Non-resolving responses to polar questions: A revision to the QUD theory of relevance

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Abstract.

The influential Question Under Discussion (QUD) theory of discourse (Roberts, 2012) formalizes Grice’s notion of relevance. In this paper, we identify a class of relevant discourse moves where Roberts’s account undergenerates, and propose a more inclusive definition of relevance. For example, if asked Should we cancel the picnic?, one can reply If it rains without fully resolving the question. However, in Roberts’s theory, all relevant responses to polar questions are predicted to fully resolve the question because a relevant answer must eliminate at least one alternative in the QUD. We propose that a non-resolving response to a polar question is relevant if it eliminates a set of worlds that overlaps with only some alternatives in the QUD. The new account turns out to make good predictions in the domain of polar questions, and beyond.

Keywords: relevance, QUD, partial answers, discourse, questions

1. Introduction

The idea that discourse is structured according to Questions Under Discussion (QUDs; Roberts, 2012) has been widely adopted in semantics and pragmatics. In recent years, the notion of a QUD has proven central in the analysis of phenomena as diverse as focus-sensitivity (Beaver and Clark, 2008), presupposition projection (Simons et al., 2010), discourse particles (Rojas-Espóneda, 2014), and donkey anaphora (Champollion et al., 2019), to name a few. One of the strengths of Roberts’s (2012) theory is its formalization of the notion of contextual relevance in terms of the comparatively well-understood semantics of questions as denoting sets of alternatives (see Hamblin, 1973; Groenendijk and Stokhof, 1984; Ciardelli et al., 2018).

Despite the centrality of the QUD theory, there has been surprisingly little empirical evaluation of its key predictions regarding relevance (though see Hyska (2015) for related discussion). In this paper, we show that Roberts’s (2012) account of relevance systematically undergenerates in several cases. The primary challenge comes from relevant responses to polar questions that do not fully resolve the question, as in (1). Under the Roberts (2012) theory, only partial answers to the current QUD can be relevant. Partial answers are answers that move the discourse toward a more informed state by eliminating alternatives. However, despite being intuitively relevant, answer A in (1) does not eliminate any alternatives to the QUD, since it leaves open whether the picnic should be canceled in the event it doesn’t rain.

(1) Q: Should we cancel the picnic?
   A: If it rains.

In fact, Roberts’s (2012) partial answer theory of relevance wrongly predicts that any relevant response to a polar QUD must fully resolve the question. This is because the notions of partial

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answer and resolving answer turn out to be equivalent for polar questions.

If the QUD is a wh-question, one can (usually) eliminate an alternative without resolving the question. For example, in (2), answer A is predicted to be relevant because it eliminates from the QUD any alternatives where neither Jane nor Sue ate the cookies, while leaving behind alternatives where Jane did and Sue didn’t, or Sue did and Jane didn’t, or both did. Whereas A1 eliminates an alternative, A2 does not, and thus fails to be relevant.

(2) Q: Who ate the cookies?
   A1: Jane or Sue did.
   A2: # Jane baked the cookies.

Polar questions, on the other hand, contain only two alternatives. If $p$ is a proposition, then the polar question $?p$ has alternatives $\{p, \neg p\}$. Eliminating either alternative from the context fully resolves the question: If $p$ is eliminated the then $\neg p$ is entailed, and if $\neg p$ is eliminated then $p$ is entailed. This is the underlying reason why equating partial answerhood with relevance cannot explain the availability of non-resolving responses to polar questions.

We observe that a wide variety of non-resolving responses to polar questions turn out to be very natural in discourse. Examples (3-6) illustrate several dialogues in which a polar question $Q$ is responded to with a relevant but non-resolving answer. For the purposes of our analysis, we will focus primarily on the semantic relation between these responses and the QUD. However, two tangentially related properties of these responses bear mentioning: the availability of ellipsis in the A2 answers, and the preference for the rise-fall-rise intonational contour (L*+H L-H%, Ward and Hirschberg, 1985) in each of these examples.

(3) Q: Is John going to Coachella?
   A1: He’s either going to Coachella or Lollapalooza.
   A2: Or Lollapalooza.

(4) Q: Should we cancel the picnic?
   A1: We should cancel if it rains.
   A2: If it rains.

(5) Q: Will Lucy win the race?
   A1: She might win.
   A2: She might.

(6) Q: Did Lucy come in first?
   A: She didn’t come in last.

First, let us discuss the various constructions that give rise to non-resolving responses to polar questions. Each example in (3-6) is weaker than some alternative in the QUD in an ordinary context. In (3), the response is a disjunction of one of the alternatives of the QUD with another proposition. In (4), an alternative is embedded in the consequent of a conditional, and as a result the QUD will only be resolved if the antecedent is determined to be true. In (5), an alternative is embedded under an epistemic possibility modal. Finally, in (6) the response is

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2 We provide examples in dialogue format in order to make the QUD unambiguous. However, in connection with implicit QUDs the facts regarding the (non-elliptical) responses should be identical.

3 As we will discuss in §4.3, this other proposition is generally not arbitrary; usually it is related to another QUD.
asymmetrically entailed by one of the alternatives on the assumption there are more than two competitors in the race. In the literature on relevance, Hyska (2015) has discussed epistemic reports like (5), and Ginzburg (1995, 2012) has discussed both epistemic reports and conditional answers. To our knowledge, this work is the first to recognize the challenge posed by disjunctive answers (3) and answers like (6) that are entailed by some alternative.

