Presuppositional Exhustification

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Abstract  Grammatical theories of Scalar Implicatures (SI) make use of an exhaustivity operator \( \text{exh} \), which asserts the conjunction of the prejacent with the negation of excludable alternatives. We present a new Grammatical theory of SI according to which \( \text{exh} \) is replaced with \( \text{p-exh} \), an operator which asserts just its prejacent, and presupposes the negation of excludable alternatives. We show that this theory resolves various extant empirical challenges faced by the old formulation of \( \text{exh} \) (as well as by neo-Gricean theories of SI). The empirical issues include the oddness-inducing SI, the behavior of SI under negation, and a new SI puzzle we name \textit{some-under-some}, which proves challenging for competing SI accounts.

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1 Presuppositional exhaustification: Basic assumptions and implementation

According to Grammatical accounts of Scalar Implicatures (SIs), SIs are triggered by an exhaustification operator, \( \text{exh} \), which asserts both its prejacent and the negation of each of its excludable alternatives (Chierchia et al. 2011, a.o.). This view is captured in (1) (some of the specifics of (1) are based on the SI account in Magri 2009 and anticipate the discussion to follow):

\[
(1) \quad \text{Given } \phi \text{ and a set of excludable alternatives } \text{Excl}(\phi), \ [\text{exh}(\phi)] \text{ expresses the conjunction of } [\phi] \text{ with the negation of each member of } \text{Excl}(\phi) \text{ that is relevant in the utterance context } c.\]

\[
a. \quad [\text{exh}(\phi)] = [\phi] \land \land [\neg [\psi]] : \psi \in \text{Excl}(\phi) \land [\psi] \in R.
\]

\[
b. \quad \text{Excl}(\phi) \text{ is a subset of the set of formal alternatives of } \phi, \text{ such that, for each } \psi \in \text{Excl}(\phi), [\psi] \text{ isn’t logically entailed by } [\phi] \text{ (or equivalently, such that } [\phi] \text{ is logically consistent with } \neg [\psi]).
\]

\[
c. \quad R = \text{a contextually assigned ‘relevance’ predicate which minimally satisfies the following two conditions:} (i) \text{ the prejacent, } \phi, \text{ is relevant, i.e. } [\phi] \in R, \text{ and } (ii) \text{ any proposition that is contextually equivalent to the prejacent is also in } R \text{ (i.e., if } [\phi] \cap c \equiv [\psi] \cap c, \text{ then } [\psi] \in R).\]

In this paper, we present an alternative Grammatical account according to which \( \text{exh} \) should be replaced with \( \text{p-exh} \), an exhaustification operator which asserts its prejacent and presupposes the negation of each of its excludable alternatives.\(^3\)

\[
(2) \quad \text{Given } \phi \text{ and a set of excludable alternatives } \text{Excl}(\phi), \ [\text{p-exh}(\phi)] \text{ presupposes the negation of each (relevant) member of } \text{Excl}(\phi) \text{ and asserts } \phi:
\]

\[
a. \quad [\text{p-exh}(\phi)] = \{ \text{presupposition: } \land [\neg [\psi]] : \psi \in \text{Excl}(\phi) \land [\psi] \in R \}
\]

\[
\text{assertion: } [\phi]
\]

---

\(^1\) Following standard practice, we will throughout the paper represent a context \( c \) as a set of worlds (‘context set’) compatible with all the propositions mutually believed by the participants in \( c \). We also sometimes use the term ‘common ground’ to describe the context set.

\(^2\) Conditions (i) and (ii) are necessary for the range of data we are concerned with in this paper. Generally (i) and (ii) are not seen as sufficient (Chierchia et al. 2011, Fox & Katzir 2011, Roberts 2012, Trinh & Haida 2014). This is compatible with our conclusions in this paper, but the weaker condition stated in the text is sufficient.

\(^3\) Bade & Sachs (2019) also propose an amendment to \( \text{exh} \) in (1). It is, however, orthogonal to the issue we discuss in this paper.
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b. $\text{Excl}(\phi)$ is defined as in (1-b).

c. $R$ is defined as in (1-c).

$p$-exh is thus the mirror image of the overt exhaustifier only, on standard accounts of the latter (Horn 1969): $p$-exh presupposes what only asserts and vice versa.\footnote{We briefly return to the relationship between only and $p$-exh in the summary section. It should be noted that our proposal for Scalar Implicatures bears some resemblance to certain proposals for the meaning of cleft sentences (e.g. Velleman et al. 2012, Büring & Križ 2013), on which the exhaustivity inference of clefts is introduced at the level of presuppositions while the assertive level conveys just the meaning of the prejacent.}

We also assume, following Magri 2009, that $p$-exh is obligatorily present in the structure. Cases where SI appear to be optional are modeled as cases in which exhaustification is rendered vacuous due to the effect of the relevance parameter $R$; in such cases, the alternative that would be responsible for the missing SI is not in $R$.

As a simple illustration, a sentence like some students passed will be parsed as in (3-a), and its alternative is in (3-b). If the alternative is also Relevant in the context of utterance (i.e., if $[(3-b)] \in R$), then the output of (3-a) is in (3-c).

(3) Some students passed.
   a. LF: $p$-exh[Some students passed]
   b. $\text{Excl}$ (some students passed) = All students passed
   c. Presupposition: not all students passed;
      Assertion\footnote{We will use the terms ‘assertion’ and ‘at-issue’ interchangeably.}: some students passed.

The central claim of this paper is that $p$-exh resolves a number of extant empirical challenges faced by the standard, non-presuppositional formulation of exh. These include the oddness-inducing implicatures studied in Magri (2009)’s work, as well as various observations about the way SI project from certain embedded environments. Based on those results, we will argued that the Grammatical theory should be formulated in terms of $p$-exh.

$P$-exh raises a worry that would be useful to address at the outset: normally, SIs don’t intuitively have the status of a presuppositional inference. To wit, asserting felicitously a sentence like some students passed does not require that it be common knowledge that not all students passed prior to the assertion (compare with a standard case of presuppositions: asserting a sentence like Mary passed again is normally felicitous only when it is common knowledge that Mary passed before). Established tests for presupposition-hood such as the ‘Hey Wait A Minute’ test (HWAM; von Fintel 2004) fail with Scalar Implicatures:

(4) A: Some students passed.
b. B: #Hey wait a minute! I didn’t know that not all of them did!

However, it is well-known that presuppositions can often be effortlessly accommodated in the course of a conversation, provided they are not incompatible with common ground knowledge (Karttunen 1974, Lewis 1979, von Fintel 2008, a.o.). Accommodation is the process by which the context is ‘adjusted’ to entail the presuppositions of an uttered sentence before evaluating its assertive content. We propose that this is what normally happens with the presuppositions delivered by **p-exh** when they are not in conflict with the common ground, but aren’t already part of it, hearers accommodate them prior to evaluating the assertive content of the sentence.

