What part of ‘no’ don’t you understand?

Some thoughts on Negative Quantifiers

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The goal of this short article is to present a novel idea that both Universal Quantifier and Existential Quantifier can be reduced to a Negative Quantifier.

In mathematics, only Universal Quantifiers and Existential Quantifiers are traditionally used. There are even special symbols for this: $\forall$ (a vertically inverted “A”) and $\exists$ (a vertically inverted “E”).

The letter $\mathbf{A}$ stands for All and the letter $\mathbf{E}$ stands for Exists.

When describing natural language, semanticists actually use more than just these two quantifiers. In English, quantifiers may compound with other words.

Universal Quantifier: everyone, everything, everywhere, …

Existential Quantifier: someone, something, somewhere, …

Negative Quantifier: noone, nothing, nowhere, …

Interestingly, we can derive the other quantifiers from the negative one. But first we need a symbol for the Negative Quantifier. For this I suggest we use a vertically inverted $\mathbf{N}$ . (Luckily it isn’t hard to find on the keyboard, since an inverted $\mathbf{N}$ is also written $\mathbf{N}$.)

$\mathbf{N}x \ P(x)$ means “there is no $x$ such that $P(x)$”.

Both the Universal Quantifier $\forall x$ and the Existential Quantifier $\exists x$ can be defined in terms of $\mathbf{N}x$ in the following way:

(1) $\forall x \ P(x) = \mathbf{N}y \ \mathbf{N}x \ P(x)$

(2) $\exists x \ P(x) = \mathbf{N}x \ \mathbf{N}y \ P(x)$

Of course, the variable $y$ must be free in $P(x)$. 
∀x and ∃x are no longer elementary.

A Regular Negation likewise can be expressed in terms of a Negative Quantifier N:\n
(3) ¬P(x) = Ny P(x)

As is well know, all other logical connectives can be expressed in terms of Conjunction and Negation. Assuming that pre-linguistic intelligence (Margaliot, 2017) already has Conjunction, adding Negative Quantifier N immediately gives us all the other quantifiers as well as all of the logical connectives.

References

Margaliot, Sasson 2017 “Minimalist Semantics and Pre-Linguistic Intelligence”, lingbuzz/003409