1.1. Data discussion: the role of ellipsis and intonation

The availability of elliptical “short answers” to polar questions suggests that the answer is relevant to the immediately preceding question, rather than to some other possible QUD. This provides indirect evidence against an alternative analysis where non-resolving answers shift the current QUD. Jacobson (2016) has argued that short answers are felicitous only if they address an explicit QUD. For example, take (7) below, adapted from Jacobson (2016: e.g. 14). If it were possible to shift the QUD prior to the response (7A1), we would not expect the response to be infelicitous. For example, a possible shifted QUD could be Who left?, which carries no presupposition that the person who left is a mathematics professor. QUD shifting is only possible with a clausal answer (7A2).

(7) Q: Which mathematics professor left the party at midnight?  
A1: #Jill, but she’s not a mathematics professor.  
A2: Jill did, but she’s not a mathematics professor.

Our analysis (to be laid out in §3) does not require any manipulation of the QUD in order to explain the data in (3-6). This greatly simplifies the treatment of non-resolving responses, since any theory that relies on shifting the QUD has to either stipulate new QUDs, or provide a method to independently test for changes in the QUD, which is a difficult problem.

Furthermore, in each of these examples the rise-fall-rise contour (L*+H L-H%, Ward and Hirschberg, 1985) is preferred. This is unsurprising, as this contour has been argued to conventionally denote non-resolving answerhood (Wagner et al., 2013; though see Constant, 2012 and Westera, 2013 for slightly different theoretical perspectives), while a falling contour generally gives rise to exhaustivity inferences (Westera, 2017). This contour is especially crucial in understanding conditional answers like (4), which we do not intend to be interpreted with conditional perfection (Geis and Zwicky, 1971), i.e. pragmatically strengthened to a biconditional. Intonation can disambiguate between these two readings, with rise-fall-rise being consistent with the non-resolving interpretation of interest.

1.2. Roadmap

We have seen that the partial answer theory of relevance encounters systematic problems in accounting for polar question responses. The goal of this paper is to propose a different criterion for relevance that maintains the strengths of the partial answer theory, while making finer-grained distinctions between response types.

Under our proposal, a response to Q can be relevant in one of two ways. It can be a partial answer to Q, or it can eliminate a partial answer from the common ground. An answer satisfying one of these conditions is called a **reductive answer**. Once the definition of relevance is updated in this way, we show that the new data can easily be accommodated, and that the updated
theory is not too permissive; it still rules out discourses that the original theory was designed to exclude. Moreover, the new theory is stated generally enough to make predictions about question types other than polar questions. In §2, we review the theories of relevance that our account builds on. In §3, we present the formal details of the analysis, and show how it works in a few concrete cases. In §4, we point out some desirable predictions of our proposal outside polar questions, and suggest some slight modifications of the account. In §5 we conclude.

2. Background

2.1. Defining Relevance

The importance of relevance to pragmatic theory can be traced back at least to Grice’s (1975), whose Maxim of Relation states that cooperative speakers should strive to make only relevant contributions to the discourse (or simply “Be relevant.”). Subsequently there have been several attempts to make Grice’s idea explicit and fully characterize the conditions under which a discourse move is relevant (Sperber and Wilson, 1986; Ginzburg, 1995, 2012; Roberts, 2012: inter alia). These accounts do not totally agree on two main points: (1) the intended scope of a theory of relevance and (2) the empirical criteria for relevance. We will discuss each of these points in turn. Ultimately we will follow Roberts (2012) in evaluating the relevance of a discourse move relative to a question.

First, “relevant” is a two-place relation (though Grice’s original maxim does not make this explicit), but there is disagreement over what objects are being related. Roberts (2012) and Ginzburg (2012) evaluate the relevance of discourse moves, or speech acts. As for the second argument in the relation, there is more variation across accounts. Sperber and Wilson (1986) evaluate relevance with respect to a context, i.e. a set of assumptions or beliefs. In some work, Ginzburg focuses on characterizing the set of relevant responses to a query (Ginzburg, 2010). Finally, Roberts (2012) evaluates relevance with respect to a question, namely the question under discussion or QUD. Crucially, the QUD need not be the semantic content of any overt query from earlier in the discourse, but it is a part of the discourse context. We adopt Roberts’s view that relevance holds between a discourse move (a speech act) and the QUD (a set of alternatives). For convenience, we will sometimes refer to the discourse move by its semantic content (i.e. a proposition or a question). Furthermore, for clarity we will generally evaluate the relevance of moves with respect to an overt QUD, i.e. direct responses to queries, though we intend for our claims to apply equally to implicit QUDs.

Second, we need some empirical test for determining whether a discourse move is relevant to a given question. One criterion is infelicity, which we take to be evidence for irrelevance. In this respect, the maxim of relation differs from Grice’s other maxims. For example, while it is uncooperative to withhold information or be needlessly verbose, it is not infelicitous. Thus, a felicitous move must generally be relevant to the QUD, with one notable class of exceptions: One can felicitously make metadiscursive moves such as assertions of ignorance (8A), queries about relevance (8B), or clarification requests (8C). We follow Roberts (2012) in considering such responses irrelevant to A’s question, as they do not make progress towards resolving the

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4We are interested in modeling relevance only for information-seeking discourse, and so we restrict our attention to assertions and queries. Different speech acts may prove relevant in other kinds of discourses.
QUD.5

(8) Q: Who is Betsy bringing to the wedding?
   A: I don’t know.
   B: Why does it matter?
   C: Who’s Betsy?

In fact, Ginzburg (1995) suggests a test that distinguishes relevant moves from metadiscursive ones. He notes that question embedding predicates like about and concerning can only be truthfully applied to relevant answers, as judged by native English speakers. This test is illustrated in (9).

