Static and dynamic exceptional scope

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**Abstract** This paper proposes a formal account of indefinites’ exceptional quantificational and binding scope properties — static and dynamic exceptional scope. I argue that minimal dynamic extensions of tools independently motivated and widely used for questions and indefiniteness offer a unified explanation of indefinites’ multifaceted exceptional scope behavior. The account improves on existing static and dynamic theories of indefiniteness, and predicts a wide range of attested exceptional scope behavior for a range of expressions, both indefinite and not.

1 Overview

Indefinites display exceptional scope in two ways. They take exceptional quantificational scope, up and out of islands for overt and covert movement. And they take exceptional binding scope, anteceding pronouns in ways ‘genuine’ quantifiers cannot (e.g., in cross-sentential and donkey-anaphora configurations).

This co-incidence doesn’t seem coincidental. For one, since quantificational scope feeds binding (Barker 2012), we should expect exceptional quantifiers to be exceptional binders! Also, the pattern recurs, e.g., with disjunction (Section 6) (and with indefinites cross-linguistically, though this paper’s focus is English). Finally, our best theories of both varieties of exceptional scope are built on closely related ideas. Static theories of exceptional quantificational scope treat indefinites as (potentially) referential, or near enough (Fodor & Sag 1982, Reinhart 1997, Kratzer 1998, Schwarzschild 2002). Dynamic theories of exceptional binding scope treat indefinites as discourse-referential — as nondeterministic analogs of definites (e.g., Karttunen 1976, Kamp 1981, Heim 1982, Barwise 1987, Rooth 1987, Groenendijk & Stokhof 1991a).

These points suggest that we are owed a unified account of indefinites’ exceptional scope properties. For the most part, though, research on indefinites can be sorted into static treatments of their quantificational properties and dynamic treatments of their anaphoric properties. While theories combining (e.g.) choice-functional accounts of quantificational scope with dynamic accounts of binding scope do exist (see, e.g., von Heusinger 2000, 2002, Chierchia 2005, Brennan 2011), such accounts inherit well-known deficits of choice-functional treatments of indefinites’ quantificational scope, and fail to explain how exceptional quantificational scope feeds exceptional binding scope (Section 7).

Alternative semantics theories of indefinites, though static, share with dynamic theories a view of indefiniteness as nondeterminism (e.g., Ramchand 1997, Kratzer & Shimoyama 2002, Kratzer 2005, Charlow 2019b). Indeed, indefinite alternatives are used in ways both static and dynamic theorists of exceptional scope can appreciate. Parallel to static theories, alternatives are used to explain how indefiniteness can bubble up out of islands; parallel to dynamic theories, quantificational operators may ‘associate with’ indefiniteness by quantifying over alternatives.
This paper presents a compositional dynamic system unifying the exceptional quantificational and binding scope of indefinites. I show how familiar tools originally developed within the literature on type-shifting and questions can be naturally and minimally extended with notions of input (assignment-sensitivity) and output (assignment modification). This yields a modular interface for dynamic meaning composition (modular in the sense that it’s directly grafted onto familiar compositional systems built on functional application), which explains exceptional quantificational and binding scope as static and dynamic manifestations of a general form of island-insensitivity that inheres in the system. I show that this theory compares favorably with existing static and dynamic treatments of indefiniteness, makes a number of attested predictions about the varieties of exceptional scope, and reveals a fundamental connection between dynamic accounts of indefiniteness and accounts based on alternatives.

I flag at the outset that this paper focuses on ‘first-generation’ compositional dynamic systems (e.g., Muskens 1996) (intensionality is suppressed too). My main concern is developing and motivating a general compositional interface for indefiniteness, and I consider other domains only insofar as they bear on the feasibility and desirability of this project. Concretely, I treat dynamic generalized quantification (Kanazawa 1994, Chierchia 1995) only cursorily, and I don’t consider the insightful techniques for generalized quantification developed within modern ‘plural’ dynamic systems (e.g., van den Berg 1996, Nouwen 2003, Brasoveanu 2007). Nevertheless, as we’ll see, the theory we end up with is extremely similar to standard first-generation compositional dynamic systems, and so the usual techniques developed for extending those systems into plural dynamic systems can in principle be straightforwardly applied.

2 Aspects of exceptional scope

2.1 Quantification

The quantificational scope of indefinites is basically unbounded (Fodor & Sag 1982, Farkas 1981, Abusch 1994, Reinhart 1997). Indefinites which appear stranded on islands for overt and/or covert movement are nevertheless able to acquire scope outside these islands: (1) allows a reading ‘about’ a specific relative, paraphrasable as a rich relative of mine is such that, if they die, I’ll inherit a house. This reading exists despite the fact that the indefinite is inside a conditional antecedent (the bracketed constituent), an island for both overt and covert movement (i.e., scope). ‘Genuinely’ quantificational expressions like every NP do not exhibit similar flexibility (e.g., May 1988): (2) doesn’t allow an interpretation paraphrasable as every rich relative of mine is such that, if they die, I’ll inherit a house (in which case, any of their deaths would be sufficient). The scope of the universal quantifier is confined to the conditional’s antecedent.

\[
(1) \text{ If } [\text{a rich relative of mine dies}], \text{ I’ll inherit a house.} \quad \exists \Rightarrow \text{if}
\]

\[
(2) \text{ If } [\text{every rich relative of mine dies}], \text{ I’ll inherit a house.} \quad \forall \Rightarrow \text{if}
\]

There is a great deal of empirical and theoretical work on the exceptional scope properties of indefinites. I will not recap most of it here (see Charlow 2009b for a recent overview). I will, for example, take it for granted that indefinites can project their existential force up and out of islands, and that the ultimate scope position of this existential force can be non-maximal (in

