \)
   
   (ii) Implicature: \( \sim [F and J] \)

b. Exhaustification above \( K \)
   
   (i) \( I^{exs} = \{ [K_s [\text{exh}_2 F], [K_s [\text{exh}_2 J], [K_s [\text{exh}_2 F and J], [K_s F], [K_s J], [K_s F and J] \}
   
   (ii) Implicatures:
   \[ \sim K_s[F \land \sim J], \sim K_s[J \land \sim F], \sim K_s[F and J], \sim K_s[F], \sim K_s[J] \]

27 For clarity, we use \( F \) and \( J \) as short forms for the sentences \textit{John speaks French} and \textit{John speaks Japanese}, respectively. As is customary, we write \( I_s(\phi) \) for ‘the speaker \( s \) is ignorant about \( \phi \), where \( I_s(\phi) \) holds if and only if both \( \sim K_s(\phi) \) and \( \sim K_s(\sim \phi) \) hold.

28 Following Meyer (2013), we assume that the matrix \( K \) operator is immune from deletion: if a sentence includes \( K \) then \( K \) is preserved in the structural alternatives to that sentence. We do not assume however that this immunity applies to the (embedded) occurrences of \textit{exh}, which can be deleted as usual by substitution with a subconstituent (for a different view, see Meyer 2013).
ignorance inferences obtain by computing the implicatures associated with the structural alternatives to $\text{exh}_x$’s prejacent corresponding roughly to its independent disjuncts, both with and without $\text{exh}_z$, all of which are innocently excludable. The resulting outcome, (77-c), delivers the pattern of inferences we were after, (76).

As Marty & Romoli (2019) show, integrating Meyer’s (2013) proposal to M&M’s system predicts the implicatures in (77-b) to be mandatory and, consequently, the resulting ignorance inferences to be obligatory. The reason for that is that, in M&M’s system, the only way an implicature can be avoided is if the alternative it is based on can be pruned from the set of relevant propositions $R$. Disjunctive sentences are well-known to be subject to additional discourse conditions. In particular, it is generally the case that for a disjunction to be felicitous its disjuncts have to be understood as relevant alternatives (Simons 2001; see also Fox 2007, Singh 2008, Fox & Katzir 2011, Marty & Romoli 2019 for discussion). In other words, whenever a disjunction is relevant so are its disjuncts (i.e., neither of the disjuncts can be pruned from $R$ if the whole disjunction is itself in $R$).29 We adopt this line of explanation to account for the general observation that disjunctions give rise to obligatory ignorance inferences in ordinary conversations, and in particular for the observation that examples like those in (41) are infelicitous in run-of-the-mill contexts:

\begin{enumerate}
\item #I have two or more children.
\item #My wife is from France or Italy.
\item #Sue had lunch with Noah or me yesterday.
\end{enumerate}

We will now see that this line of explanation extends to disjunctive presuppositions, accounting for our novel cases of presupposed speaker’s ignorance.

29 This rule of thumb holds for all the cases which matter for us, namely those cases in which disjunctive sentences readily give rise to seemingly non-cancellable ignorance inferences. There are yet two main exceptions to this rule having to do with cases like (i) and (ii):

(i) A: Did the candidate take Logic or Algebra?  
B: Yes, she did (take Logic or Algebra).

(ii) Either the candidate took Logic, or I’m the King of France.

In the first case, a disjunctive sentence is uttered as an answer to a Yes-No question. In this case, the partition of the context set associated with the question has two cells, one in which the candidate didn’t take either subject, and one in which she took at least one of them and possibly both. Given this partition, the disjunction is relevant (Logic or Algebra) but neither its independent disjuncts (Logic, Algebra), nor their conjunction (Logic and Algebra) are. Crucially, as expected on a relevance-based approach, we observe that no exclusivity or ignorance implicature is drawn from B’s reply in this context. The same observation holds of the second case in (ii): these are cases where, given the evident falsity of the second disjunct, the hearer will conclude that the candidate took Logic (see among others Simons 2001, Chemla 2010).
7.2.2 Presupposed speaker’s ignorance

Given some natural assumption about the Matrix $K$ operator, Meyer’s (2013) proposal can be integrated to the M&M system so as to account for the presuppositional variants of (76). In Marty & Romoli (2019), this is done by refining the semantics of $K$, as in (78), so as to offer a proper treatment of presuppositions: (78) simply states that presuppositions project universally under $K$. As Marty & Romoli (2019) note, this refinement corresponds to what is predicted by standard accounts of presupposition projection under attitude predicates (see Heim 1992 among others).

\[
[[K_S]] = \lambda w: \forall w' \in Dox(w)[p(w')] . \forall w' \in Dox(w)[[[S_p]](w')]
\]