(9) Q: When is the train leaving? (adapted from Ginzburg, 1995: e.g. 98)
   a. Jill: At 2:58./In about an hour./In a short while.
      Jill provided information about when the train is leaving.
   b. Jill: I haven’t got a clue./We should be informed of this soon./It doesn’t matter.
      Jill did not provide information about when the train is leaving.

Ginzburg uses this as a test for a notion he calls aboutness, which is closely related to Roberts’s notion of relevance. Ginzburg’s definition of aboutness does not reduce to exhaustive answerhood in the case of polar questions, which distinguishes his notion from Roberts’s relevance. However, the definition is not stated in a possible worlds semantics, making it difficult to integrate with standard dynamic semantics in the tradition of Stalnaker (1978) and Roberts (2012).

2.2. Technical Background

We adopt Stalnaker’s (1978) dynamic model of discourse in which the discourse context is a set of worlds called the context set. Following Groenendijk and Stokhof (1984), we take a question to denote a set of propositional alternatives that form a partition over the context set. Groenendijk and Stokhof first define an equivalence relation over the pairs of worlds where the question abstract has the same extension. We call this relation $$R_Q$$ (10a). From this relation, there is guaranteed to be a unique partition of the context set (10b). We use the term question to refer to this partition, and interrogative to refer to a sentence whose denotation is a question.

(10) **Definition: Question Denotation** If $$Q$$ is an interrogative with LF $$wh_1, ..., wh_n(\beta)$$, where $$wh_i$$ is the $$i^{th}$$ wh-word in $$Q$$, $$D(wh_i)$$ is the domain of $$wh_i$$ (e.g. $$D(who)$$ is the set of humans), and $$\beta$$ is the intension of an n-ary relation whose argument slots correspond to the $$wh$$-words in the question. When $$n = 0$$ (i.e. when $$\beta$$ is a proposition), the resulting partition has at most two cells (a polar question).

In the partition view of questions, we can define several degrees of answerhood.

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5Note that Ginzburg (2010, 2012) does apply the term “relevant” to metadiscursive moves. This may in fact be more faithful to Grice’s original intent, as such moves are arguably cooperative. See Hyska (2015) for discussion of ways to integrate moves of this kind into Roberts’s (2012) QUD framework.
Figure 1: Illustration of the dialogues in (12). The large box represents the context set, divisions correspond to cells in the QUD, and the gold region represents answers. ‘J’ labels the cells in which Jane ate the cookies, etc.

(11) a. **Definition: Resolving Answer**
    Proposition $a$ is a resolving answer to $Q$ iff $\exists q \in Q[a \subseteq q]$
a entails an alternative in $Q$.
b. **Definition: Partial Answer**
    Proposition $a$ is a partial answer to $Q$ iff $\exists q \in Q[a \cap q = \emptyset]$
a eliminates (is inconsistent with) at least one alternative in $Q$.
c. **Definition: Non-eliminating answer**
    Proposition $a$ is a non-eliminating answer to $Q$ iff $\forall q \in Q[a \cap q \neq \emptyset]$
a eliminates no alternative in $Q$.

In (12) below, A, B, and C are three possible responses to the question in $Q$. These different response types are illustrated in Figure 1. A is a resolving answer because it entails exactly one of the alternatives in $Q$. B is a partial answer because it eliminates several alternatives in $Q$ (note that A is technically a partial answer for the same reason). C is a non-eliminating answer because it is consistent with every alternative in A.

(12) Q: Who (of Jane, Lucy, and Steve) ate the cookies?
    A: (Only) Jane did.
    B: Jane or Lucy, but not Steve.
    C: # Jane ate the cake.

Quantified answers (13) are also partial answers. This is because the partition denoted by a *wh*-question is in one-to-one correspondence with the subsets of the *wh*-domain. Thus, any quantifier over that domain picks out a unique set of cells in the partition.

(13) Q: Who ate the cookies?
    A: Every first grader.
    B: Some second grader.
    C: At most one teacher.

The definitions of resolving and partial answers are formulated to include over answers, i.e. answers that strictly entail a union of cells in the partition but are not equivalent to any particular union of cells. For example, in (14) the response in A eliminates all alternatives in which Jane did not eat the cookies. But it also eliminates worlds in the remaining cells in which Jane ate the cookies for breakfast.

(14) Q: Who ate the cookies?
A: Jane had them for lunch.

Roberts’s (2012) QUD theory provides the most influential formal account of relevance to date, as given in (15). Crucially, this definition is built around the notion of partial answerhood. We assume, as Farkas and Bruce (2010) do, that the goal of the conversational participants in information-seeking discourse is to empty the QUD stack. From this perspective, any partial answer makes progress towards this goal by eliminating alternatives from the current QUD. Empirically, this account makes many good predictions for *wh*-questions. For instance, the partial answers in in (12B) and (13) are all correctly predicted to be relevant, and the non-eliminating answer in (12C) is correctly predicted to be irrelevant.

(15) **Definition: Relevance (Roberts, 2012: to be revised)**

Move \( m \) is relevant to the QUD \( Q \) iff \( m \)'s content is (a) proposition that is a partial answer to \( Q \), or (b) a question whose alternatives are partial answers to \( Q \).

The condition in (15) also applies to followup questions. If the QUD \( Q_1 \) cannot be resolved easily, one strategy to answer \( Q_1 \) is to ask an easier followup question \( Q_2 \) which, once resolved, is guaranteed to simplify \( Q_1 \). In Roberts’s account, the stereotypical and rational way to do so is to ensure that \( Q_2 \)'s alternatives are no stronger than \( Q_1 \)'s alternatives.\(^6\)

Sometimes, all the alternatives of the followup question \( Q_2 \) are weaker than the alternatives of \( Q_1 \). For instance, in (16), \( Q_1 \) is a pair-list question whose alternatives are those propositions that specify for each person what that person did and did not eat. On the other hand, the alternatives in \( Q_2 \) are all strictly weaker because they specify what Jane did and did not eat, but say nothing about other individuals.