On a prominent approach to exceptional quantificational scope (Reinhart 1997, Winter 1997), indefinite determiners denote variables over choice functions, and existential closure operators bind these variables, as schematized in (3). An $f : (e \to t) \to e$ is a choice function iff $f$ maps any non-empty $P : e \to t$ into one of $P$'s members, as in (4) (here and throughout, I’ll freely mix property and set talk). Putting these pieces together, (i) can be assigned the meaning in (3) if the $\exists^n$ binding $\alpha_n$ is base-generated with maximal scope. This meaning can be paraphrased, roughly, as follows: there’s a way of choosing things (including relatives of mine), such that if that relative of mine dies, I’ll inherit a house. This provides a reasonable approximation of (i)’s exceptional scope meaning (though not a perfect one; see Section 7.1 for critical discussion of choice functions), without requiring the indefinite to evacuate its island.

A second approach to exceptional quantificational scope, though one not typically applied to English indefinites, is alternative semantics (Ramchand 1997, Kratzer & Shimoyama 2002, Alonso-Ovalle & Menéndez-Beníto 2013, Charlow 2019b). In such systems, meanings of type $a$ are systematically replaced with sets of $a$’s, as in (6) and (7). In order to compositionally assemble sets of meanings, compositional operations are lifted into ‘point-wise’ versions operating on sets, as in (8), where $\emptyset$ is the operation relevant for composing $\alpha$ and $\beta$ normally (forwards or backwards functional application, predicate modification, and so on). Notice that (8) does not tell us how to upgrade syncategorematic compositional rules like predicate abstraction. Indeed, binding is problematic in alternative semantics. See Section 7.1 for discussion.

As in choice-functional theories, indefinites in alternative semantics aren’t semantically different in kind from definites. The meaning of Polly is a singleton set, $\{p\} : S e$. Alternative-

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1 Here’s a summary of notational conventions. First, ‘$a := b$’ means that type $a$ is defined as $b$; ‘$a \to b$’ names the type of functions from type $a$ to type $b$; and ‘$x :: a$’ means $x$ has type $a$. Second, parentheses are dropped whenever possible, under the following two conventions: types associate to the right, such that $a \to b \to c$ is equivalent to $a \to (b \to c)$; and application associates to the left, such that $f \langle x y \rangle$ is equivalent to $\langle f (x) \rangle (y)$. Finally, $\lambda_x \Delta$ names the function $f$ such that for all type-appropriate $d$, $f d = \Delta^{x=d}$ (that is, $\Delta$ with $d$ replacing all free occurrences of $x$).

semantic indefinites, in other words, are nondeterministic analogs of definites. Like definites, but unlike truly quantificational expressions, they can always be interpreted in situ.

Given (8), indefinite alternatives compositionally expand into sets of alternative propositions, which predicts a form of exceptional scope. Consider the exceptional scope reading of (i). The antecedent is associated with the set of alternative propositions in (9). Assuming the conditional operator’s meaning is \(\{\lambda l, l \Rightarrow r\} \cdot S (t \rightarrow t \rightarrow t)\), this expands into the set of alternative conditional propositions in (10). This gives the indefinite something akin to wide scope, even as it is interpreted in situ: there is a true element of this set iff for some relative of mine, dies \(x \Rightarrow\) house.

\[
\begin{align*}
(9) & \quad \{\text{dies } x \mid \text{rel } x\} \Rightarrow S t \\
(10) & \quad \{\text{dies } x \Rightarrow \text{house } \mid \text{rel } x\} \Rightarrow S t
\end{align*}
\]

The alternatives introduced by indefinites may be counterbalanced by closure operators, which quantify over alternatives. For example, the entry for negation in (11) requires that its prejacent’s meaning contains no true members (i.e., is wholly false). A closure-operator variant of the conditional, (12), can be defined in terms of negation and conjunction, drawing on the familiar equivalence of the material implication \(p \Rightarrow q\) and \(\neg(p \land \neg q)\).

\[
\begin{align*}
(11) & \quad \{\text{not } a\}^S := \{T \notin \{a\}^S\} \\
(12) & \quad \{\text{if } a \beta\}^S := \{\text{not } [a \land \neg \beta]\}^S = \{T \in \{a\}^S \Rightarrow T \in \{\beta\}^S\}
\end{align*}
\]

I don’t mean to imply that alternative semanticists adopt entries like (12). For one, this semantics is inconsistent with exceptional scope: with (12), (10) can’t be derived if the indefinite remains inside the conditional! I include (12) for illustration, and since it bears a close resemblance to corresponding conditional meanings in other frameworks that we’ll examine soon.

2.2 Anaphora

There is a second way in which indefinites display exceptional scope: they function more like definites than ‘genuine’ quantifiers with respect to anaphoric processes:

\[
(13) \quad \begin{cases} 
\text{Polly}^p \\
\text{A linguist}^n \\
\text{Every linguist}^n \\
\text{No linguist}^n
\end{cases}
\text{walked in the park. She, whistled.}
\]

Like the definite proper noun, it’s natural to hear the indefinite and the pronoun as covalued, though it’s unlikely the indefinite could ever come to have scope over a pronoun in a separate sentence. Tellingly, the same pattern is seen if the two sentences are conditionally connected, as in if a linguist\(^n\) walked in the park, she, whistled. This sentence has a grammatical ‘donkey’ reading on which the indefinite antecedes but does not scope over the pronoun (i.e., meaning that if there is a linguist who walked in the park, that linguist whistled). ‘Genuine’ quantifiers pattern differently. Cross-sentential anaphora and donkey anaphora are degraded or impossible if the indefinite determiner is replaced with every or no.
Figure 1: Dynamic sentence meanings as relations on assignment functions.