With this refinement in place, we turn to show that a similar explanation as that given above for (76) extends to speaker-oriented presupposed ignorance cases like (79):

\[
\text{Noah is unaware that John speaks French or Japanese.}
\]

a. Presupposed exclusivity implicature:
   \sim the speaker believes that John doesn’t speak French and Japanese

b. Presupposed ignorance inferences:
   \sim the speaker is ignorant whether John speaks French
   \sim the speaker is ignorant whether John speaks Japanese

The sentence in (79) conveys the same exclusivity and ignorance implicatures as the ones in (76). However, those implicatures can only be derived in the case of (79) on the basis of its presuppositional competitors. In a system combining M&M’s approach with Meyer’s (2013) proposal, call it the M&M&M system, the pattern of inference in (79) can be derived in a way completely parallel to what we saw in (77) by means of the parse in (80).

\[
[\text{exh}_\beta] [K_s \text{exh}_\beta [\text{Noah is unaware that John speaks French or Japanese}}]
\]

To explain why (80) gets us the desired results, consider first the logically non-weaker alternatives to $\text{exh}_\beta$’s prejacent schematised in (81-a): these alternatives are all Strawson-entailed by $\text{exh}_\beta$’s prejacent, i.e. $[\text{Noah is unaware that } F \text{ or } J][F \text{ or } J]$, and therefore they all qualify as presuppositional alternatives. Of those presuppositional alternatives, only the one with the conjunctive presupposition, namely $[F \text{ and } J]$, is innocently excludable, (81-b). The outcome of this first exhaustification process is thus as shown in (81-c): it is asserted that Noah is unaware that John speaks French or Japanese, and it is presupposed that John speaks French or Japanese but not both (i.e., $[F \text{ or } J] \land \neg[F \text{ and } J]$).

---

30 Here and in the following, we use the notation ‘$S_p$’ to represent any sentence $S$ with presupposition $p$. This notation is only meant to facilitate the understanding of the examples we will work through.


(81) a. \( \text{ALT}^{prs}(\mathbb{2}) = \{ [\text{Noah is unaware that F}], \\
[\text{Noah is unaware that J}], \\
[\text{Noah is unaware that F and J}] \} \)

b. \( \text{IE}^{prs}(\mathbb{2}) = \{ [\text{Noah is unaware that F and J}] \} \)

c. \( \text{EXH}_{\mathbb{2}} [\text{Noah is unaware that John speaks French or Japanese}] \)

(i) Presupposition: \( [F \text{ or } J] \land \neg [F \text{ and } J] \)

(ii) Assertion: Noah is unaware that F or J

As before, this result does not account by itself for the full pattern of inferences in (79). However, once we put it together with the rest of the LF in (80), we derive the presupposed ignorance inferences we are after. To see this, consider first the set of structural alternatives to \( \text{exh}_{\mathbb{2}} \)'s prejacent in (82), all of them are logically non-weaker than \( \text{exh}_{\mathbb{2}} \)'s prejacent and thus potential candidates for exclusion.

(82) \( \text{ALT}(\mathbb{1}) = \{ [K_s \ [\text{exh}_{\mathbb{2}} [\text{Noah is unaware that F}]]], \\
[K_s \ [\text{exh}_{\mathbb{2}} [\text{Noah is unaware that J}]]], \\
[K_s \ [\text{exh}_{\mathbb{2}} [\text{Noah is unaware that F and J}]]], \\
[K_s \ [\text{Noah is unaware that F}]], \\
[K_s \ [\text{Noah is unaware that J}]], \\
[K_s \ [\text{Noah is unaware that F and J}]] \} \)

The last three alternatives in (82), those without \( \text{exh}_{\mathbb{2}} \), are presuppositional alternatives: they carry stronger presuppositions and they are Strawson-entailed by \( \text{exh}_{\mathbb{2}} \)'s prejacent. By contrast, the first three alternatives, those with \( \text{exh}_{\mathbb{2}} \), are assertive alternatives: they all carry stronger conjunctive presuppositions, but they are not Strawson-entailed. Thus for instance, on its strengthened meaning, the alternative \( [K_s \ [\text{exh}_{\mathbb{2}} [\text{Noah is unaware that F}]]] \) is equivalent to \( [K_s \ [\text{Noah is unaware that F}] \land [\text{Noah is aware that J}]] \) which, by the projection rules we assume, presupposes \( K_s [F \land J] \). The sets of excludable presuppositional and assertive alternatives to \( \text{exh}_{\mathbb{2}} \)'s prejacent are thus as shown in (83-a) and (83-b).