(16) \( Q_1: \) Who ate what?
    \( Q_2: \) What did Jane eat?

In other cases, \( Q_1 \) and \( Q_2 \) may share some alternatives. For example, in (17), the positive answer to \( Q_2 \) is one of the alternatives to A's question (i.e. it is an exhaustive answer), while the negative answer is the union of the remaining alternatives.

(17) \( Q_1: \) Where did Sammy go?
    \( Q_2: \) Did Sammy go to the store?

By contrast, (18) is an irrelevant followup question because knowing the time that Freddie is coming to dinner is not usually sufficient to eliminate any possibilities regarding what he is bringing.

(18) \( Q_1: \) What is Freddie bringing to the dinner?
    \( Q_2: \) # What time is Freddie coming?

2.3. Problems

Despite some good predictions for *wh*-questions, Roberts’s (2012) theory is inadequate to account for relevance to polar questions. The problem stems from the fact that partial and re-

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\(^6\) If \( Q_2 \) contains alternatives that are stronger than those in \( Q_1 \), then \( Q_2 \) may still be relevant to \( Q_1 \) according to (15) (recall that any resolving answer to \( Q_1 \) is also a partial answer to \( Q_1 \)). However, \( Q_2 \) cannot be considered part of a rational strategy to answer \( Q_1 \) if it turns out to be harder to answer.
solving answerhood are equivalent for polar questions by the definitions in (11). If \( Q \) is a polar question, it contains exactly two alternatives. If \( A \) is a partial answer to \( Q \), it eliminates at least one alternative. But this implies that \( A \) entails a single alternative to \( Q \), and is thus a resolving answer. Thus, any non-resolving answer to a polar question is predicted to be irrelevant under the Roberts (2012) theory. In light of clear counterexamples in (3)-(6), the theory clearly needs to be resolved.

Concretely, the partial-answer theory of relevance does not distinguish between dialogues like (19) and (20). In both cases, the response \( A \) is true at some worlds in the positive answer to the question \( Q \), and some worlds in the negative answer to \( Q \). Because \( A \) overlaps with both alternatives in \( Q \), \( A \) is predicted to be irrelevant to \( Q \).

(19) Q: Is John going to Coachella?
A: He’s going to Coachella or Lollapalooza.

(20) Q: Is John going to Coachella?
A: # Mary is going to Sinn und Bedeutung.

The next section refines the QUD theory of relevance to classify non-resolving responses like (19A) as relevant, but correctly rule out non-resolving responses like (20A). Once the analysis is adopted, we find that it makes desirable predictions in a wider set of cases. We will highlight the predictions about felicitous responses to \( \text{wh} \)-questions and followup questions in §4.

3. Analysis

The guiding intuition of our account is that felicitous non-resolving responses are not logically independent from the QUD alternatives. While learning that (19A) is true does not resolve the QUD (19Q), learning the negation of (19A) would in fact resolve the QUD. In other words, the question of (19A)’s truth is useful to the discourse because the information that (19A) is false would be sufficient to eliminate an alternative in the QUD. We argue that this observation holds true of all the relevant non-resolving responses to polar questions we have discussed. From the perspective that discourse participants are trying to find a strategy to empty the QUD stack, this observation is unsurprising. If resolving the question of whether \( a \) is true has the possibility to resolve the QUD, then asserting \( a \) ought to be considered part of a rational strategy to achieve this conversational goal.

If \( a \) meets this condition, defined formally in (21), then we call \( a \) a reductive answer. One can check whether \( a \) is a reductive answer by checking if either \( a \) or its contextual negation is a partial answer (22). Figure 2 illustrates how reductive answers compare to the other three answer types defined in (11). Clearly, if \( a \) is a partial (or resolving) answer, then \( a \) is a reductive answer. Furthermore, one can see in Figure (2d) that a non-eliminating answer can be a reductive answer as long as its contextual negation (the white region) is a partial answer.

(21) **Definition: Reductive Answer**

Proposition \( a \) is a reductive answer to \( Q \) in context \( c \) iff \( \exists q \in Q \exists b \in \mathcal{A} [c \cap b \cap q = \emptyset] \)

(22) **Equivalent restatement of reductive answerhood**

Proposition \( a \) is a reductive answer to \( Q \) in context \( c \) iff either (a) or (b) holds:

\[
\text{(a) } \exists q \in Q \exists b \in \mathcal{A} [c \cap b \cap q = \emptyset]
\]

\[
\text{(b) } \exists q \in Q \exists b \in \mathcal{A} [c \cap b \cap q = \emptyset]
\]
Figure 2: Different answer types. The large box represents the context set, divisions correspond to cells in the QUD, and the gold region represents answers.

a. \[ \exists q \in Q \left[ c \cap a \cap q = \emptyset \right] \]  
(a is a partial answer to \( q \))

b. \[ \exists q \in Q \left[ (c \setminus a) \cap q = \emptyset \right] \]  
(a’s contextual negation is a partial answer to \( q \))

We are now in a position to redefine relevance as in (23). We follow Roberts (2012) in saying that whether a response is relevant to the QUD depends on the logical relationship between the response and the alternatives in the QUD. However, we build our definition around reductive, rather than partial, answerhood. Thus, our definition is identical to Roberts’s (2012), but with partial answerhood replaced with reductive answerhood.