But what if B has some reason to refuse to accommodate the presupposition that not all students passed? Here it should be remembered that SI computation is subject to Relevance. Specifically, unlike standard presupposition triggers (e.g. *again, knows*), **p-exh** is non-deterministic concerning what information exactly is presupposed, due to the effect of the Relevance parameter R (cf. (2-a),(1-c)). Suppose it’s not common knowledge that not all students passed, A just uttered *some students passed*, and B is reluctant to accommodate the presupposition that not all students passed. Instead of voicing a ‘HWAM!’ response, B has another strategy available at her dispense, namely she can simply assume that the alternative *all students passed* of A’s utterance is not relevant. In that case B would entertain an LF in which **p-exh** is vacuous, i.e. *not all students passed* would not be a predicted presupposition of (4-a). We assume that mentally removing a proposition from the set of Relevant alternatives is much less disruptive to the flow of conversation than explicitly challenging a presupposition with a ‘HWAM!’ response. Therefore the latter will be dispreferred, assuming that interlocutors try to minimize the disruption of information exchange as much as possible. This explains why HWAM will be odd in such a case. Crucially, there is no parallel move available in the case of triggers like *know* and *again*, whose presuppositions are entrenched and cannot be pushed aside as irrelevant: if a hearer refuses to accommodate the presupposition of those triggers, they have no choice but to halt the conversation and protest. The flexibility of **p-exh** regarding what exactly is presupposed thus explains what makes **p-exh** special compared to the other presupposition triggers with respect to the ‘HWAM!’ test.

Having dispelled this possible objection to **p-exh**, the rest of the paper is devoted to showing the advantages of **p-exh** over **exh**. In section 2 we argue that **p-exh** makes better predictions than standard **exh** relative to oddness patterns generated by mismatches between SIs and the common ground (Magri 2009). In section 3 we show that **p-exh** explains the limited availability of scalar implicatures in downward entailing contexts by unifying it with the phenomenon of presupposition cancellation. In section 4 we show that **p-exh** accounts for the complex pattern of SIs in *some-*
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under-some’ constructions, of which no satisfactory account has been given to date. In section 5 we summarize and point out the connection between our proposal and broader questions regarding the cut between presuppositional and assertive content (the ‘triggering’ problem of for presuppositions).

2 Oddness and accommodation

The sentences in (5-a)-(5-b) are odd, given common world knowledge.

(5)  

<table>
<thead>
<tr>
<th>C: standard adult background information</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. #Some Italians come from a beautiful country.</td>
</tr>
<tr>
<td>b. #Mary is sometimes tall.</td>
</tr>
</tbody>
</table>

According to Magri (2009), the oddness of the sentences in (5) is due to a clash between the implicatures they give rise to, by way of exh, and the common ground. We accept the general architecture of Magri’s explanation, but argue based on a broader set of oddness cases that the explanation should be cast in terms of p-exh rather than exh.

To do that, we first go over Magri’s basic account of the oddness in (5) and its implications for the general theory of SI, in section 2.1. In section 2.2 we point out empirical problems for his specific proposal, and in section 2.3 we proceed to show the advantages of incorporating p-exh to overcome the problems.

2.1 A review of Magri’s theory

Magri explains the pattern in (5) as follows. A sentence like (5-a) has a scalar alternative all Italians come from a beautiful country. This alternative is obligatorily exhaustified (negated), and the result of the exhaustification entails that not all Italians come from the same country – an inference which directly conflicts with the common ground, since all Italians come from the same country. The same line of explanation applies to (5-b): the scalar alternative to this sentence, namely Mary is always tall, undergoes exhaustification, leading to the odd inference that Mary is sometimes but not always tall.

As Magri points out, the observation that implicatures can clash with the common ground and generate oddness is problematic for neo-Gricean, non-grammatical theories of SI. Neo-Griceans conceive of SI as computed to ensure that speakers come out as cooperative and rational conversational partners. So why compute an implicature that is inconsistent with the common ground, and generates oddness? From this perspective, why not choose, in cases like (5-a)-(5-b), the literal interpre-
tations which do not clash with the common ground and can even convey useful information such as that Italy is beautiful and that Mary is tall?

Magri argues that the Grammatical account of SI is best suited to incorporate the notion of common ground-mismatching implicatures. Working with standard \textit{exh} (see (1)), his Grammatical account rests on two central stipulations concerning the distribution and computation of \textit{exh}:

(I) \textit{exh} is obligatorily present in the structure.\textsuperscript{6}

(II) Any alternative that is \textit{contextually} equivalent to the prejacent \(\phi\), but \textit{logically} stronger than it, must be negated in the course of computing \textit{exh}(\(\phi\)) (i.e. contextually-equivalent formal alternatives cannot be pushed aside as irrelevant for the purposes of \textit{exh}).\textsuperscript{7}

In addition, Magri proposes an oddness filter that works together with (I) and (II) to determine which expressions are marked as odd:

(III) \textbf{Oddness filter.}

\textbf{If} \textit{exh}(\(\phi\)) mismatches the common ground in context \(C\), then \textit{exh}(\(\phi\)) is odd in \(C\).

The basic idea, then, is that sentences like (5-a)-(5-b) are obligatorily parsed as in (6-a)-(6-b). These LFs generate enriched readings in (7-a) and (7-b) respectively, which in turn clash with the common ground \(C\). Applying the oddness filter (III), we then get that (5-a)-(5-b) are predicted to be odd in \(C\).

(6)  
\begin{enumerate}
  \item \textit{exh} [Some Italians come from a beautiful country]
  \item \textit{exh} [Mary is sometimes tall]
\end{enumerate}

(7)  
\begin{enumerate}
  \item Some Italians come from a beautiful country \(\land \neg\text{all Italians come from a beautiful country}
  \item Mary is sometimes tall \(\land \neg\text{Mary is always tall}
\end{enumerate}

Why do we need stipulations (I)-(III)? If we gave up on the obligatoriness of \textit{exh} (I), we could select parses without \textit{exh}, thereby rescuing these expressions from adding the problematic enrichments. If we gave up on the notion that \textit{exh} blindly negates logically-stronger alternatives without taking contextual knowledge into account.

\textsuperscript{6} And moreover at every proposition taking site. We gloss over this since we don’t discuss SI in embedded positions in this section.