(83) a. \( \text{E}^{prs}(\mathbb{1}) = \{ [K_s \ [\text{Noah is unaware that F}]], \\
[K_s \ [\text{Noah is unaware that J}]], \\
[K_s \ [\text{Noah is unaware that F and J}]] \} \)

b. \( \text{E}^{asr}(\mathbb{1}) = \{ [K_s \ [\text{exh}_{\mathbb{2}} [\text{Noah is unaware that F}]]], \\
[K_s \ [\text{exh}_{\mathbb{2}} [\text{Noah is unaware that J}]]], \\
[K_s \ [\text{exh}_{\mathbb{2}} [\text{Noah is unaware that F and J}]]] \} \)

The next step is to determine the sets of \( \text{IE}^{prs} \) and \( \text{IE}^{asr} \) alternatives. For the presuppositional alternatives, it is easy to verify that the presuppositions of the alternatives in (83-a) can be consistently negated all together without contradicting \( \text{exh}_{\mathbb{2}} \)'s prejacent. Therefore, the alternatives in (83-a) are all \( \text{IE}^{prs} \) alternatives, (84-a). The presuppositional implicatures
associated with them are given in (84-b).

\[(84)\]

\[\text{IE}^{prs}(\, \overline{3} \, ) = \left\{ \\
K_s [\text{Noah is unaware that F}], \\
K_s [\text{Noah is unaware that J}], \\
K_s [\text{Noah is unaware that F and J}] \right\} \]

b. Presuppositional Implicatures: \(\neg K_s[F], \neg K_s[J], \neg K_s[F \text{ and } J] \)

Recall now that the set of \(\text{IE}^{asr}\) alternatives corresponds to the intersection of all maximal sets of assertive alternatives that can be negated consistently with the prejacent and the negation of the presupposition of all \(\text{IE}^{prs}\) alternatives. In the present case, all the assertive alternatives in (83-b) carry the conjunctive presupposition \(K_s[F \text{ and } J]\). Since this presupposition is inconsistent with the presuppositional implicatures above, none of them is innocently excludable (i.e., \(\text{IE}^{asr}(\, \overline{3} \, ) = \emptyset\)) and therefore, \(\text{EXH}_3\)'s prejacent doesn’t give rise to any assertive implicature. We end up for (80) with the final representation in (85): it is asserted that the speaker believes that Noah is unaware that John speaks French or Japanese, and it is presupposed that (i) the speaker believes that John speaks French or Japanese but not both (from (81-a) by projection), and that (ii) the speaker is ignorant as to whether John speaks French and as to whether John speaks Japanese (from (i) and (84)).

\[(85)\]

\[\text{EXH}_3 \left[ K_s \left[ \text{EXH}_2 \left[ \text{Noah is unaware that John speaks French or Japanese} \right] \right] \right] \]

a. Presupposition: \(K_s[[F \text{ or } J] \land \neg [F \text{ and } J]] \land L[F] \land L[J] \)

b. Assertion: \(K_s[\text{Noah is unaware that F or J}] \)

Just as before, our assumptions about the calculation of relevance for disjunctive sentences make it so that, since the (disjunctive) prejacent of \(\text{EXH}_3\) has to be relevant for the whole sentence to be felicitous, its presuppositional alternatives presupposing the independent disjuncts must also be relevant; consequently, those alternatives and their presuppositions cannot be pruned from the domain of quantification of \(\text{EXH}_3\), which makes in turn the presuppositional implicatures \(\neg K_s[F] \) and \(\neg K_s[J] \) obligatory.\(^{31}\) The obligatory generation of those inferences in run-of-the-mill contexts accounts for Marty & Romoli’s observation that the infelicity effects in (41) reproduce in embedded cases like (40):

\[(40)\]

\[\begin{array}{l}
\text{a. } #\text{Noah is unaware that I have two or more children.} \\
\text{b. } #\text{Sue didn’t realize that my wife is from France or Italy.} \\
\text{c. } #\text{Mary was sorry that Sue had lunch with Noah or me yesterday.}
\end{array}\]

\(^{31}\) This result relies on the fact that, if a sentences \(S_p\) is relevant given a partition \(Q\), then its presuppositions \(p\) is also relevant given \(Q\). A short proof of this fact can be given by \textit{reductio ad absurdum}. For the argument, suppose that \(S_p\) is relevant given \(Q\), but that \(p\) is not. Since \(p\) isn’t relevant, then, for some cell \(q \in Q\) and some worlds \(w, w' \in q\), we have \(p(w) \neq p(w')\). It follows then that, for some cell \(q \in Q\) and some worlds \(w, w' \in q\), we also have \(S_p(w) \neq S_p(w')\). But it follows now that \(S_p\) is not relevant given \(Q\) and a contradiction obtains. Therefore, \(p\) must be relevant given \(Q\) if \(S_p\) is.
In sum, integrating Meyer’s (2013) proposal to the M&M approach, as recently proposed by Marty & Romoli (2019), offers a way to derive in the grammar presupposed ignorance inferences and to account at the same time for the fact that those inferences are speaker-oriented. We will now see that the M&M&M approach also captures the cases of presupposed ignorance involving too as well as its problematic variants, thus addressing the presupposed ignorance challenge in full.