(23) **Definition: Relevance (ours)**

Move \( m \) is relevant to the QUD \( Q \) iff \( m \)’s content is (a) a proposition that is a reductive answer to \( Q \), or (b) a question whose alternatives are all reductive answers to \( Q \).

The definitions in (21-23) form the core of our proposal. In what follows, we show how to capture the data in (3-6), and explore some other consequences of the account.

3.1. Accounting for the key data

In this section we show how the updated theory (23) predicts that the responses in (3-5) are in fact relevant, despite being non-eliminating answers according to (11). The examples are repeated below as (24-26).

(24) Q: Is John going to Coachella?  
A: He’s either going to Coachella or Lollapalooza.

(25) Q: Will we cancel the picnic?  
A: We’ll cancel if it rains.

(26) Q: Will Lucy win the race?  
A: She might win.

In each case, the negation of the response in B entails an alternative in the polar question A. This is shown graphically in Figure 3. In each diagram in the figure, the negation of the answer (the white region) lies entirely within a single alternative in the QUD. For example, the worlds in the negation-set of (24A) are those worlds where John is going to neither Coachella nor Lollapalooza. In all such worlds, John is not going to Coachella. For (25A), the negation worlds
are those where it rains and the picnic is not cancelled. Crucially, the picnic is cancelled at those worlds. Finally, the negation worlds for (26A) are the worlds where there is no possibility that Lucy wins. In fact, those are all worlds where Lucy will not win.

Although our revised definition of relevance is strictly more inclusive than Roberts’s (2012), it still correctly excludes intuitively irrelevant responses like (27A). (27A) does not count as a reductive answer because neither (27A) nor its negation entails either the positive or negative resolution of (27Q).

(27) Q: Is John going to Coachella?
A: Mary is going to Sinn und Bedeutung.

3.2. Relevance is context-sensitive

Reductive answers, and by extension relevance, are defined in (21) in terms of their dynamic effect on the context set. Accordingly, we predict that whether or not an answer turns out to be relevant depends on the prior beliefs of the discourse participants. This kind of context sensitivity turns out to be necessary to capture certain types of relevant responses. For instance, in example (28) (repeated from (12C)), the response in A sounds deviant because it asserts a proposition that is logically independent from every alternative given by the QUD.

(28) Q: Who ate the cookies?
A: Jane ate the cake.

However, as (29) shows, it is easy to construct a context in which the same response is relevant. If it is common ground that Jane ate no more than one dessert, then this assumption plus the new information that Jane ate the cake jointly entail that Jane did not eat the cookies in the posterior context.

(29) Context: It’s mutually known that Jane is cutting calories and will only eat one dessert.
Q: Who ate the cookies?
A: Jane ate the cake.

Here we assume the conditional response has the truth conditions of the material conditional. In the Appendix, we discuss this example in a restrictor theory of conditionals following Kratzer (1981).

We get this result as long as ¬⋄p entails ¬p. The Appendix discusses this assumption in the context of Kratzer’s (1981) theory of modality.
Thus, the richer context in (29) ensures that A does contextually entail an alternative to the QUD, and is predicted to be relevant.

3.3. Informativity doesn’t matter

A prediction of the account is that even answers with very little informative content can be considered relevant. This situation arises where the contextual negation of the answer has exceptionally low probability, as in (30). As Figure 4 shows, the answers in these examples overlap with almost the entire context set. This means that they provide very little new information. Nonetheless, we predict they are relevant because the eliminated worlds in the white region lie entirely inside one alternative.

(30) a. Context: It’s mutually known that John is in Boston or New York 99% of the time.  
   Q: Is John in New York?  
   A: Or Boston.

b. Child: Can I have a pony?  
   Parent: If I win the lottery.

We consider this a good prediction. However, judging the relevance of these responses is difficult because assertions with low information content have low utility and may give the impression of being a rhetorical responses. For example, the parent’s response in (30b) may come off as a negative answer. Nonetheless, if we assume these are genuine answers, they are intuitively relevant.

3.4. Relation to previous work

It turns out that an answer is reductive (in our sense) just in case it is at-issue in the sense of Simons et al. (2010). Despite the resemblance between these two notions, our proposal differs from Simons et al.’s (2010) in several respects. First, Simons et al. do not change the definition of relevance for assertions, retaining essentially the theory from Roberts (2012). For Simons et al. (2010), not all at-issue responses are relevant, and the authors do not take a position on whether at-issue-ness determines the felicity of discourse moves. Second, Simons et al. are motivated by explaining projective content, while we take no position on whether reductive answerhood plays a role in projection.

Their footnote 3 briefly presages some of the problems for the QUD theory that we identify in this paper, but their proposal does not include an account of data like (3-6). Our goal in the
present work has been to clarify the scope of the problem for Roberts’s theory and explicitly relate our preferred solution to the data.

Others have proposed widening the class of relevant responses by defining relevance without reference to entailment. For example, Büring (2003) and Hyska (2015) suggest that a response can count as relevant as long as it shifts the probability weights of the alternatives in the QUD in a certain way. We find this approach appealing, and more work is needed to compare the predictions of the probabilistic theory to the entailment-based theory advanced here.

4. Further predictions

In this section, we discuss consequences of our proposal beyond answers to polar questions. We find that our account makes good predictions regarding answers to wh-questions and followup questions. We also address certain cases of overgeneration by making other independently needed constraints explicit.