\textsuperscript{7} Note that stipulation (II) is already incorporated in how \textit{exh} was defined in (1). Specifically, it was stipulated there that \(R\), the Relevance parameter that restricts the propositions among \(\text{Excl}(\phi)\) that \textit{exh} negates, always includes any contextually-equivalent alternative to the prejacent \(\phi\). It follows that any contextually-equivalent but logically-stronger alternative than the prejacent must be negated by \textit{exh}.
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consideration (II), we could say that the application of **exh** for (6-a)-(6-b) is vacuous (in a normal context): in each case, the prejacent is contextually equivalent to its logically stronger alternative, so the latter would not be negated and no clash with the common ground would ensue. And if we dropped the oddness filter in (III), we could say that when a speaker asserts any expression which clashes with the common ground, a listener should revise their beliefs about what is in the common ground prior to assessing the truth value of the utterance; the utterance might still be judged false by the listener, but no oddness need result.⁸

2.2 An overgeneration problem

Although Magri’s account of oddness is promising, it faces an overgeneration problem. It predicts that any exhaustified expression which mismatches with the common ground will be filtered out as odd, but this is not borne out. Consider first the following contrast:

(8)  

\[ C : \text{normal adult background information} \]

a. #Some lions are animals.

b. Only SOME lions are animals.

c. Some but not all lions are animals.

While (8-a) has the signature of a Magri-style oddness, (8-b)-(8-c) feel markedly different: intuitively, it seems more appropriate to classify them as downright false rather than as odd. Magri’s theory, as currently formulated, predicts that all the examples in (8) should be equally odd in \( C \); they are all overtly or covertly exhaustified, and the resulting interpretation is inconsistent with the common ground, so the oddness filter in (III) predicts them to all be odd.⁹

One might initially take this data to suggest that an exhaustivity inference that mismatches the common ground will be judged odd if it is contributed by a covert

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⁸ For a comprehensive discussion of Magri-style oddness cases that focuses on the crucial role of logical entailment in the definition of exhaustification, and includes a critical discussion of recent neo-Gricean-friendly attempts to explain oddness without appealing to the notion of mismatching implicatures, see Del Pinal (2019).

⁹ In fact Magri’s theory is too strong even for more mundane sentences that don’t involve exhaustive inferences, such as “California is in Germany”. This is a false sentence which mismatches the common ground, and since every sentence is obligatorily appended with **exh** in Magri’s theory, the oddness filter in (III) predicts it to be odd, whereas the intuition is that it is simply false (even when uttered by someone who knows that California is not in Germany). Nevertheless, we abstract away from this more basic problem for Magri because it could possibly have a local and uninteresting fix within an **exh** theory. Below we focus on cases for which we believe the same local fix would not be available and which call for a more substantive solution: cases in which the mismatching inferences are contributed by **exh**. Our solution in section 2.3 will also cover the more mundane cases.
operator, using \textbf{exh} (as in \text{(8-a)}), but merely false if it is triggered by an overt operator/material (as in \text{(8-b)-(8-c)}). This is not the correct generalization however. The minimally different data in \text{(9)} shows a contrasting pattern compared to \text{(8)}:

\begin{itemize}
  \item \text{(9)} \quad C: \textit{normal adult background information}
  \begin{itemize}
    \item a. Some lions are robots.
    \item b. \textit{Only SOME} lions are robots.
    \item c. Some but not all lions are robots.
  \end{itemize}
\end{itemize}

In \text{(9)} the sentence with the covert exhaustification sounds merely false, while the one with \textit{only} sounds odd – the opposite of \text{(8)}. The key difference from \text{(8)} is that the prejacent here (\textit{some lions are robots}) is held in the common ground to be false rather than true. Note that this difference does not affect the (c) examples – \text{(9-c)} is judged not odd (but false), like \text{(8-c)}.

In our view, the pattern in \text{(8)-(9)}, and especially the flipped behavior in the (a) and (b) examples, reveals that oddness-inducing inferences are sensitive to different ingredients of meaning. Specifically, we claim that oddness arises when the common-ground-conflicting information is contributed at a presuppositional level. The non-odd sentences—\text{(8-b),(8-c), (9-a) and (9-c)}—are not odd because they convey the false information (i.e. \textit{not all lions are animals/some lions are robots}) at the assertive/at-issue component of meaning alone, without any presupposed content. The contrast in the \textit{only} examples \text{(8-b)-(9-b)} is thus explained on the basis of the fact that while \text{(8-b)} merely has a false assertion, \text{(9-b)} has a false presupposition. In \text{(9-a) (and (9-c))} the false information is, again, conveyed with at-issue material, which is why they are not odd. At this point the reader can already see how the data in \text{(8-a)} fits with this generalization: its oddness will be explained if the exhaustive inference is conveyed at a presuppositional level, which is predicted by our hypothesis that SI are presuppositions.

To sum up, the problem for Magri’s system in light of \text{(8)-(9)} is that it is not attuned to the distinction between presuppositional content and at-issue content for the purpose of explaining oddness. This problem can be easily fixed by reformulating Magri’s theory in terms of presuppositional exhaustification, \textbf{p-exh}.

2.3 Oddness and \textbf{p-exh}

According to our account of exhaustification, \textbf{p-exh} triggers an asserted vs. presupposed entailment structure that is the mirror image of that triggered by \textit{only}. The definition of \textbf{p-exh} is repeated from \text{(2)} in a simplified form:
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(10) Given $\phi$ and a set of excludable alternatives $\text{Excl}(\phi)$, $\mathbf{p-exh}(\phi)$ presupposes the negation of each (relevant) member of $\text{Excl}(\phi)$ and asserts $\phi$:

$$\mathbf{p-exh}(\phi) = \left\{ \begin{array}{ll} \text{presupposition:} & \neg \psi : \psi \in \text{Excl}(\phi) \cap R \\ \text{assertion:} & \phi \end{array} \right.$$ 

Using $\mathbf{p-exh}$ we can formulate a refined version of Magri’s theory that doesn’t overgenerate oddness. Specifically, we can maintain his (I) and (II) from section 2.1—the core elements of Magri’s theory—yet drop the ad hoc and overly strong oddness filter in (III). Instead, we introduce the weaker and more principled oddness filter in (11):

(11) **Revised oddness filter.** Let $\phi$ be a sentence with a presupposition $p$ and assertion $p'$ (notation: $\phi = pp'$). Then asserting $\phi$ is odd in $c$ if (i) $p$ is inconsistent with $c$, or (ii) $p \land p'$ is inconsistent with $c$ while $p'$ alone would be consistent with $c$.

According to (11), an assertion of $\phi$ is odd if it has a presupposition that is, either by itself or when conjoined with the at issue content, inconsistent with the common ground. Unlike Magri’s filter, (11) can be derived as a reasonable and independently justified constraint on accommodation. As various theorists have pointed out, presupposed information that is inconsistent with the common ground cannot be accommodated. If a speaker wants to convey information that challenges commonly-held beliefs, one must do so fully at the at-issue level of meaning. Conspiracy theorists are allowed to try to convince others to revise their beliefs about what information should be in the common ground, but they are not allowed to systematically insert their conspiracy theories as presuppositions (cf. Heim 1992).