7.2.3 Presupposed common ignorance and its variants

The M&M&M approach permits us to explore in detail the interactions between too and the ignorance implicatures discussed in (76)/(77) and, in particular, to study how the additional use of too affects the presupposition of these implicatures and subsequently the presuppositional strength of the whole sentence. So consider the variant of (76) where, as before, the focus-sensitive particle too associates with the focused element John:

(86) John, too, speaks French or Japanese.

On the M&M&M approach, this sentence allows different parses which differ in terms of the scopal relations between too and the mechanisms responsible for ignorance inferences. As a starting point, consider the parse in (87) where too occurs below K, while an instance of exh occurs below it to associate with the disjunction or.32

(87) \[
\text{exh}_3 [K_s [too [John_F \lambda x \text{exh}_2 [x \text{speaks French or Japanese}]]]]
\]

a. Exhaustification below K and too:
   (i) \(I_E^{\text{asr}} = \{F \land J\}\)
   (ii) Implicature: \(\neg[F \land J]\)

b. Presupposition projecting under too following (a):
   \(\sim\) some salient \(x\) distinct from John speaks \(F\) or \(J\), but not both

c. Exhaustification above K:
   (i) \(I_E^{\text{asr}} = \{[K_s \text{exh}_2 \text{too F}], [K_s \text{exh}_2 \text{too J}], [K_s \text{exh}_2 \text{too F and J}], [K_s \text{too F}], [K_s \text{too J}], [K_s \text{too F and J}]\}\)
   (ii) Implicatures: \(\neg K_s [\text{too } F \land \neg J], \neg K_s [\text{too } J \land \neg F], \neg K_s [\text{too } F], \neg K_s [\text{too } J]\)

d. Additional presuppositions triggered by too following (c):
   \(\sim\) some salient \(x\) distinct from John speaks \(F\), but not \(J\).
   \(\sim\) some salient \(y\) distinct from John speaks \(J\), but not \(F\).

On this parse, too and our set of implicatures interact in two different ways. First, an exclusivity implicature is computed in the scope of too, (87-a), which gives us the presupposition that some salient \(x\) distinct from John speaks French or Japanese, but not both, (87-b). Second, ignorance inferences are computed on the basis of alternatives including too, (87-c), which

32 We thank Uli Sauerland for discussion of these data and analytical options.
gives us by projection the additional presuppositions that some salient x distinct from John speaks French but not Japanese and some salient y distinct from John speaks Japanese but not French. (87-d). On this parse, the sentence in (86) presupposes therefore that there are two salient individuals x, y, distinct from John, such that x speaks French but not Japanese and y speaks Japanese but not French. This analysis correctly predicts (86) to be felicitous in a context where a split conjunctive antecedent is available, as in (88-a), and correctly predicts (86) to be infelicitous in contexts where only a simple or a non-split conjunctive antecedent is available, (88-b) and (88-c). However, this parse does not explain why (86) is also felicitous with genuine disjunctive antecedents, (88-d).

(88) Predictions from the first parse in (87)

a. Split conjunctive antecedent
   Context: Mary speaks French and Bill speaks Japanese.
   John, too, speaks French or Japanese.
   ✔ Predicted: Ok, Observed: Ok

b. Simple antecedent
   Context: Mary speaks French.
   #John, too, speaks French or Japanese.
   ✔ Predicted: Odd, Observed: Odd

c. Non-split conjunctive antecedent
   Context: Mary speaks French and Japanese.
   #John, too, speaks French or Japanese.
   ✔ Predicted: Odd, Observed: Odd

d. Disjunctive antecedent
   Context: Mary speaks French or Japanese.
   John, too, speaks French or Japanese.
   ✗ Predicted: Odd, Observed: Ok

The felicity of (86) with disjunctive antecedents is yet accounted for once we consider another possible parse for (86) on which too takes scope over the whole sentence as follows:

(89) too ∪ [exh₁ [ks₁ John ↑ λx exh₂ [x speaks French or Japanese]]]

a. Exhaustification below K:
   (i) IEassr = { [F and J] } 
   (ii) Implicature: ¬[F and J]

b. Exhaustification above K:
   (i) IEassr = { [ks₂ [exh₂ F]], [ks₂ [exh₂ J]], [ks₂ [exh₂ F and J]], [ks₁ F], [ks₁ J], [ks₁ F and J] } 
   (ii) Implicatures: ¬ks₂ [F∧¬J], ¬ks₂ [J∧¬F], ¬ks₁ [F], ¬ks₁ [J]

c. Presuppositions triggered by too following (a) and (b):
   ¬some salient x distinct from John speaks F. or J. but not both, but the speaker is ignorant whether x speaks F. and whether x speaks J.
On this alternative parse, all implicatures are computed in the scope of *too*, which results in the presupposition in (89-c): *some salient* *x* *distinct from John speaks French or Japanese but not both, and the speaker is ignorant whether* *x* *speaks French and whether* *x* *speaks Japanese*. As a result, this parse is only compatible with antecedents expressing similar forms of ignorance on the part of the speaker towards some individual other than John. The predictions for split conjunctive and disjunctive antecedents in (87) are now reversed, while, crucially, the predictions for the simple antecedent and the non-split conjunctive antecedents are the same as before, that is, they are both predicted to be infelicitous under this parse as well.