4.1. Wh-questions

Our proposal makes good predictions about responses to wh-questions, despite being formulated with polar questions in mind. Recall that partial answerhood as defined in (11b) is not trivial when applied to wh-questions: a partial answer can be obtained by taking a disjunction of any number of resolving answers, as long as at least one of the cells of partition is eliminated, as illustrated in (12B-C). However, some relevant answers do not fit this disjunctive pattern (31).

(31) Q: Who will come to the picnic?
   A: Jane will, if it doesn’t rain.
   B: Jane might.

First, a partial answer in the consequent of a conditional (31A), is correctly predicted to be relevant in our account, but not Roberts’s (2012). One can show that it is not a partial answer, because it is consistent with every cell in the partition. Indeed, if it doesn’t rain, any cell in which Jane doesn’t come can be eliminated, but if it does rain, any cell in the partition is still an epistemic possibility. By contrast, response A is a reductive answer because it eliminates from consideration any worlds in which it doesn’t rain and Jane won’t come to the party. More generally, embedding a partial answer in the consequent of a conditional does not preserve partial answerhood. However, it does preserve reductive answerhood. If p is a partial answer to Q, then q → p must be a reductive answer to Q, since its negation ¬q ∧ p is stronger than p.

Second, our account correctly predicts a partial answer under a possibility modal (31B) to be relevant. As in §3.1, we assume that p ⊆ ♦ p. The answer in (31B) is not a partial answer, as it fails to eliminate any cells in the partition (any cell in which Jane does or does not come is still an epistemic possibility). However, its negation is a partial answer since ¬♦ p is logically stronger than ¬p, making the answer itself a reductive answer. This reasoning applies in general to any partial answer under a possibility modal. For a more detailed and precise explanation, please see the Appendix §6.
4.2. Followup questions

We find that the original QUD theory of relevance is too restrictive with followup questions—as with answers—when the QUD is a polar question. In Roberts’s (2012) definition of relevance (15), a question is considered relevant iff it is a subquestion as defined in (32).

\textbf{(32)} \textbf{Definition: Subquestion (paraphrased from Roberts, 2012: 6:15; to be revised)}

\[ Q \text{ is a subquestion of } Q' \text{ if and only if every complete answer to } Q \text{ contextually entails a partial answer to } Q'. \]

We consider the follow-up questions in (33) to be relevant. However the negative answer to Q2 is consistent with both answers to Q1, and thus Q2 is not a subquestion according to (32).

\textbf{(33)}

\begin{align*}
\text{Q1: Does Sam have any pets?} \\
\text{Q2: Does she have a dog?}
\end{align*}

In fact, under definition (15), any polar question \(Q_2\) can only follow a polar \(Q_1\) if \(Q_1 = Q_2\), assuming that both questions partition the same context set. The reasoning goes as follows: \(Q_2\) must be a subquestion of \(Q_1\), therefore both alternatives in \(Q_2\) must be a partial answer to \(Q_1\). However, the only partial answers to polar questions are resolving answers, so both alternatives in \(Q_2\) must be a subset of an alternative in \(Q_1\). The only way for this to be true is if \(Q_2\) and \(Q_1\) contain identical alternatives.

By contrast, our account allows for polar questions with non-trivial relevant followups. We revise the notion of a subquestion in (34) simply by replacing partial answerhood with reductive answerhood. According to our definition of relevance in (23), a question is relevant iff it is a subquestion in this new sense. Q2 in (33) comes out as relevant on this account because the positive answer to Q2 is a resolving answer to Q1 (Sam’s having a dog entails her having a pet), while the negative answer is a reductive answer to Q1.

\textbf{(34)} \textbf{Definition: Subquestion (revised)}

\[ Q \text{ is a subquestion question of } Q' \text{ if and only if every cell in } Q \text{ contextually entails a reductive answer to } Q'. \]

However, this account interestingly predicts that for any two polar questions \(?p\) and \(?q\), \(?p\) is a subquestion of \(?q\) iff \(?q\) is a subquestion of \(?p\). All that is required for \(?p\) to be a subquestion of \(?q\) is that one of the alternatives to \(?q\) entails one of the alternatives to \(?p\). But this implies that \(?q\) will also be a subquestion of \(?p\). Hence, we predict that the ordering of the questions in (33) should be reversible. We judge this to be a good prediction, as example (35) shows, though notably we get the additional inference that B was biased towards a negative answer.

We leave an explanation of this inference to future work.

\[9\]In the following examples, it is best to evaluate the felicity of followup questions in a multilogue setting, where the followup question is directed at a participant other than the original asker. The original asker has already indicated ignorance about the original question, hence it is not rational to address a followup to them.

\[10\]There are some theories such as Isaacs and Rawlins’s (2008) in which not all questions on the QUD stack partition the same context set. In particular conditional questions are represented as a partition on the set of worlds satisfying the antecedent of the conditional. While the predictions regarding conditional questions would have to be reevaluated in such an analysis, the predictions for (33) are unchanged.

\[11\]Note that under the new definition, the subquestion relation no longer gives rise to a partial order.
Who went where?

Where did Jane go?  Where did Sally go?

Did Jane go to Lollapalooza?  Did Jane go to Coachella?  ...

Figure 5: A subquestion tree, visualizing the evolution of the QUD stack over time.

(35) A (to B): Does Sam have a dog?
    B (to C): Does Sam have any pets?
    C: No.
    B: Yeah, I didn’t think so.

4.3. Additional constraints on relevance

In this section, we will show that considering QUDs other than the current QUD allows us to explain why some reductive answers are intuitively irrelevant. For example, consider (36), which is a variant of (3) with a different answer. In both dialogues, a question of the form $p$ is answered with a proposition of the form $p \lor q$. According to our definition of relevance (23), $p \lor q$ is relevant regardless of the $q$. Clearly, this is incorrect for (36).