Given $\mathbf{p-exh}$ and the revised oddness filter in (11), we can explain the target oddness patterns. Consider first examples (8-a) and (9-a), repeated in (12) and (13), given their LFs with $\mathbf{p-exh}$ in (12-a) and (13-a). In each case, the underlined part represents the part of the content that is presupposed. (12-a) presupposes that it is not the case that all lions are animals. Since this presupposition is inconsistent with the common ground in $C$, it is correctly predicted to be odd by the revised oddness filter. In contrast, (13-a) presupposes that it is not the case that all lions are robots. This presupposition is consistent with (in fact entailed by) the common ground in $C$; it is the assertive component alone in (13-a) that mismatches the common ground, so the revised oddness filter does not rule it odd (even if it is judged to be obviously false by the interlocutors).

(12) #Some lions are animals.

a. $\mathbf{p-exh}[\text{some lions are animals}]$
   
   $= \neg \text{all lions are animals} \land \text{some lions are animals}$
Some lions are robots.
   a. \( \text{p-exh}[\text{some lions are robots}] \)
      = \( \neg \text{all lions are robots} \land \text{some lions are robots} \)

Consider next the only examples in (8-b) and (9-b), repeated in (14) and (15). According to standard accounts, only \( \phi \) presupposes its prejacent, \( \phi \), and asserts the negation of each excludable focus alternative of \( \phi \). In the case of (14), we get the presupposition that some lions are animals, which is consistent with (in fact entailed by) the common ground in \( C \). It is thus easy to see that (14) is not filtered out by the revised oddness filter. In the case of (15), we get the presupposition that some lions are robots, which is inconsistent with \( C \) and is thus marked as odd by the revised oddness filter. Both of these predictions are adequate.

Only SOME lions are animals.
   a. only [SOME lions are animals]
      = some lions are animals \( \land \neg \text{all lions are animals} \)

Only SOME lions are robots.
   a. only [SOME lions are robots]
      = some lions are robots \( \land \neg \text{all lions are robots} \).

The cases in (8-c) and (9-c), repeated in (16)-(17), are also handled adequately. The key observation, in these cases, is that all the common ground-mismatching content is asserted, and so the revised oddness filter doesn’t predict them to be odd:

Some but not all lions are animals.
   a. = some lions are animals \( \land \neg \text{all lions are animals} \)

Some but not all lions are robots.
   a. = some lions are robots \( \land \neg \text{all lions are robots} \)

Finally, let us go back to Magri’s original examples, (6-a)-(6-b). We focus on (6-a), repeated here in (18):

Some Italians come from a beautiful country.
   a. \( \text{p-exh} [\text{some Italians come from a beautiful country}] \)
      = \( \neg \text{all Italians beautiful country} \land \text{some Italians beautiful country}. \)

\[ ^{10} \text{Strictly speaking (14-a) and (15-a) are also parsed with p-exh in the matrix. But we assume that in this case p-exh doesn’t negate any Relevant alternative (since the alternative triggered by some are already captured by only), so we ignore it. The same applies to (16)-(17). } \]
Based on (18-a), we get the presupposition that not all Italians come from a beautiful country. The common ground \( C \) entails that Italians come from the same country, but does not entail that Italy is or is not beautiful. Accordingly the presupposition in (18-a), taken on its own, is strictly compatible with the common ground. In addition the at issue content of (18-a) is not itself incompatible with \( C \). Crucially, however, the conjunction of the presupposed and at issue content of (18-a) is incompatible with \( C \). Accordingly, the revised oddness filter (Specifically clause (ii) of it) predicts correctly that it should be filtered out.

At this point, it is worth reemphasizing the basic intuition behind our revised oddness filter: information that is inconsistent with the common ground, that aims to revise it, should be expressed as at-issue content alone, and as we saw this was indeed the case in all the intuitively non-odd examples (13), (14), (16) and (17). Accordingly, a Magri-style sentence that has the same entailments as (18), in \( C \), except that it makes the controversial information fully at-issue, is expected to be intuitively less odd than (18). This prediction is attested by examples like (19-a), which feels like a more direct and less odd way to call for revision of \( C \) than (18):

\[(19) \quad C: \text{stable background knowledge}\]

\[a. \quad ?\text{Some but not all Italians come from a beautiful country.}^{11}\]

In this section we provided an argument for the hypothesis that SI arise on a presuppositional level, based on one well-known feature of presuppositional material: when it is inconsist with common-ground knowledge, the result is a feeling of oddness. The next two sections will develop arguments for \(\text{p-exh}\) based on a different well-known feature of presuppositional material: the fact that it ‘projects’ from various embedded positions in a structure. We will show that at least some SI inferences in embedded positions can be made sense of under the current proposal.

3 \(\text{p-exh and the restricted distribution of SI under negation}\)

This section discusses the distribution of SI under negation. We will point at the advantages of \(\text{p-exh}\) in explaining both why SI are not normally computed under negation (section 3.1), and why sometimes they do, by unifying the latter cases with the phenomenon of ‘presupposition cancellation’ (section 3.2).

\(^{11}\) Speakers might vary with respect to how felicitous they judge (19-a) to be. We suggest that the variability reflects the extent to which one is willing to imagine a person who would be in a position to utter (19-a), for example a person who believes that some Italians do not come from Italy.
3.1 The vacuity of p-exh under negation

SI are not normally computed in the scope of negation. The sentences in (20), for examples, if read with neutral intonation, do not allow the indicated readings which can be described as enriching a weak scalar item (or, some) with a SI in the scope of negation. This is corroborated by the infelicity of the continuation sentences in parentheses, which would only be compatible with the enriched meaning.

\[\text{(20) } \begin{align*}
a. \text{Alex didn’t talk to Mary or Sue. (#She talked to both)} & \quad \neg \text{it is not true that Alex talked to just one of them} \\
b. \text{I don’t think someone cheated. (#I think they all did)} & \quad \neg \text{it is not true that (I think that) some-but-not-all cheated}
\end{align*}\]

Existing grammatical theories have to say something special to block this, since if \(\text{exh}\) was freely available in the syntax there is no apriori reason why in (20) it couldn’t be embedded under negation, resulting in the problematic parses in (21):

\[\text{(21) } \begin{align*}
a. \text{not } [\text{exh [Alex talked to Mary or Sue]}] & \quad (\text{not } > \text{or-but-not-both}) \\
b. \text{not (believe) } [\text{exh [someone cheated]}] & \quad (\text{not } > \text{some-but-not-all})
\end{align*}\]

In order not to overgenerate this, Fox & Spector (2018) propose an economy condition which generally bans the syntactic insertion of \(\text{exh}\) under negation.\(^{12}\)

Given the \(\text{p-exh}\) theory, a condition on syntactic insertion is not needed. Due to the revised semantics of \(\text{p-exh}\), the correct result is obtained even with an embedded occurrence of \(\text{p-exh}\).