(90) Predictions from the second parse in (89)

a. **Split conjunctive antecedent**
   Context: Mary speaks French and Bill speaks Japanese.
   John, too, speaks French or Japanese.
   ✗Predicted: Odd, Observed: Ok

b. **Simple antecedent**
   Context: Mary speaks French.
   #John, too, speaks French or Japanese.
   ✓Predicted: Odd, Observed: Odd

c. **Non-split conjunctive antecedent**
   Context: Mary speaks French and Japanese.
   #John, too, speaks French or Japanese.
   ✓Predicted: Odd, Observed: Odd

d. **Disjunctive antecedent**
   Context: Mary speaks French or Japanese.
   John, too, speaks French or Japanese.
   ✓Predicted: Ok, Observed: Ok

In sum, we have shown that S&S’s proposal fails to extend to some variants of their second motivating case. In reply to these challenges, we have proposed an alternative account of the previous and new data based on the interactions of *too* with the ignorance inferences associated with disjunctive sentences, and show that those interactions allow different parses which can license either split conjunctive or disjunctive antecedents, but neither simple nor non-split conjunctive antecedents. The resulting account was shown to reproduce the predictions of S&S’s system for the basic case of presupposed common ignorance and to extend to its variants.

In the next and final subsection, we turn to briefly discuss how Meyer’s proposal can be integrated to the PIP and LI approaches and how the resulting systems fare with respect to the the presupposed ignorance cases.
7.3 Other approaches

We discussed how adding Meyer’s (2013) proposal to M&M’s approach accounts for the presupposed ignorance cases. Can we achieve the same results with the other two approaches? That is, can we add K to the system of S&S or to that of Anvari (2018) to account for those cases? To explore this question, consider adding K to a system that includes the exhaustivity operator from S&S (see definition in (10)) together with a pragmatic principle like the PIP or LI, and consider again a case of speaker-oriented ignorance like (91):

(91) #Noah is unaware that I speak French or Japanese

As a first step, we can look at the parse in (92), which involves K but no exh, the presupposition of which is that the speaker believes that he speaks French or Japanese.

(92) [Ks [Noah is unaware that I speak French or Japanese]]K,F or J

The alternatives to consider on this parse also include K, as illustrated in (93-a) and (93-b). Note that the presuppositions of those alternatives are stronger than that (92) and logically independent from their counterparts without K: they presuppose that the speaker believes that he speaks French, and that the speaker believes he speaks Japanese, respectively.

(93) a. [Ks [Noah is unaware that I speak French]]K,F
    b. [Ks [Noah is unaware that I speak Japanese]]K,J

It is easy to imagine a context in which the presuppositions of (93-a) and (93-b) are not satisfied, i.e., a context in which it is not common knowledge whether the speaker believes that he speaks French and whether he believes that he speaks Italian. While (91) remains infelicitous in such contexts, a principle like the PIP cannot account for its infelicity because the presuppositions of (93-a) and (93-b) are not satisfied. Similarly, LI cannot account for the infelicity of (92) either: in such contexts, (92) does not contextually entail either of (93-a) and (93-b).33 Crucially, we note that adding an occurrence of S&S’s exhaustivity operator below and above K, as shown in (94), does not change the situation since their exhaustivity operator is predicted to be vacuous here, in both positions.

(94) EXH,F [Ks [EXH,F [Noah is unaware that I speak French or Italian]]]

33 It is conceivable that something along the lines of (i) be naturally assumed by the interlocutors in a normal conversation. Yet as far as we can see, making that assumption common knowledge would not solve the issue at hand: (i) does not permit the presuppositions of (93-a) and (93-b) to be satisfied in the context, nor does it permit (92) to contextually entail (93-a) or (93-b).

(i) If an individual x speaks a language L, then x knows that x speaks L.
Finally, let us consider the presupposed common ignorance cases. For those cases, adding K to S&S’s system helps with only one of the problematic variants of presupposed common ignorance, repeated below. Specifically, it helps in correctly predicting the felicity of (71-b), but not with the incorrect prediction about (71-a).

(71) a. **Variant 1: non-split conjunctive antecedent**  
    Context: Mary speaks French and Japanese.  
    #John, too, speaks French or Japanese.

b. **Variant 2: split conjunctive antecedent**  
    Context: Mary speaks (only) French and Bill speaks (only) Japanese.  
    John, too, speaks French or Japanese.