(36) A: Did Jane go to Lollapalooza?
    B: # Or there is life on Mars.

This issue is not actually unique to reductive answers. When relevance is defined in terms of partial answers, a similar problem arises with conjunctive over answers, as in (37). The answer $p \land q$ is always relevant to the question $?p$ because it entails the positive answer $p$.

(37) A: Did Jane go to Lollapalooza?
    B: #Yes, and there’s life on Mars.

We suggest that the irrelevance of the answers (36) and (37) is not due to the logical relationship between the answer and the question, but rather to the structure of discourse. Namely, they irrelevant because they are built from units of meaning that do not come from a question on the QUD stack, as defined in (38).

(38) Ordering of the QUD stack (adapted from Roberts, 2012)
    a. $Q_1 < Q_2$ iff $Q_1$ was asked prior to $Q_2$, and $Q_1$ and $Q_2$ are both unanswered and accepted QUDs.
    b. If $Q_1 < Q_2$ then $Q_2$ must be a subquestion of $Q_1$.

In case that $Q_1 < Q_2$ we say that $Q_1$ is a higher QUD. Generally, a higher question on the stack is more finely divided partition of the context set than the current question, as depicted in Figure 5. We use this notion to define a new constraint Stack Relevance, which responses must satisfy in addition to QUD Relevance (defined in (23)).

(39) Stack Relevance: A felicitous assertion must provide an exact partial answer (a union of alternatives) to some QUD on the stack, either the current QUD, or a higher QUD.
**Formal Statement:** An assertion with content \( p \) is relevant only if there is some \( Q \) on the stack such that \( p = \bigcup R \) where \( R \) is a subset of the partition given by \( Q \).

Stack Relevance is independently needed no matter which version of QUD Relevance we adopt. If we adopt the original notion based on partial answers, Stack Relevance is still needed to rule out (37).\(^{12}\) If we adopt our proposal in (23), then Stack Relevance is also needed to rule out (36) as well. Thus, the problem we address in this section is not unique to our account. Nevertheless, we think it is worth solving, and Stack Relevance is a plausible solution.

With Stack Relevance in place, we are now in a position to explain why (36) is infelicitous while (40) is felicitous. The felicity of (40) depends on whether the QUD stack contains a higher QUD \( Q' \) such that the response (40A) is an exact partial answer to \( Q' \).

(40) \begin{align*}
Q: & \text{Is John going to Coachella?} \\
A: & \text{He’s either going to Coachella or Lollapalooza.}
\end{align*}

A good candidate for \( Q' \) is the \( wh \)-question *Where is John going?*. If we assume that \( Q' \) is this question, and that it satisfies all the conditions to be a higher QUD for (40Q), then (40A) provides an exact partial answer to \( Q' \). In this situation, Stack Relevance is satisfied.

However, we also predict that there is a different possible discourse context in which (40) does not satisfy Stack Relevance. This is because without a richer description of the context, nothing ensures that *Where is John going?* is a higher QUD. For example, the asker might have heard about a terrorist plot at Coachella, in which case their immediate goal in asking (40Q) is to find out whether John will be safe. In this alternative context, the answer (40A) would not meet the conditions for Stack Relevance. The predictions of our account in this area are subtle, and we think more work is needed to understand the judgements.

5. **Conclusion**

The key claim of this paper is that data involving polar questions force us to distinguish carefully among responses that are not partial answers. Some of these non-resolving responses are intuitively irrelevant, and produce incoherent discourses, while others are perfectly natural. All of the natural non-resolving responses (reductive answers) update the context in a particular way, eliminating worlds that verify some but not all of the alternatives given by the QUD. The unnatural responses eliminate worlds from every alternative, or none.

Our theory shares with Roberts (2012) the idea that the logical structure of the QUD determines the set of felicitous next moves at any point in a discourse, though we differ on the exact logical relationship that is required. By eliminating worlds from only some alternatives, reductive answers respect the partition given by the QUD, even though they fail to eliminate any alternative completely. We show in this paper that reductive answerhood proves to be a useful generalization of partial answerhood. By updating the theory of relevance, our aim is to strengthen the empirical foundations of the QUD theory.

\(^{12}\)Note that in order for stack relevance to be non-trivial, we must assume that the Big Question is not present on the stack. In a context \( c \), the Big Question is the partition in which each world in \( c \) occupies its own cell. In other words, it is the question corresponding to "Which world is the actual world?". Since any proposition can be made up as a union of cells in the Big Question, no response is ruled out by stack relevance if the Big Question is on the stack.
6. Appendix: Truth Conditions of Modals and Conditionals

This appendix shows how the results proposed in this paper can be maintained within a more articulated theory of modals and conditionals. Throughout this section, we will consider several responses to a polar question $Q$ whose alternatives are $q$ and $\neg q$. In each case, we will show that the response in question is a reductive answer. Recall the definition of a reductive answer, which we restate in a useful equivalent form in (41).

\begin{equation}
\text{Reductive answer} \quad \text{Proposition } p \text{ is a REDUCTIVE ANSWER to } Q \text{ in context } c \iff \exists q \in Q \left[ (c \cap p \cap q = \emptyset) \lor (c \setminus p \cap q = \emptyset) \right].
\end{equation}


\text{either } p \text{'s contextual meaning or } p \text{'s contextual negation is a partial answer to } Q

In each case below, $a$ will stand for the response to $Q$ under consideration. For each response $a$, we will show that $c \setminus a \subseteq \neg q$.