To see this, consider the LF of (21-b) under the \(\text{p-exh}\) theory, in (22-a).\(^{13}\) The embedded \(\text{p-exh}\) will derive the presupposition that not all cheated, (22-b). Since it is a presupposition, it will project through negation to the matrix level. The presupposition is entailed by the assertive component of the whole sentence, which is just a negated existential – the basic semantics of the sentence.

\[\text{(22) } \begin{align*}
a. \text{[not } [\text{p-exh [someone cheated]]]} \\
b. \text{Projected presupposition: not all cheated} \\
c. \text{At-issue content of (22-a): no one cheated}
\end{align*}\]

\(^{12}\) Their condition can be obviated in certain conditions, cf. our discussion surrounding (23). Fox & Spector (2018) build their economy condition on the observation that computing a SI under negation weakens the overall interpretation of the sentence, and weaker meanings are generally dispreferred to stronger ones. Enguehard & Chemla (to appear) propose a similar constraint on the insertion of \(\text{exh}\), though they motivate their principle on different grounds than Fox & Spector (2018).

\(^{13}\) We continue to assume that matrix \(\text{p-exh}\) is also present, but in this case it will be vacuous so it isn’t represented. we also abstract away from the contribution of ‘believe’ in (21-b) and drop it from the representation for the purpose of simplicity.
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Thus, in these simple cases a structure with \textbf{p-exh} embedded under (one) negation is not distinguishable from its \textbf{p-exh}-less parse (modulo the entailed presupposition), which—on Grammatical theories of SI—explains without further stipulations why SIs are not normally detectable in these environments.

3.2 \textbf{p-exh} and Presupposition Cancellation

This perspective allows us to account for another known fact about SI, namely that under special conditions they \emph{do} seem to appear under negation. Specifically, when the sentences are pronounced with the ‘contradiction contour’ (Horn 1989)—H* pitch on negation, L+H* on the scalar item and LH% as the boundary tone—we get the reading that was missing in (20) (see e.g. Meyer 2016 for recent discussion).

(23) a. Alex didn’t\textsubscript{H*} talk to Mary OR\textsubscript{L+H*} Sue\textsubscript{LH%}\ldots she talked to both!
b. I don’t\textsubscript{H*} think SOME\textsubscript{L+H*} one cheated on the exam\textsubscript{LH%}\ldots everyone did!

This pattern bears a striking resemblance to the phenomenon known as 
\textbf{presupposition cancellation}. In presupposition cancellation, presuppositional content in the scope of negation does not project through it, but appears to be at-issue. Some illustrative examples are in (24):

(24) a. Mary isn’t\textsubscript{H*} late to the meeting AGAIN\textsubscript{L+H*} LH%\ldots she has never been late before!
    (negation denies the presup. of [again \(\phi\)], i.e. “\(\phi\) happened before”)
b. Mary didn’t\textsubscript{H*} STOP\textsubscript{L+H*} smoking\textsubscript{LH%}\ldots she never used to!
    (negation denies the presup. of \textbf{stop}, namely “used to”)
c. Mary can’t\textsubscript{H*} climb THE\textsubscript{L+H*} tree in the garden\textsubscript{LH%}\ldots because there are two of them!
    (negation denies the uniqueness presupp. of \textbf{the})
d. Chris didn’t\textsubscript{H*} MANAGE\textsubscript{L+H*} to solve the problem\textsubscript{LH%}\ldots it was quite easy for him
    \hfill \textbf{(Horn 1989)}
    (negation denies the ‘it-was-hard’ presupp. of \textbf{manage})

(23) and (24) have the same prosodic signature, and moreover they both require the continuation sentence to be felicitous with this prosody. This similarity suggests that the two phenomena—SI under negation and presupposition cancellation—should be accounted for by the same mechanism. Crucially, our hypothesis about \textbf{p-exh}, on which implicatures are presuppositions, allows for such a unified account.

Assume following Heim (1983) that presupposition cancellation is possible because under certain conditions presuppositions can be “locally accommodated”
under negation. Let us further model local accommodation as is standard within trivalent logic, using the $\mathcal{A}$ operator (Beaver & Krahmer 2001). $\mathcal{A}$ turns a trivalent proposition into a bivalent one by collapsing undefinedness with falsity:

\begin{equation}
(25) \quad \text{With } \phi = pp':
\end{equation}

$$
[\mathcal{A}(\phi)] = \begin{cases}
1, & \text{if } [\phi] = 1 \quad \text{(i.e. if } p \land p') \\
0, & \text{if } [\phi] = \# \text{ or } 0 \quad \text{(i.e. if } \neg p \text{ or } \neg p')
\end{cases}
$$

When (25) is embedded under negation, the whole sentence is predicted to be true if the presupposition is false; this derives presupposition cancellation:

\begin{equation}
(26) \quad [\text{not}(\mathcal{A}(\phi))] = \begin{cases}
1, & \text{if } [\phi] = \# \text{ or } 0 \quad \text{(i.e. if } \neg p \text{ or } \neg p') \\
0, & \text{if } [\phi] = 1 \quad \text{(i.e. if } p \land p')
\end{cases}
$$

On our p-exh proposal, SI under negation are derived with the same mechanism: these are cases where an $\mathcal{A}$ operator takes scope in between negation and p-exh. We illustrate the analysis in (27).

\begin{equation}
(27) \quad \text{I don’t think SOME_LH+one cheated on the exam_LH... they all did!}
\end{equation}

a. not [\mathcal{A} [p-exh [someone cheated]]]

b. $\sim$ it is not true that some-but-not-all cheated

Indeed, in our theory deriving embedded SI under negation in fact requires application of $\mathcal{A}$ immediately below negation, as we showed in (22), without $\mathcal{A}$ the information that the relevant alternative is locally exhaustified will project through negation rather than enter the at-issue content that negation targets. This might provide an explanation for the restricted distribution of embedded implicatures: local accommodation under negation is known to be possible only as a last-resort mechanism, to rescue an otherwise inconsistent discourse (Gazdar 1979, Heim 1983). On the hypothesis that local accommodation involves the insertion of $\mathcal{A}$ under negation, this means that inserting $\mathcal{A}$ under negation is possible only to rescue an inconsistent discourse. Notice that the continuation sentences in (23) threaten the consistency of the whole discourse, so inserting $\mathcal{A}$ is possible, and the embedded SI reading arises, which accounts for why the continuation sentence is necessary for the embedded SI reading to arise. We do not have a particularly illuminating explanation for the link to the prosody facts, but we would like to hypothesize that embedding

---

14 This assumes standard semantics for negation which flips truth- and falsity-conditions.
15 Note that such configuration is equivalent to embedding the bivalent exh under negation without $\mathcal{A}$: $[\text{not}(\mathcal{A}(p-exh(\psi)))] = [\text{not}(\text{exh}(\psi))]$. 