To illustrate, consider the non-split conjunctive antecedent case first, and assume that it is common knowledge that Mary speaks French and Japanese and that the speaker believes that Mary speaks French and Japanese. Recall that the problem for S&S was that while (71-a) appears infelicitous in this context, there is a parse of it under which it is incorrectly predicted to be felicitous, namely (73).

(73)  

If we add K to this parse, as in (95), the assertion becomes that the speaker believes that John speaks French or Japanese but not both, and the presupposition becomes that some salient individual other than John speaks both French and Japanese. This presupposition obtains by projecting the presupposition of the negated conjunctive alternative to exf’s prejacent, which includes the presupposition trigger too. Once again, that presupposition is satisfied in the context of (71-a) and no alternative has a stronger presupposition. Hence, under this parse, the sentence is still incorrectly predicted to be felicitous.

(95) K [exf [too [John speaks French or Japanese]]]

Let us now turn to the split conjunctive antecedent case, (71-b), and see how K can help in this case. Recall that the problem there was that, under all parses, (71-b) was incorrectly predicted to be infelicitous. The question in this case is whether adding K to any of those parses would help. To address this question, let us refine the context above by assuming that the speaker knows that Mary speaks only French and that Bill speaks only Japanese, and consider each of the parses in (96) in turn.

(96) a. K [exf [too [John speaks French or Japanese]]]  
    b. K [exf [too [John speaks French or Japanese]]]  
    c. exf [K [too [John speaks French or Japanese]]]

It is easy to see that (96-a) and (96-b) are still predicted to be infelicitous. (96-a) is deemed infelicitous by the PIP because of the alternatives in (97-a) and (97-b), both of which have
stronger presuppositions which are satisfied in the context (i.e., the speaker knows that someone other than John speaks French and that some other individual speaks Japanese). Adding \( \text{exh} \) below \( K \) does not improve on that result: if an exclusivity implicature is computed in the scope of \( K \), then (96-b) gives rise to the presupposition that the speaker believes that some salient individual other than John speaks both French and Japanese, but that presupposition is not satisfied; if that implicature is suspended, then (96-b) amounts to (96-a) and, therefore, it is deemed infelicitous by the PIP for the same reason as (96-a).

\[
(97) \begin{align*}
\text{a. } & [K_s \text{ [too [John speaks French]]}]_{K_s[F]} \\
\text{b. } & [K_s \text{ [too [John speaks Japanese]]}]_{K_s[J]}
\end{align*}
\]

The last parse in (96-c), however, has the potential to make the right prediction: if the exhaustification process above \( K \) is performed against the alternatives in (98), we can now derive implicatures such as \( \neg[K_i\text{ [too F]}]_{K_i[F]} \) and \( \neg[K_i\text{ [too J]}]_{K_i[J]} \), which convey that the speaker isn’t sure that that John (also) speaks French and that the speaker isn’t sure that John (also) speaks Japanese while adding the presupposition that the speakers believes that someone other than John speaks French and the presupposition that the speaker believes that someone other than John speaks Japanese.\(^{34}\) Those presuppositions are satisfied in the context and, crucially, they contribute to strengthen the meaning of the target sentence in a way that renders the application of the PIP vacuous. In particular, one can verify that the sentences in (97-a) and (97-b) do not have stronger presuppositions.

\[
(98) \quad \text{ALT} = \left\{ [K_s \text{ [too [John speaks French]]}]_{K_s[F]}, [K_s \text{ [too [John speaks Japanese]]}]_{K_s[J]} \right\}
\]

In sum, integrating \( K \) to S&S’s system does not help with the incorrect prediction in the non-split antecedent case in (71-a), but it allows to correctly predict the felicity of (71-b) on a parse along the lines of (96-c). This is essentially similar to the line of explanation we proposed and discussed in (87) when exploring the predictions of the M&M&M approach. We take this last result to provide further evidence that integrating \( K \) to S&S’s system may in fact be a necessary addition in order to temper further the application of the PIP (see fn.22 for discussion). Nonetheless, such an addition remains insufficient to solve all of the problematic cases. We therefore conclude for now that the resulting systems do not fully address the presupposed ignorance challenge.

8 Discussion

Maximize Presupposition! (MP), as originally proposed in Heim 1991 and developed in

\(^{34}\) Note that it is critical in (98) to assume that the conjunctive alternative \( [K_i\text{ [too F and J]]}]_{K_i[F \text{ and J}]} \) is pruned from the domain of \( \text{exh} \) for the implicature associated with that alternative would otherwise add the false presupposition that the speaker believes that some contextually salient individual other than John speaks both French and Japanese.
subsequent works, offers an account of the otherwise mysterious (un)assertability of a variety of sentences against given contextual information. At the core of MP is the idea that speakers are urged to use a sentence $\phi$ over a sentence $\psi$, if, in the context of use, $\phi$ contributes the same new information as $\psi$, yet carries a stronger presupposition. As such, MP is based on two main ingredients: (i) contextually equivalent competitors, (ii) the stronger presuppositions of which is satisfied in the context. Classical cases motivating MP involve for instance competing sentences with an indefinite article and its definite counterpart in contexts where the uniqueness presupposition of the latter is satisfied and thus where both sentences convey the same information (e.g., #A sun is shining vs. The sun is shining). While MP has been refined and extended in different ways throughout the years, most (if not all) of its formulations had retained those two ingredients.