In dynamic theories of anaphora, a sentence $\phi$ denotes a relation on assignments $[\phi]$, type $g \rightarrow Sg$ (with $g$ the type of assignments). The sentences in (i) are associated with the dynamic meanings below. For example, the relation (i) maps an input assignment $g$ into a set of outputs, each associating the index $n$ with a linguist who walked in the park (and otherwise just like $g$).\(^3\)

(14) $[\text{Polly}^n \text{ walked in the park}] = \lambda_g \{ g^{n \rightarrow p} | \text{witp} p \}$

(15) $[\text{a linguist}^n \text{ walked in the park}] = \lambda_g \{ g^{n \rightarrow x} | \text{ling} x, \text{witp} x \}$

(16) $[\text{every linguist}^n \text{ walked in the park}] = \lambda_x \{ g | \forall x \in \text{ling} : \text{witp} x \}$

(17) $[\text{no linguist}^n \text{ walked in the park}] = \lambda_g \{ g | \neg \exists x \in \text{ling} : \text{witp} x \}$

The intuition is that these meanings model how sentences come to update the value associated with $n$ in context. Processing (14) means coming to associate $n$ with Polly (conditional on Polly having walked in the park). Processing (15) means coming to associate $n$ with a linguist who walked in the park. And processing (16) or (17) adds nothing to the conversational scoreboard apart from requiring that every/no linguist walked in the park in order to return an output at all. This process is represented pictorially for (14) and (15) in Figure 1.

Dynamic meanings allow for binding, even in the absence of scope: the relations in (14) and (15) store information about the values of variables in their output assignments. This stored information can then be passed between sentences. For example, dynamic conjunction, in (18), links two dynamic sentence meanings together by composing them qua relations: the left conjunct outputs a set of updated assignments $lg$, which are fed as inputs to the right conjunct, yielding new sets of updated assignments, which are finally collected to complete the update.

(18) $l ; r := \lambda_g \bigcup_{h \in lg} r h$

Dynamically conjoining $l$ and $r$ gives $r$ access to the anaphoric information stored in $l$ (but not vice versa). If she$_n$ whistled has the meaning in (19) (where 'g$_n$' names the individual $g$ associates with the index $n$), dynamically conjoining (14) and (19) yields (after a few simplifications) the meaning in (20), which covalues the walking linguist and the whistler, as desired. Binding transpires because the left conjunct outputs assignments that associate $n$ with a linguist who walked in the park, which dynamic conjunction passes as inputs to the right conjunct.

Thus, the indefinite comes to have a kind of 'semantic scope' over the pronoun, even as the indefinite’s 'syntactic scope' remains confined to its minimal tensed clause.

\[ \lambda_x \{ g \} \text{ if whistle}_n \text{ else } \emptyset \quad \lambda_x \{ g^{m-x} \mid \text{ling}_x, \text{witp}_x, \text{whistle}_x \} \]

Dynamic sentence meanings return outputs only when their truth conditions are met. Accordingly, we can say that \( m \) is true at an input \( g \) if \( m \) is non-empty: \((15)\) is true at an input \( g \) if a linguist walked in the park; \((19)\) is true at \( g \) if \( g_n \) whistled; and \((20)\) is true at \( g \) if a linguist who walked in the park whistled. All as expected. Note here the connection to alternative semantics, where truth conditions also involve existentially quantifying over outputs.

There are many compositional routes to dynamic sentence meanings. One straightforward option is to identify the dynamic propositional type \( T \) with the type of relations on assignment functions, \((21)\). (Again, this paper completely sets aside intensionality.) Then verb phrase meanings are modeled as dynamic properties, functions from individuals into dynamic propositions, \((22)\). Here and subsequently, 'simple' meanings are set in sans and 'fancy' or dynamic ones in bold sans; thus, \( \text{witp} : e \rightarrow t \), but in this section \( \text{witp} : e \rightarrow T \).

\[ T := g \rightarrow Sg \]
\[ \text{witp} := \lambda_x \lambda_y \{ g \} \text{ if witp}_x \text{ else } \emptyset \]
\[ \text{witp} : e \rightarrow T \]

Correspondingly, proper names and indefinites denote functions from dynamic properties to dynamic propositions, as in \((23)\) and \((24)\): \( \text{polly}^n \) feeds an individual and a modified assignment to a scope argument \( f \), while \( \text{a.ling}^n \) does the same thing nondeterministically, once per linguist. To derive \((14)\) or \((15)\), apply \((23)\) or \((24)\) to \((22)\). This delivers the desired propositional meanings after a few \( \lambda \)- and set-theoretic simplifications.

\[ \text{polly}^n := \lambda_f \lambda_x \{ g \} \rightarrow Sg[\rightarrow T] \]
\[ \text{polly}^n : (e \rightarrow T) \rightarrow T \]
\[ \text{a.ling}^n := \lambda_f \lambda_x \{ g \} \rightarrow Sg[\rightarrow T] \]
\[ \text{a.ling}^n : (e \rightarrow T) \rightarrow T \]

As in alternative semantics, the possibilities created by indefinites are counterbalanced by operators that quantify over these possibilities. We again focus on negation and the conditional (this will be a running theme). Negation, in \((25)\), requires that its prejacent has no outputs, i.e., that there is no way of making the prejacent true, cf. \((11)\). A dynamic conditional is defined in \((26)\) using negation and conjunction, cf. \((12)\). It requires that every assignment output by the antecedent survives in the consequent, i.e., that each way of making the antecedent true makes the consequent true. (I do not focus on donkey anaphora in this paper, but it’s easy to see given the semantics in \((26)\) that the consequent \( r \) has access to the anaphoric information in \( l \).)