10
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\( \mathcal{A} \) under negation requires the contradiction contour (perhaps just the \( L + H^* \) part of it), accounting for the contour’s obligatoriness in both (23) and (24).16

4 Some under some

In this section we analyze a pattern of inference that arises from sentences in which a weak scalar item is embedded in the scope of another (hence ‘some under some’). The pattern poses a problem for existing theories of SIs, and to our knowledge has never been seriously addressed. We will show that p-exh, coupled with a certain theory of presupposition projection (‘Strong Kleene’), successfully derives the data, thus providing another argument in favor of p-exh.

4.1 Data

Consider (28), where a weak scalar item (some) is embedded under another, and two potential implicatures it gives rise to, labeled ‘Local’ and ‘Strong Global’ SIs (terminology adapted from Gotzner & Benz 2018):

(28) Some of the girls found some of their marbles.
   a. Local SI: Some of the girls found some but not all of their marbles
   b. Strong Global SI: None of the girls found all of their marbles

Gotzner & Benz (2018) report experimental evidence that suggests that subjects readily derive the strong global implicature for some under some sentences like (28). Note that the strong global SI, taken together with the prejacent, entails the local SI (i.e., the conjunction of (28) and (28-b) entails (28-a)). This pattern of inference is predicted by virtually all existing theories of implicatures, most straightforwardly by any theory that allows (28) to trigger the alternative some of the girls found all of her marbles.

To this we add the observation that, as shown in (29), a some under some sentence can also trigger the local SI side by side with what we call a ‘Weak Global’ SI, which is of the general form ‘not all... some...’.

16 A potential problem is that the correlation between local accommodation and the contradiction contour is not exception-free. Cancelling the existence presupposition of ‘the’ (as opposed to its uniqueness presupposition, cf. (24-c) doesn’t seem to require or even allow a \( L + H^* \) accent on the: (i) Mary didn’t\( _{H^*} \) meet the king of France... because there is no king of France.

We are not sure why the existence presupposition of the is different than other presuppositions in this respect. It is worth mentioning that Coppock & Beaver (2015) argue that the existence inference is not hard-coded as a presupposition of the, as opposed to the uniqueness inference.
(29) Some of the girls found some of their marbles. And some found all of them.
   a. **Local SI**: Some of the girls found some but not all of their marbles
   b. **Weak Global SI**: Not all of the girls found some of their marbles
      \[\equiv \text{Some of the girls didn’t find any of their marbles}\]

The Weak Global SI together with the prejacent does *not* entail the Local SI. Note importantly that the second sentence of (29) is not compatible with computing the Strong Global SI, yet both the inferences in (29-a)-(29-b) still arise; therefore the pattern in (29-a)-(29-b) does not depend for its existence on deriving the Strong Global SI.

This means that for a ‘some under some’ sentence, a theory of Scalar Implicatures needs a way to derive both the local and the weak global inferences, **without** necessarily computing the strong global SI. This is summarized in (30).

(30) **Desideratum**: Given a sentence of the form \(\text{Some}_x[... \text{some}_y ... P(x, y)]\), derive the following implicatures:
   a. \(\exists x[... \exists y \wedge \neg \forall y [... P(x, y)]]\) (Local SI)
   b. \(\neg \forall x[... \exists y [... P(x, y)]] \equiv \exists x[... \neg \exists y [... P(x, y)]]\) (Weak-global SI)

As it turns out, (30) presents a difficulty for all existing theories of SIs known to us, Grammatical and Neo-Gricean alike.\(^{18}\) Below we focus on the predictions of the grammatical \textbf{exh} theory and show that they are inadequate.

4.2 An \textbf{exh} paradox

In an \textbf{exh} theory, it is possible to derive just the Local SI, and it is possible to derive just the Weak-Global SI. But deriving both is, as far as we can see, not possible without ad-hoc stipulations. The structure in (31) at first glance looks like the correct LF representation to derive (30):

(31) \textbf{exh}_1[\text{some of the girls} \lambda_x \textbf{exh}_2[\text{some of the marbles} \lambda_y [x \text{ found } y]]]

(31) seems promising because each \textbf{exh} operator in it is meant to deliver one of the two inferences we’re after: \textbf{exh}_2 is supposed to deliver the Local SI, by targeting the embedded ‘some’, and \textbf{exh}_1 is supposed to deliver the Weak Global SI, by...\textbf{exh} theory and show that they are inadequate.

---

17 The terms ‘Strong’ and ‘Weak’ might be misleading in this context, since (28-b) as a whole is not logically stronger than (29-b) as a whole. Rather this terminology is meant to reflect just the logical relationship between the quantifiers ‘none’ and ‘not all’.

18 The first author is grateful to Danny Fox for pointing out to him the pattern in (29) and its significance, and to Paul Marty for helpful discussion.
targeting the matrix ‘some’. However, the interpretation (31) actually yields, which is given in (32), is too weak, in particular the part in (32-b).

(32) Inferences predicted from the parse in (31):
   a. *some of the girls founds some but not all of their marbles*
      (prejacent of exh₁)
   b. *not all of the girls founds some but not all of their marbles*
      (output of exh₁)

While (32-a) is equivalent to the desired local SI, (32-b) is weaker than the desired Weak Global SI: it only guarantees that some girls found either none or all of their marbles, but what is needed is to guarantee that some girls found none of the marbles.

To get the desired weak global SI we would want to replace ‘some but not all’ in (32-b) with a plain ‘some’. The culprit for that ‘some but not all’ is of course the presence of exh₂ downstream; but we cannot just choose to remove exh₂ from the structure (or to make its contribution vacuous in some other way), because its presence is required to generate the local SI in (32-a).\(^{19}\)

This then is the conundrum for the standard exh theory in trying to account for (30): the embedded exh is required for one task (local SI), but its contribution interferes with achieving the other task (weak global SI).

4.3 A p-exh solution

We now show that p-exh can derive our desideratum. The key point is that a separation between the presuppositional and at-issue content of p-exh allows the presence of an embedded p-exh to not necessarily affect the contribution of material higher-up in the structure; specifically, an embedded p-exh here will be, in some sense

\(^{19\text{ One might suggest that exh can optionally delete/disappear from alternatives. Then the (relevant) alternative to the higher exh could have the embedded exh deleted from it, as shown in (i-b):}}\)

(i) a. *Prejacent of exh₁*: some of the girls \(\lambda_1\) exh₂ [some of the marbles \(\lambda_2\) [x found \(y\)]]
   b. *Alternative*: all of the girls \(\lambda_3\) exh₂ [some of the marbles \(\lambda_4\) [x found \(y\)]]

That would work, but it is mysterious why exh could disappear from alternatives. Note that the overt exhaudtifier only doesn’t have this option; (ii) cannot have the relevant inference, which would be possible if only could be deleted in an analogous way. (ii) in fact strongly conveys that all girls found some of their marbles.

(ii) Some girls only found SOME of their marbles.
   a. \(\sim\) Not all girls found *some-but-not-all* of their marbles
   b. \(\not\sim\) Not all girls found *some* of their marbles.
to be percisified soon, semantically transparent when it comes to evaluating the
contribution of the higher exhaustifier.