Recently, however, the empirical adequacy of the classical principle has been questioned upon the discovery of novel cases which appear very similar to the classical ones, yet do not seem to meet the conditions in regard of the underlying competition which could account for their infelicity. This has led Spector & Sudo 2017 to propose a novel, less restrictive principle, the Presupposed Ignorance Principle (PIP), which extends the MP-competition to sentences which are not contextually equivalent. That is, the PIP drops the first ingredient of the MP principle, by allowing competitors which are not contextually equivalent to the base sentence, as long as their stronger presupposition is satisfied in the context. And this principle, in combination with a theory of scalar implicatures, is shown by Spector & Sudo 2017 to account for the novel cases and the classical ones as well.

In this paper, we have presented a body of evidence showing that this novel principle is both too liberal and too restrictive: the PIP overgenerates infelicity for a variety of felicitous cases involving existential presuppositions, cardinal partitives, and restrictors, and it undergenerates for variants of S&S’s original presupposed ignorance cases. We next turned our attention to two recent alternative accounts, both of which re-introduce stricter conditions on competing alternatives while broadening the scope of application of MP, covering the classical MP cases and the asymmetry discussed in Spector & Sudo (2017).

The first approach is based on the principle proposed in Anvari (2018, 2019), Logical Integrity (LI), whose formulation appeals to the weaker notion of contextual entailment. This approach was found to lead to similar overgeneration issues as S&S’s proposal and more subgeneration issues than S&S’s proposal. The second approach is based on the grammatical theory of scalar implicatures stemming from Magri (2009) and further developed in Marty (2017, 2019b) and Marty & Romoli (2019), which subsume the conditions on contextual equivalence and presupposition satisfaction from MP under the broader notion of relevance. This approach was found to account for one of the overgeneration cases we raised, the case of cardinal partitives, but the other two remain challenging for this approach as well. We have shown that, once combined with Meyer’s (2013) approach to ignorance implicatures, this approach also accounts for the presupposed ignorance challenge in full, capturing the original cases by Spector & Sudo (2017) and the problematic variants that we unveiled in this paper. As we discussed, Meyer’s (2013) proposal is also compatible with the PIP and LI, but the resulting systems, as they stand, remain unable to achieve similar results.
In the end, the present investigation offers a systematic comparison of four approaches to MP and MP-like phenomena against a wide range of old and novel cases. Each of those approaches differs in critical ways from one another with respect to how they define the presuppositional competitors to a given sentence and consequently with respect to the conditions under which they predict infelicity effects to arise, as summarised in Table 1.

<table>
<thead>
<tr>
<th>Competitors</th>
<th>Logical criterion</th>
<th>Contextual criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP</td>
<td>stronger presupposition</td>
<td>satisfaction and contextual equivalence</td>
</tr>
<tr>
<td>PIP</td>
<td>stronger presupposition</td>
<td>satisfaction</td>
</tr>
<tr>
<td>LI</td>
<td>non-weaker alternatives</td>
<td>contextual entailment</td>
</tr>
<tr>
<td>Implicature</td>
<td>1E^(prs) alternatives</td>
<td>relevance of presupposition</td>
</tr>
</tbody>
</table>

Table 1 Comparison of the different approaches investigated in this paper along the logical and contextual criteria determining what counts as a competitor on each approach.

The predictions of the approaches against each other and against each of the cases we studied are synthesized in Table 2. One potential benefit of these results is that they provide concrete baselines for evaluating the advantages and drawbacks of each approach and, more generally, for considering the question of the trade-off between the kind of criteria imposed on competing alternatives and the resulting empirical coverage of the different approaches, e.g. by identifying the theoretical moves that permit to account for additional phenomena sometimes at the cost of subsequent overgeneration issues.