\[ \text{not} m := \lambda_x \{ g \} \text{ if } m g = \emptyset \text{ else } \emptyset \]
\[ \text{not} : T \rightarrow T \]
\[ \text{if } r := \text{not} \{ l ; (\text{not} r) \} \]
\[ \text{if } : T \rightarrow T \rightarrow T \]

Closure operators are externally static: they always return the input assignment \( g \) unchanged. This accounts for the observation that closure operators delimit the anaphoric scope of indefinites. For example, it is impossible to continue if a linguist walked in the park. shea whistled with shea skipped, at least on the natural reading where the indefinite scopes inside the conditional.
We round out this basic setup with the entries for indefinite and universal determiners in (27) and (28). The universal is defined in terms of negation and the indefinite (drawing on the usual first-order equivalence between $\forall$ and $\neg \exists$); a definition for no$^n$ can be obtained by dropping the inner not. Because every$^n$ and no$^n$ contain a wide-scoping negation, they are externally static, and therefore cannot bind pronouns outside their scope. This is consistent with the data in (16) and the relational meanings in (17).

(27) \[ a^n \cdot k \ := \lambda_g \bigcup_{x: e, h \in c x g^{m-x}} k x h \quad a :: (e \to T) \to (e \to T) \to T \]

(28) \[ \text{every}^n \cdot k \ := \text{not} \left( a^n \cdot (\lambda_x \text{not} (k x)) \right) \quad \text{every} :: (e \to T) \to (e \to T) \to T \]

\[ = \lambda_g \{ g \} \text{ if } \forall x, h : h \in c x g^{m-x} \Rightarrow \exists i : i \in k x h \text{ else } \emptyset \]

Crucially, standard dynamic theories do not predict exceptional quantificational scope. Consider the exceptional scope reading of (i), if [a rich relative of mine dies], I’ll inherit a house. This reading is modeled by (29), and not by (30) (the latter’s truth conditions are that every way of making the antecedent true results in a house). So in standard dynamic systems, exceptional quantificational scope for indefinites requires them to actually be assigned wide scope; it is unclear why this option should be available to indefinites but not (e.g.) universals.

(29) \[ a\.rel^n \ (\lambda_x \text{if} (\text{dies} x) \text{house}) = \lambda_g \{ g^{m-x} \mid \text{rel} x, \text{dies} x \Rightarrow \text{house} \} \]

(30) \[ \text{if} \ (a\.rel^n \ (\lambda_x \text{dies} x)) \text{ house} = \lambda_g \{ g \} \text{ if } \forall x \in \text{rel} : \text{dies} x \Rightarrow \text{house} \text{ else } \emptyset \]

Because the conditional is externally static, only (29), with the indefinite scoping over the conditional, allows anaphoric reference back to the indefinite from outside the conditional; that is, only (29) outputs a modified assignment. This is empirically correct (though, to emphasize, why a reading akin to (29) should be available in the first place is mysterious): continuing (i) with she is an oil baron is acceptable given an exceptional wide-scope reading for the indefinite (and, as noted previously, unacceptable otherwise) — exceptional quantificational scope, in other words, feeds exceptional binding scope (see Abusch 1994: 112ff, Chierchia 2005: 166ff).

2.3 Taking stock

Dynamic treatments of indefinites bear a strong family resemblance to alternative-semantic treatments. Both perspectives model indefiniteness as nondeterminism — in both theories, indefinites cause us to contemplate a number of separate meanings (or contexts) in parallel. In both systems, the introduction of nondeterminism by indefinites is counterbalanced by operators that quantify over or ‘capture’ alternatives.

This similarity is striking indeed, but is it a mirage? Following Groenendijk & Stokhof (1990), Muskens (1996), and many others, the dynamic system we assume is compositionally austere, based solely on functional application. (This austerity is offset by relatively complex lexical meanings, which build in the work of passing around assignments and managing nondeterminism.) This contrasts starkly with alternative semantics, in which the grammar is nondeterministic ‘all the way down’ — every expression is associated with a set of alternatives (possibly a singleton), and meaning composition itself is nondeterministic (i.e., point-wise).
Moreover, while dynamic systems explain aspects of indefinites’ exceptional anaphoric properties well, they seem to offer no insight about why indefinites take exceptional quantificational scope: because dynamic grammars are oriented around functional application, there’s no obvious way to project an indefinite’s scope out of an island, let alone relate its exceptional quantificational scope properties to its exceptional binding scope properties. (Reminder: accounts that treat existential quantification over choice functions dynamically will be considered in Section 7.) Conversely, though alternative semantics has something to say about exceptional quantificational scope, it does not offer any insight into exceptional binding scope.

2.4 Previewing the account

Relations on assignments are pure, uncut context change. This is a feasible sort of semantic representation for dynamic propositions (since, e.g., truth can be identified with having an output), but there is no clear sense in which dynamic individuals, dynamic properties, and so on can be represented as such — except derivatively, as unsaturated dynamic propositions.

This paper proposes a more general characterization of dynamicity, appropriate to values of any type: a dynamic $a$ is a mapping from an input assignment to a set of $a$’s paired with output assignments, as in (31). $D$ carries the usual markers of dynamicity: output and nondeterminism. Indeed, $D_a$ is close in form to the standard dynamic-semantic propositional type $T_F \rightarrow S_g$. But whereas a $T$ only nondeterministically returns updated assignments (and is therefore only appropriate to dynamically modeling sentential meaning), a $D_a$ nondeterministically returns a value of type $a$, alongside updated assignments.  