Let us look at the relevant LF in (33), where \textbf{p-exh} replaces \textbf{exh}:

(33) \textbf{p-exh}_1[\text{some girls } \lambda \text{x } \textbf{p-exh}_2 \text{ [some of the marbles } \lambda \text{y } [x \text{ found y}]]]

To see what is predicted for (33), we need to know how quantifiers handle
presuppositions in their scope, because (33) features a presupposition trigger—
\textbf{p-exh}_2—in the scope of the matrix quantifier (and it also features the same \textbf{p-exh}_1
at the level of the alternative that \textbf{p-exh}_1 negates; see below). Our analysis
will henceforth rest on a specific (yet independently argued for) framework for
presupposition projection: the Strong Kleene trivalent logic (George 2008, Fox 2013
a.o.).

In Strong Kleene, the method for computing the presuppositions of a complex
expression \( \phi \) consists of first specifying its bivalent truth conditions (Truth conditions
and Falsity conditions). These are the conditions under which, roughly speaking,
\( \phi \) is guaranteed to have a bivalent truth value, even if some of its sub-constituents
do not have a bivalent truth value. The conditions under which it is not possible to
determine such a bivalent value for \( \phi \) are the conditions under which \( \phi \) is assigned
the third truth value, \#. The predicted presupposition of \( \phi \) is the disjunction of its
truth and falsity conditions (i.e., the conditions under which it is not \#). See George
2008 and Fox 2013 for a more thorough discussion and technical implementation;
this general description is sufficient for our purposes.\(^{20}\)

We also need to revise our lexical entry for \textbf{p-exh} from (2-a) to respect the switch
to a trivalent framework. The most natural way of doing so that would be in line
with our core proposal is given in (34):

\[
\textbf{[p-exh(\phi)]} = \begin{cases} 
1, & \text{if } [\phi] = 1 \land (\exists \psi : \psi \in \text{Excl} (\phi) \land [\psi] \in R) \\
0, & \text{if } [\phi] = 0 \\
\#, & \text{otherwise}
\end{cases}
\]

This entry says that the conditions under which \textbf{p-exh}(\phi) is true are the expected
ones, but the conditions under which \textbf{p-exh}(\phi) is false are identical to the conditions
under which \( \phi \) is false. As long as we’re restricting our attention to alternatives of
\( \phi \) that are strictly stronger than \( \phi \), which is what we do throughout the paper, the
predicted presupposition here (i.e. the disjunction of truth and falsity) derive our core

\(^{20}\) We work with the the Strong-Kleene system because we could not see a way to derive the desideratum
with a different logic of presuppositions (not given the LF in (33), anyway). Thus, if our account
of some under some in terms of \textbf{p-exh} is correct, it also serves as an indirect argument that Strong
Kleene is the right system for modeling presupposition projection in general.
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proposal, namely the presupposition is that all the excludable relevant alterantives are false.\textsuperscript{21} As a simple illustration:

(35) She found some of her marbles.

\begin{align*}
\text{a. } & \llbracket \text{p-exh}(35) \rrbracket = \begin{cases} 
1, & \text{if she found some but not all her marbles} \\
0, & \text{if she found none of her marbles} \\
\# , & \text{otherwise}
\end{cases} \\
\text{b. Predicted presupposition:} \\
\text{either she found none of her marbles, or some but not all her marbles (} \equiv \text{she didn’t find all her marbles).}
\end{align*}

Armed with these assumptions, we now go back to our LF in (33). The task is to specify its truth and falsity conditions. According to (34), this LF is true if the prejacent of the matrix exhaustifier (\text{p-exh} \text{h}_1) is true and the alternative to it is false; false if the prejacent is false; and # otherwise. For convenience, the prejacent and alternative for \text{p-exh} \text{h}_1 are shown in (36), so the interpretation of the whole LF is succinctly represented in (37).

(36) The prejacent and (relevant) alternative of \text{p-exh} \text{h}_1 in (33):

\begin{align*}
\text{a. } & \text{prejacent: some girls } \lambda_x \text{ p-exh}_2 \text{ [some of the marbles } \lambda_y [x \text{ found } y] \\
\text{b. } & \text{alternative: all girls } \lambda_x \text{ p-exh}_2 \text{ [some of the marbles } \lambda_y [x \text{ found } y]]}
\end{align*}

(37) \llbracket (33) \rrbracket = \begin{cases} 
1, & \text{if } \llbracket (36-a) \rrbracket = 1 \land \llbracket (36-b) \rrbracket = 0 \\
0, & \text{if } \llbracket (36-a) \rrbracket = 0 \\
\# , & \text{otherwise}
\end{cases}

To unpack (37), we need to know the truth and falsity conditions of (36-a), and the falsity conditions of (36-b). Starting with (36-a), this is an existential statement; Strong Kleene’s logic produces the following 3-valued semantics for such statements:

\text{21 One could imagine an alternative entry where the exclusion of alternatives is duplicated in the falsity conditions (the addition is underlined):}

\begin{align*}
\text{(i) } & \llbracket \text{p-exh}(\phi) \rrbracket = \begin{cases} 
1, & \text{if } \llbracket \phi \rrbracket = 1 \land \bigwedge(\llbracket \psi \rrbracket = 0) ; \psi \in Excl(\phi) \land \llbracket \psi \rrbracket \in R \\
0, & \text{if } \llbracket \phi \rrbracket = 0 \land \bigwedge(\llbracket \psi \rrbracket = 0) ; \psi \in Excl(\phi) \land \llbracket \psi \rrbracket \in R \\
\# , & \text{otherwise}
\end{cases}
\end{align*}

But as long as only strictly stronger alternatives are considered, (i) comes down to the same thing as (34): in these cases the falsity conditions in (i) can be shortened to just its first conjunct (the underlined conjunct is redundant because if the prejacent is false, all stronger alternatives are necessarily false). We leave for another occasion studying cases where the alternatives that \text{p-exh} operates over might not be stronger than the prejacent, cases for which the choice between (i) and (34) might make different predictions.
(38) **Strong Kleene semantics for existential quantifiers.** An expression of the form $[\text{Some } \xi_x : \psi_x]$ (\(\xi\) is the restrictor and \(\psi\) the scope of the quantifier) is **True** if (i) holds, **False** if (ii) holds, and # if neither (i) nor (ii) hold.