<table>
<thead>
<tr>
<th></th>
<th>MP</th>
<th>PIP</th>
<th>LI</th>
<th>Implicature</th>
<th>M&amp;M</th>
<th>MM&amp;M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical MP cases</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>The asymmetry</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Presupposed common ignorance</td>
<td>×</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Presupposed speaker ignorance</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Cardinal partitives</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Existential presuppositions</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Restrictors</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

Table 2 Comparison of the four approaches with respect to the cases discussed: a green ‘✓’ indicates that the approach makes correct predictions for the relevant case, a blue ‘×’ that it leaves the case unaccounted for (subgeneration issue), and a red ‘×’ that it makes incorrect predictions (overgeneration issue).
One interesting finding of this investigation is that none of the accounts we have investigated, with their possible variants, can satisfyingly capture all the cases discussed in the paper. In other words, the issue of how to properly restrict the competition for MP-like cases, accounting for the classical cases, those of Spector & Sudo (2017) and the novel ones we presented here, remains an open challenge for all accounts in the literature. In particular, two cases remain challenging for all three non-MP accounts: the case of existential presuppositions and the case of restrictors, repeated below.

\[(46)\] **Existential Presuppositions**

*Context: all of my students used to smoke.*

If some of my students stopped smoking, Jane will be happy.

\[(48)\] **Restrictors**

*Context: some students met with both Danny and Irene; it is possible that Jane is one of them.*

a. All/Most/The students who met with Danny or Irene understood the puzzle.

b. If Jane met with Danny or Irene, she understood the puzzle.

As we explicitly stated, the problem of existential presuppositions is a problem only if one assumes that presuppositions do not project universally from the scope of existential quantifiers. Experimental support for this assumption is found for instance in Chemla 2009 which found endorsement of universal projection to vary among quantifiers and, crucially, to be lower in cases involving the scope of quantifiers like *few/less than 3/many/most* than others like *each* and *none*, suggesting that presuppositions project existentially or at least less-than-universally from the scope of the former. We note, however, that these findings remain in principle compatible with a theory like the one proposed in Schlenker 2009 in which presuppositions uniformly project universally from all quantifiers’ scope, while some other mechanism would be responsible for the differences observed between quantifiers (e.g., local accommodation). On this alternative view, one could thus argue that, in cases like \[(46)\], what is relevant for the application of the different principles or mechanisms we discussed (PIP, LI, EXH) is not the observed existential presupposition but rather the predicted universal one, solving then the problem of existential presuppositions.

The problem of restrictors may yet remain more recalcitrant. A possible way out would be to adopt a non-presuppositional analysis of the non-emptiness requirement on restrictors, as proposed in Abusch & Rooth (2005) and Schlenker (2012a). One of the arguments for a non-presuppositional analysis comes for instance from examples like \[(99)\], adapted from Schlenker (2012a: 62). The observation here is that \[(99)\] does not convey that *there is at some future time a student who gets a perfect score on the test*, which is yet what \[(99)\] is expected to presuppose if *will* is used for future time reference in that sentence.\[35\]

\[35\] Very similar considerations apply to analysing the compatibility inference of conditionals not as a presupposition (cf. Gazdar 1979).
We note, however, that cases like (99) raise an issue for a presuppositional analysis insofar as one assumes that will must receive a temporal reading. If we concede instead that will may also be used as a modal here, e.g. in creating a nonveridical epistemic space (e.g., Giannakidou 2012, Giannakidou & Mari 2013, 2018) and conveying a conjectural reading (a.o., Palmer 1986, Sarkar 1998, Huddleston 1995, Enç 1996), the issue then fades away. On this modal use, (99) presupposes instead that in the most ideal worlds from the modal base, there is a student who gets a perfect score on the test, expressing the speaker’s weak epistemic commitment to there being a student scoring perfectly on the next test as much as a positive bias towards that unsettled event. In support of this modal use of will in (99), we note for instance that its variant in (100) is infelicitous if it is common knowledge that no student may get a 100 on the next test (it is ok, of course, if it is understood as a touch of irony, precisely because of the speaker’s harmless commitment).

(100) *Context: the next test will be marked out of 90, with no extra credits*
*I’ll give a bottle of wine to every student who gets a 100 on the next test.*

We further hint that cases like (99) may only work that way with future-oriented eventualities, precisely because of the possible modal use of will: in contrast to (99), its past-variant in (101) strongly suggests that there are students who got a perfect score on the last test.

(101) *I gave a bottle of wine to every student who got a perfect score on the last test.*

Finally, and perhaps most importantly, while this line of explanation would solve the problem of restrictors for quantifiers like every, it would not extend to the restrictors of the or most, for which the proper restriction on presuppositional competitors remains an interesting challenge for all non-MP approaches. We hope therefore that the present investigation will invite further works on the relationships between presupposition and scalar strengthening to engage with the issues left open here.
References


Anvari, Amir. 2019. Logical Integrity Ms., IJN-ENS.


Fox, Danny. 2016. On why ignorance might not be part of literal meaning. Commentary on Marie-Christine Meyer at the MIT Workshop on Exhaustivity.


Fox, Danny & Benjamin Spector. 2009. Economy and embedded exhaustification Handout from a talk at Cornell, MIT & ENS.


Magri, Giorgio. 2013. An account for the homogeneity effects triggered by plural definites and conjunction based on double strengthening. *Ms., CNRS*.


Singh, Raj. 2010. Oddness and ignorance inferences. Handout presented at Modularity Reading Group at MIT.