$D_a : g \rightarrow S(a \times g)$

(31) To build a grammar around $D$, we provide a way to upgrade values into dynamic ones, and a characterization of how dynamic meanings interact, or compose. This requires two mappings, respectively (32) and (33). The first, $\eta$, ‘lifts’ a value into a trivially dynamic meaning, a singleton with an unchanged assignment. $\eta$ is a dynamic analog of Partee’s (1986) IDENT, or Karttunen’s (1977) Proto-Question Rule (Section 3.1). The second, $\star$, characterizes how dynamic meanings interact; $\star$ extracts $m$’s value-output pairs (at $g$), feeds them component-wise to $f$, and collects the sets of new pairs that result. The potentially updated assignments $m$ outputs are thus visible in $f$, reminiscent of dynamic conjunction, (18). Like $\eta$, $\star$ is a dynamic analog of an operation familiar from the questions literature (again, Section 3.1 gives details).

$\eta x : x \mapsto (x, g)$

(32) $\eta : a \rightarrow D a$

$m \star f : x \mapsto \{ (x, h) \in m g \mapsto f x h \}$

(33) $\star : D a \rightarrow (a \rightarrow D b) \rightarrow D b$

These two functions, minimal dynamic extensions of tools independently motivated and widely used for questions and indefiniteness, offer a unified and complete explanation of  

4 An interesting subtlety bears mentioning. Because assignments can be identified with sequences or lists (products of arbitrary length), the distinction in principle between $a \times g$ and $g$ is somewhat blurry. Indeed, $a \times g$ can be seen as a ‘pointed’ assignment, one whose first position holds the current ‘compositionally active’ value (cf. Grosz, Joshi & Weinstein 1995, Hardt 1999, Bittner 2001, Murray 2014, Beaver & Condoravdi 2007, AnderBois, Brasoveanu & Henderson 2015.) Perhaps, then, $S(a \times g)$ and $Sg$ are not fundamentally different in kind. (These remarks presuppose that assignments can harbor values with types other than $e$. See Muskens 1995, 1996 for pertinent discussion.)
static and dynamic exceptional scope. The rest of the paper shows how. Section 3 reviews the account of static exceptional scope defended in Charlow 2009b. Section 4 shows how to replace the static treatment of binding relied on there with a more general version that incorporates a notion of output, upgrading the static alternative semantics into a dynamic system, albeit one with DNA extremely close to standard dynamic systems, as well as static theories based on alternatives. I use this semantics to treat a variety of exceptional scope constructions in Sections 5 and 6, including the ability of exceptional scope to feed binding and the static and dynamic exceptional scope properties of disjunction, indefinite and bare plurals, dynamic generalized quantifiers, and definite expressions. I compare the theory to existing static and dynamic accounts of exceptional scope in Section 7, before concluding.

3 The scope of alternatives

This section summarizes the main results of Charlow 2009b. The presentation is self-contained, but all details not directly relevant to the current investigation are omitted or compressed. Readers interested in a more leisurely exposition, as well as considerations of intensionality, composition inside DP, etc., are referred to that work.

3.1 Indefinite composition

Partee (1986) (see also Chierchia 1998, Winter 1998) argues that nominals have multiple faces: referring, type e; set-denoting, type Se; and quantifying, type (e → t) → t. Partee proposes that languages provide a suite of mappings for moving between these guises. Mappings that aren’t overtly lexicalized may be available as silent type-shifters.

For example, to go from e to Se, Partee provides an IDENT operation mapping any entity x into a singleton set containing just x (see Chierchia 1998, Winter 1998, Fox 2002, Champollion 2016, and many others). See (34) for the definition, where I rename this operation η, and give it a maximally general polymorphic type. The η mapping is also invoked in approaches to question composition following Karttunen (1977); in that context, it maps a declarative meaning (a proposition) into a trivial interrogative meaning (a singleton set of propositions).

η x := {x}  
η :: a → Sa

To go from Se to (e → t) → t, Partee provides an Α operation that introduces existential quantification over the members of its input set. I rename this operation ∃, and again give it a maximally polymorphic type:

∃m := λx. ∃x ∈ m : f x  
∃ :: Sa → (a → t) → t

∃(η x) = LIFT₁ x := λf. f x  
LIFT₁ :: a → (a → t) → t

η and ∃ form a decomposition of LIFT₁, the mapping from values of type a into generalized quantifiers over a’s, as in (36) (the t subscript here indicates the ‘result type’ presupposed by LIFT). The left-lower portion of Figure 2 summarizes the relationships between η, ∃, and LIFT₁.

Karttunen builds η into his Proto-Question Rule, and further requires that (proto-) questions contain only true answers. Following Dayal (1996) and many others, we drop the restriction to true answers.
Figure 2: Summarizing the relationships between various type-shifting operations. Previous work uses \( \eta, \exists, \) and \( +\text{wh} \) to compositionally build sets of alternatives. My proposal is to use \( \eta \) and \( \star \).