6.1. Epistemic possibility responses

The main claim of this subsection is that $\text{might } q$ is always a reductive answer to the polar question $?q$. We will adopt a Kratzerian theory where the flavor of a natural language modal expression is modeled as a set of constraints on the modal base of the expression (Kratzer, 1981).

Let $B$ be a \textbf{modal base}, a contextually-supplied function of type $\langle s, \langle s, t \rangle \rangle$. We define possibility modals as follows:

\begin{equation}
\text{Possibility modals} \quad \diamond_B q = \lambda w. \exists w'. B(w)(w') \land q(w')
\end{equation}

For epistemic modals like $\text{might}$, we assume that the modal base is \textbf{realistic}, meaning that $B(w)(w)$ is true for any world $w$.

\begin{equation}
\text{might}-\text{responses} \quad \left[ \text{might } q \right] = \diamond_B(q) = \lambda w. \exists w'. B(w)(w') \land q(w') \quad \text{(where for all } w, B(w)(w) = 1)
\end{equation}

We can now prove the following:

\begin{equation}
\text{Suppose } a \text{ is a response to } ?q \text{ of the form } \text{might } q. \text{ Then } c \setminus a \subseteq \neg q, \text{ and therefore } a \text{ is a reductive answer to } ?q.
\end{equation}

\text{Proof:} If $a$ is as above, then every world $w \in c \setminus a$ satisfies the following formula:

\[ \neg \exists w' [B(w)(w') \land q(w')] \]

In words, there are no worlds $w'$ that are $B$-accessible from $w$ where $q(w')$ is true. But recall that $B$ is a realistic modal base, so $B(w)(w) = 1$ ($w$ is accessible from itself). It follows that $q(w)$ is false, so $\neg q(w)$ is true. Thus, $c \setminus a \subseteq \neg q$.

\footnote{Here we adopt the shorthand where $p$ refers to both the function of type $\langle s, t \rangle$ and the set of worlds $w$ satisfying $p(w)$.}
6.2. Conditional antecedents

The theory we are considering is one in which conditional antecedents serve as restrictors of modal operators (Kratzer, 1981). First, we will define some useful notation. As before, $B$ is a modal base, a contextually-supplied function of type $\langle s, \langle s, t \rangle \rangle$. If $p$ and $q$ are propositions (type $\langle s, t \rangle$), then we define binary and unary necessity modal operators $\Box$ as follows.

\begin{align*}
\text{(45) Necessity modals} \\
\Box_B p(q) &= \lambda w. \forall w'. [B(w)(w') \land p(w')] \rightarrow q(w') \\
\Box_B q &= \Box_B (\top)(q), \text{ where } \top \text{ is the tautological proposition.}
\end{align*}

In words, $\Box_B p(q)$ is true at a world $w$ if and only if: for every world $w'$ that is $B$-accessible from $w$ and where $p$ is true at $w'$, $q$ is also true at $w'$. Simply put, $q$ must be true at all accessible worlds that satisfy the restrictor $p$.

\begin{align*}
\text{(46) Whenever } p \text{ entails } p', \Box_B p(q) \text{ entails } \Box_B p(q). \text{ In other words, } \Box_B \text{ is downward-entailing in its first argument. Thus, } \Box_B q &= \Box_B (\top)(q) \text{ entails all formulas } \Box_B p(q), \text{ for any } p.
\end{align*}

According to the conditionals-as-restrictors framework, every conditional sentence has an implicit (or explicit) modal operator whose flavor is given by the context-sensitive modal base $B$.

Our key dialogue (4) is a question-answer pair where the question contains an explicit modal auxiliary. The example is repeated here for clarity.

\begin{align*}
\text{(47) Q: Should we cancel the picnic?} \\
\text{A: If it rains.}
\end{align*}

We assume that the response involves ellipsis, and its non-elliptical counterpart is (48) below.

\begin{align*}
\text{(48) If it rains, we should cancel the picnic.}
\end{align*}

We further assume that the modal base in (48) must be the same as the modal base in the question. Assuming that $\text{rain}$ and $\text{cancel}$ are propositions, the positive alternative in the question (47) can be paraphrased in logic as (49a) and the response can be paraphrased as (49b).

\begin{align*}
\text{(49) a. Q: } &\Box_B (\lambda w'. \text{cancel}(w')) \\
&= \lambda w. \forall w'. B(w)(w') \rightarrow \text{cancel}(w') \\
\text{b. A: } &\Box_B (\lambda w'. \text{rain}(w')) (\lambda w'' \text{cancel}(w'')) \\
&= \lambda w. \forall w'. [B(w)(w') \land \text{rain}(w')] \rightarrow \text{cancel}(w')
\end{align*}

The key claim is that (49b) is a reductive answer to (49a), though it is not a partial answer. We must show that the negation of (49b) is enough to answer the question negatively. That is, the negation of (49b) entails the negation of (49a).

This result is easily obtained by contraposition: the fact (46) above gives us that (49a) entails (49b), so $\neg (49b)$ entails $\neg (49a)$. To see a more intuitive explanation, consider the negation of (49b). (50) below shows that $\neg (49b)$ entails that there is some $B$-accessible world $w'$ where we don’t cancel the picnic. It should be clear that this entailment contradicts (49a) above, and therefore entails the negative answer to the question.
\( (50) \lambda w. \neg \forall w'. [B(w)(w') \land \text{rain}(w')] \rightarrow \text{cancel}(w') \)

\[
\begin{align*}
\lambda w. & \exists w'. B(w)(w') \land \text{rain}(w') \\
\rightarrow & \lambda w. \exists w'. B(w)(w') \land \neg \text{cancel}(w')
\end{align*}
\]

### References