(i) \(\exists x \in \lbrack \xi \rbrack : \llbracket \psi \rrbracket (x) = 1\) \hspace{1cm} \text{(Truth conditions)}
(ii) \(\forall x \in \lbrack \xi \rbrack : \llbracket \psi \rrbracket (x) = 0\) \hspace{1cm} \text{(Falsity conditions)}

Applying (38) to (36-a), we get that for (36-a) to be True it is necessary that some girls find some-but-not-all of their marbles, and for it to be False it is necessary that all girls don’t find any marbles. Notice that \(\psi\) in our case is of the form ‘p-exh\(_2\)(x found some of x’s marbles)’, and its 3-valued semantics was given in (35-a).

\[
\llbracket \text{(36-a)} \rrbracket = \begin{cases} 
1, & \text{if some girls founds some-but-not-all marbles} \\
0, & \text{if all girls didn’t find any marbles} \\
\# & \text{otherwise} 
\end{cases}
\equiv \text{no girls founds any marbles}
\]

This allows us to update the meaning for the whole LF in (37) as follows:

\[
\llbracket \text{(37)} \rrbracket = \llbracket \text{(33)} \rrbracket = \begin{cases} 
1, & \text{if some girls founds some-but-not-all marbles} \\
\& \llbracket \text{(36-b)} \rrbracket = 0 \\
0, & \text{if no girls founds any marbles} \\
\#, & \text{otherwise} 
\end{cases}
\]

Turning to (36-b), this is a universal statement; Strong Kleene’s system produces the following 3-valued semantics for universal statements:

(41) **Strong Kleene semantics for Universal quantifiers.** An expression of the form $[\text{All } \xi_x : \psi_x]$ (\(\xi\) is the restrictor and \(\psi\) the scope of the quantifier) is **True** if (i) holds, **False** if (ii) holds, and # if neither (i) nor (ii) hold.

(i) \(\forall x \in \lbrack \xi \rbrack : \llbracket \psi \rrbracket (x) = 1\) \hspace{1cm} \text{(Truth conditions)}
(ii) \(\exists x \in \lbrack \xi \rbrack : \llbracket \psi \rrbracket (x) = 0\) \hspace{1cm} \text{(Falsity conditions)}

Applying (41) to (36-b), we get that for (36-b) to be False it is necessary that some girls don’t find any of their marbles. This result obtains crucially due to the fact that \(p\)-exh\(_2\) does not contribute anything to the Falsity condition of its prejacent. In other words, \(p\)-exh\(_2\) is vacuous here because ‘p-exh\(_2\)(x found some of x’s marbles)’ (our \(\psi\)) is false just in case the existential statement ‘x found some of x’s marbles’ is false, namely iff x found none of x’s marbles. It is this fact that is responsible for deriving the weak-global SI that was problematic for the exh theory (see below).

We now reached the final truth and falsity conditions for the whole LF. They are as follows:
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\[
[(33)] = \begin{cases} 
1, & \text{if some girls founds some-but-not-all of their marbles} \\
& \land \text{some girls didn’t find any of their marbles} \\
0, & \text{if no girls founds any of their marbles} \\
#, & \text{otherwise} 
\end{cases}
\]

The description of the truth conditions in (42) are exactly the two inferences we were after: the first line of it is the local SI and the second line the weak-global SI. So if we just concentrate on the inferences coming from the truth conditions of the LF, we have derived our desideratum, repeated in (43):

\[
(43) \quad \text{Desideratum (repeated from (30))}: \text{Given a sentence of the form } \text{Some}_x[... \text{some}_y... P(x,y)], \text{derive the following implicatures:} \\
a. \quad \exists x[... \exists y \land \neg \forall y [...P(x,y)]] \quad \text{(Local SI)} \\
b. \quad \neg \forall x[... \exists y [...P(x,y)]] \quad (\equiv \exists x[... \neg \exists y [...P(x,y)]] \quad \text{(Weak-global SI)}
\]

The Strong Kleene system we relied on to obtain this result makes predictions not only about the truth(/falsity) conditions of LFs, but also about their presuppositions. Recall that the presupposition of an LF according to Strong Kleene are the disjunction of its truth conditions and falsity conditions. The disjunctive presupposition predicted for (42) is thus:

\[
(44) \quad \text{Presupposition predicted for (42):} \\
\text{Either none of the girls found any of her marbles,} \quad \text{(F conditions)} \\
or some girl found some but not all of her marbles and some girl didn’t find any of her marbles.22 \quad \text{(T conditions)} 
\]

In section 1 we proposed that the presuppositions stemming from \text{p-exh} will normally not be detected as presuppositions because they will be quietly accommodated prior to evaluating the LF. This proposal applies here too. Note though that in our case a minimal accommodation of (44) is not very plausible to begin with, since it would entail a rather odd common ground against which to evaluate the assertive component.23 This, we propose, motivates hearers not to accommodate a presupposition here at all, but rather to just compute the truth conditions of the LF and derive inferences based on them.

22 With the help of some logic, (44) turns out to be equivalent to the following (material) implication:

\[
(i) \quad \text{Presupposition predicted for (42):} \\
[\text{If some girl found some of her marbles, then [some girl didn’t find any of her marbles and not all girls found all of their marbles.]}]
\]

23 Such a common ground would exclude worlds in which every girl found some of her marbles as well as worlds where some girls found none of their marbles but the other girls found all of their marbles.
5 Summary and Outlook

To summarize, there are two kinds of facts about Scalar Implicatures that motivated our proposal in this paper to encode SI as presuppositions, rather than as at-issue material, that a grammatical exhaustivity operator contributes. The first fact is pragmatic in nature: we argued that such conception of SI admits a more natural and empirically more adequate explanation of Magri (2009)’s facts regarding mismatches between SI and common ground knowledge, exploiting the close connection between the concept of presuppositions and the concept of common ground. The second fact is more semantic in nature, and concerns the behavior of SI when triggered under negation: the fact that embedding p-exh under negation is mute helped us explain both why SI usually don’t arise under negation, why when they do arise they show a behavior that closely resembles the way that presuppositions become at-issue under negation, and finally it was a crucial ingredient in explaining the some under some puzzle.

One important question is how our proposal relates to the triggering problem for presuppositions and more generally the division between assertive and other content. The triggering problem is the question how to predict which aspects of the meaning of a lexical items are presupposed. How to solve this problem is still an open question (Abrusán 2011, Tonhauser et al. 2013, Tieu et al. 2019). But observe that our p-exh is less problematic for approaches to the triggering problem inspired by Schlenker’s (2007) work than standard exh is. If the presupposition-assertion division is ignored, our p-exh, the standard exh, and the standard lexical entry for only are all equivalent. Schlenker develops a pragmatic analysis that implies that a lexical item with a conjunctive meaning such as pq can only occur if either p or q is presupposed. Both only and p-exh are consistent with Schlenker’s proposal, but exh is not: Only presupposes its prejacent and asserts exhaustivity; p-exh asserts the prejacent and presupposes exhaustivity; and exh presupposes nothing and asserts both the prejacent and the exhaustivity component. Schlenker’s (2007) intuition that a contribution must not lead to multiple novel inferences or answer different questions remains one of the leading intuitions guiding current approaches, and as long as that is the case, our argument for p-exh stands. We speculate that the difference between only and p-exh might in fact be a useful test case for theorizing on the typology of inferences.

References

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