Karttunen (1977), Cresti (1999), and Heim (2011) offer accounts of questions (and in Heim’s case, indefiniteness) which furnish a \( +\text{wh} \) mapping, from \( (e \to t) \to t \) to \( (e \to S t) \to S t \). The definition is provided in (37) (again, with general types). An example of how \( +\text{wh} \) turns an existential quantifier into a ‘\( \text{wh} \)-quantifier’, something that expects to scope over and return a set of meanings, is provided in (38). In concert with \( \eta \), this allows us to project alternatives from existential quantification: \( +\text{wh} \, (\exists \text{ling}) \, (\lambda_x \eta (\text{witp} \, x)) = \{\text{witp} \, x \mid \text{ling} \, x\} \).

\[
\begin{align*}
\text{(37)} & \quad +\text{wh} \, \emptyset := \lambda_f \{y \mid \emptyset (\lambda_x \, y \in f \, x)\} \\
& \quad +\text{wh} :: ((a \to t) \to t) \to (a \to S b) \to S b \\
\text{(38)} & \quad +\text{wh} \, (\exists \text{ling}) = \lambda_f \{y \mid \exists x \in \text{ling} : y \in f \, x\} \quad \text{type:} (e \to S b) \to S b \\
& \quad = \lambda_f \bigcup_{x \in \text{ling}} f \, x
\end{align*}
\]

As Figure 2 bears out, the addition of \( +\text{wh} \) exposes some terra incognita in the diagram’s eastern half, highlighting the possibility of a mapping from \( S a \) directly to \( (a \to S b) \to S b \), which together with \( \eta \) forms a decomposition of \( \text{LIFT}_{sb} \). Let’s call this mapping \( \star \). In fact, Figure 2 already tells us how it should be defined: as the composition of \( \exists \) and \( +\text{wh} \) (see also (38)).

\[
\begin{align*}
\text{(39)} & \quad \lambda_m +\text{wh} \, (\exists m) = \lambda_m \lambda_f \bigcup_{x \in m} f \, x = \star
\end{align*}
\]

In Charlow (2019b), I propose treating indefinites as generators of \textit{alternatives}, type \( S e \), in line with alternative-semantic (Ramchand 1997, Kratzer & Shimoyama 2002) and inquisitive (Ciardelli, Roelofsen & Theiler 2017) treatments of indefiniteness:

\[
\begin{align*}
\text{(40)} & \quad \llbracket \text{a linguist} \rrbracket := \text{a.ling} := \{x \mid \text{ling} \, x\} \\
& \quad \text{a.ling} :: S e
\end{align*}
\]

Instead of semantically integrating indefinites by mapping them into existential quantifiers via \( \exists \), I propose to directly use \( \star \), which turns an indefinite into something that expects to scope over and return a new set of alternatives. The \( \eta \) mapping is retained for conjuring up initial

\begin{footnotesize}
\footnotesize
\begin{itemize}
\item These authors do not treat \( +\text{wh} \) as a type-shifter per se. Karttunen builds its effects into a rule for \( \text{wh} \)-quantification; Cresti posits a \( +\text{wh} \) morpheme such that \( \llbracket +\text{wh} \rrbracket := +\text{wh} \); and Heim sprinkles \( +\text{wh} \)'s effects across an interrogative LF.
\end{itemize}
\end{footnotesize}
sets of alternatives (much as in Karttunen 1977). The proposed compositional interface for indefiniteness can thus be summed up in (41) and (42) (note the use of infix notation for $\star$).

$$\eta \ x \ := \ \{ \ x \ \}$$

$$\star \ f \ := \ \bigcup_{x \in m} \ f \ x$$

I use $\eta$ and $\star$ to derive sets-of-alternative meanings for John met a linguist (left) and a linguist met a philosopher (right). The indefinites denote sets of alternatives, which take scope via $\star$. $\eta$ conjures up an initial, maximally boring set of alternatives, over which the scope-takers scope.

$$= \ \{ \ \text{met} \ x \ j \mid \ \text{ling} \ x \ \}$$

$$= \ \{ \ \text{met} \ y \ x \mid \ \text{ling} \ x, \ \text{phil} \ y \}$$

**Figure 3**: Using $\eta$ and $\star$ to derive sets-of-alternatives meanings for *John met a linguist* (left) and *a linguist met a philosopher* (right). The indefinites denote sets of alternatives, which take scope via $\star$. $\eta$ conjures up an initial, maximally boring set of alternatives, over which the scope-takers scope.

To emphasize, $\star$ is equivalent to the composition (i.e., successive application) of $\exists$ and $+wh$. It can therefore be recovered in any system that allows $\exists$ and $+wh$ to apply freely. In a real sense, then, this most basic form of our proposal does not require any new technology — $\eta$ and $\star$ have been implicit in the work on questions and type-shifting for decades.

---

7 For now, the reader should think of these trees as semantic objects, i.e., as vertical expansions of $\lambda$-terms, spotting me some mechanism for effecting scope of the $\star$-shifted indefinites. I expand on this point in Section 4.1 and Appendix A.
3.2 Monads

It’s straightforward to check, as Figure 2 claims, that \( \eta \) and \( \star \) are a decomposition of \( \text{LIFT}_{b} \), i.e., that \( \lambda_{f} \eta x \star f = \lambda_{f} \bigcup_{y \in \{x\}} f y = \lambda_{f} f x \). This fact is restated in (44) as \text{Left Identity}, along with a couple other properties of interest. By \text{Right Identity}, scoping a \( \star \)-shifted \( m \) over \( \lambda_{x} \eta x \) is semantically idle; by \text{Associativity}, grouping doesn’t matter when iteratively scoping via \( \star \) (the reader is invited to check that \text{Right Identity} and \text{Associativity} hold for \( \eta \) and \( \star \)). \text{Associativity}, as we will see, is fundamental to our explanations of exceptional quantificational and binding scope. (Actually, you can already get a taste for why this is so. According to \text{Associativity}, the term on the left with a narrow-scoped \( m \) is equivalent to the term on the right with a wide-scoped \( m \): the appearance of wide scope without wide scope!)

\[
(44) \quad \text{Left Identity: } \quad \eta x \star f = f x \\
\text{Right Identity: } m \star \lambda_{x} \eta x = m \\
\text{Associativity: } \quad (m \star \lambda_{x} f x) \star e = m \star (\lambda_{x} f x \star e)
\]

This represents a highly general strategy for modularly extending a grammar to compositionally handle various kinds of enriched structures: identify a notion of enrichment \( T \), such that enriched meanings have types of the form \( T a \); find an operation \( \eta :: a \to T a \) for upgrading values in a trivial way, and an operation \( \star :: T a \to (a \to T b) \to T b \) for characterizing how enriched meanings interact, making sure they obey the laws in (44) (which ensures that \( \eta \) and \( \star \) are ‘well-behaved’ in a technical and practical sense). This suite of \( T \), \( \eta \), and \( \star \) is a \textit{monad}, and carrying out derivations like those in Figure 3 is doing monadic semantics.

Monads, a standard concept in category theory (e.g., Mac Lane 1971, Barr & Wells 1985), were used to study computational ‘side effects’ by Moggi (1989), Spivey (1990), Wadler (1992), and others, and first applied to linguistic semantics by Shan (2001a). Wadler (1994) points out a close connection between monads and delimited continuations, an important basis for the present work (delimited continuations are closely related to linguistic scope; see, e.g., Barker 2002, Barker & Shan 2014). Accessible introductions to the use of monads in functional programming can be found in Wadler 1992, 1995. Monads underlie a great deal of semantic theorizing, often implicitly. Since Shan’s (2001a) pioneering work, an increasing amount of research uses monads explicitly (see, e.g., Giorgolo & Unger 2009, van Eijck & Unger 2010, Giorgolo & Asudeh 2012, Unger 2012, Charlow 2014, 2019b, Barker & Shan 2014, Bumford 2015, Asudeh & Giorgolo 2016).

3.3 Exceptional scope and selectivity

Taking on \( \eta \) and \( \star \) as compositional primitives has an important upshot: alternative propagation can apply iteratively. An application of \( \star \) ultimately produces a set of alternative meanings, which can itself serve as the input to another instance of \( \star \). For this reason, even though the theory (like Karttunen’s) is oriented around scope, it ultimately predicts that alternatives may expand beyond island boundaries: the alternative-generating expression takes scope at the edge of the island, producing a set of alternatives which may itself take scope.

\footnote{‘Modular’ in the sense that the enrichment is grafted onto a base grammar, such that we are not required to rewrite our entire lexicon and rewire all our compositional operations.}
To see this in action, we’ll need (as in alternative and dynamic semantics) to define some closure operations to quantify over alternatives, so that we can see how \( \eta \) and \( \star \) allow indefiniteness to evade closure. Definitions for negation and a (material) conditional are given in (45) and (46). Negation requires a set of propositional alternatives to be uniformly false (compare alternative-semantic negation \( \eta \) and dynamic negation \( \eta \)).

The \( \neg(p \land q) \)-style definition of \( \text{if} \) is given in lieu of the simpler expression on the second line because this definition will be repurposed wholesale as we incorporate static and then dynamic treatments of binding: we will simply offer new monadic characterizations of \( \eta \), \( \star \), and \( \text{not} \), and be automatically rewarded with an upgraded conditional. Semantic clauses for indefinite and universal determiners, with the universal again drawing on dynamic treatments of binding: we will simply offer new monadic characterizations of \( \eta \), \( \star \), and be automatically rewarded with an upgraded conditional. Semantic clauses for indefinite and universal determiners, with the universal again drawing on dynamic treatments of binding: we will simply offer new monadic characterizations of \( \eta \), \( \star \), and be automatically rewarded with an upgraded conditional.

First, we derive a set-of-alternatives meaning for the conditional antecedent, the scope island on which the indefinite is stranded, by scoping \( \text{a.rel} \) to the island’s edge via \( \star \) and deriving an initial set of alternatives \( \{\text{dies} x \mid \text{rel} x\} \). Because this result is a new set of alternatives, \( \star \) can apply once more, allowing the island itself to take scope up and over the conditional, ultimately deriving a set of conditional propositions, precisely as in alternative semantics:

\[
\{\text{dies} x \mid \text{rel} x\} \star \text{if} \ (\eta \ p) \ (\eta \text{house}) = \bigcup_{p \in \{\text{dies} x \mid \text{rel} x\}} \{p \Rightarrow \text{house}\} = \{\text{dies} \Rightarrow \text{house} \mid \text{rel} x\}
\]

The reason iterative scoping via \( \star \) is successful in simulating wide scope is the associativity of \( \star \), which guarantees that the tree on the left of Figure 4, with the indefinite taking local scope, is equivalent to the tree on the right, in which the indefinite scopes out of the island.⁹

Arguments that this represents a general account of exceptional scope for indefinites can be found in Charlow 2019b. Informally and in broad strokes, it will always be possible for an indefinite to take scope via \( \star \) to the nearest island’s edge, producing a set of alternatives.

---

⁹ This type of iterated scope-taking is known, colorfully, as covert snowballing (or roll-up) pied-piping. See Charlow 2019b for arguments that this theory avoids common criticisms of covert pied-piping (Nishigauchi 1990; see von Stechow 1996 for the criticisms). Szabolcsi (2009) observes that the same kind of pied-piping is observed overtly in Hungarian and Telugu indeterminate quantification. Interestingly, an alternative compositionalization of monads allows us to derive the same meanings as \( \eta \) and \( \star \), while leaving sets of alternatives in situ. Appendix A gives details.
Figure 4: Associativity in action. The tree on the left, a derivation of if [a rich relative of mine dies], I’ll inherit a house with only local scope of the indefinite (to the edge of the highlighted island), is equivalent to the tree on the right, in which the indefinite scopes out of the highlighted island.

This island can itself take scope via ★, transmitting the indefinite’s indefiniteness upward. If further islands occur higher in the tree, scope-taking of the island may target the next-nearest island’s edge, producing a new set of alternatives, which itself is subject to further scope-taking via ★. This bank-shot route to apparent wide scope is unavailable to genuinely quantificational determiners: once, e.g., every takes scope, it produces a singleton set, containing only a universally quantificational truth condition. Further scoping via ★ will be semantically idle.

An important advantage of using η and ★ to treat indefinite’s exceptional quantificational scope is that it predicts this scope to be fully selective: multiple indefinites on an island can always be scopally distinguished outside the island. For example, (50) allows a reading with the linguist indefinite scoping outside the conditional, and the philosopher indefinite scoping inside (a specific, philosopher-hostile linguist), as well as a reading switching the indefinites’ scopes.

(50) If [a famous linguist sees a noteworthy philosopher], all hell will break loose.

Here the island has two indefinites, as in Figure 3, right. However, if we derive a meaning for the island along those lines, we have just two options: we can leave the island un-scoped, resulting in across-the-board narrow scope, or scope it via ★, resulting in across-the-board wide scope.

The full range of readings for (50) can be generated by appealing to a higher-order meaning for the island (cf. Dayal 1996, 2002, Hagstrom 1998, Fox 2012 on higher-order questions). This only requires an extra invocation of η in between the scope positions of the two ★’d indefinites,
as in (51). This derives a higher-order meaning, a set of sets. Call it $M$. Then the specific-linguist, nonspecific-philosopher reading of (50) can be generated as in (52), where the inner layer of alternatives is semantically reconstructed into the scope of if. The other selectivity reading (specific-philosopher, nonspecific-linguist) just requires swapping the indefinites’ scopes in (51).

(51) \[ \text{a.ling} \star \lambda_x \xi (\text{a.phil} \star \lambda_y \eta \text{ (sees } y \times)) = \{ \{\text{sees } y \times \mid \text{phil } y \} \mid \text{ling } x \} \quad \text{type: } S (S t) \]

(52) \[ M \star \lambda_m \text{if } m (\eta \text{hell}) = \bigcup_{m \in M} \{T \in m \implies T \in \{\text{hell}\}\} \quad \text{type: } S t \]

\[ = \{\{\exists y \in \text{phil} : \text{sees } y \times \} \implies \text{hell} \mid \text{ling } x\}\]

Selectivity is a significant feature of monadic semantics, one ultimately owing to the possibility of flexibly applying $\eta$ and $\star$. Alternative semantics, on the other hand, inflexibly associates (50)’s antecedent with a meaning of type $S t$, and is thus a poor fit for selective indefinite scope.¹⁰

### 4 From statics to dynamics

#### 4.1 Statics (adding input)

$S$’s $\eta$ and $\star$ are the foundation on which the coming analyses rest. In this subsection, we briefly explore how to extend $S$ into a monad for alternatives that also incorporates a static treatment of anaphora, laying the groundwork for the dynamic grammar in Section 4.2.

Our first job, as set out in Section 3.2, is to get clear on the kind of enrichment we wish to take on. To handle alternatives alongside static binding, we may simply mimic the alternative-semantic grammar characterized in (7), and (8), where alternative sets are compositionally determined relative to an assignment (i.e., with a notion of input, but no output). Adopting this perspective (for the moment), we take our enriched meanings to be functions from assignments into sets of alternatives, naming it ‘$A$’ (for alternative semantics):

(53) \[ \text{A } a := g \rightarrow S a \]

With $A$ in view, all that remains is to (i) define a $\eta$ mapping for trivially embedding values in the enriched space; (ii) define a $\star$ mapping for characterizing how enriched meanings interact; (iii) specify some meanings for things like indefinites, pronouns, and closure operators, which essentially live in, or manipulate things in, the enriched type-space.

$A$’s $\eta$ and $\star$ are straightforward to define: aside from the sprinkled references to assignments, the definitions in (54) and (55) just are $S$’s $\eta$ and $\star$. It is also straightforward to check that (54) and (55) obey the monad laws. E.g., for LEFT IDENTITY: $\eta x \star f = \lambda_y \bigcup_{y \in x} f x g = \lambda_y f x g = f x$.

(54) \[ \eta x := \lambda_y \{x\} \]

(55) \[ m \star f := \lambda_y \bigcup_{x \in m} f x g \]

On to the lexicon. We model indefinites and definites as assignment-relative sets of individuals (as in alternative semantics). The entries in (56) and (57) exploit different features of $A$: the

---

¹⁰ This is just a sketch of the argument, the full version of which appears in Charlow 2009b. It is generally assumed that using alternatives to explain exceptional scope is incompatible with selectivity (e.g., Krifka 1991, 2006, Rooth 1996, Wold 1996, Kratzer & Shimoyama 2002). The considerations here demonstrate that this is not so.
indefinite meaning ignores the assignment but returns a non-trivial (i.e., non-singleton) set of alternatives, while the pronoun meaning depends on the assignment but is fully deterministic.

\begin{align*}
\text{a.ling} & := \lambda_x \{ x \mid \text{ling} \ x \} & \text{a.ling} & := \text{A}\ e \\
\text{she}_{a} & := \lambda_x \{ g_{a} \} & \text{she}_{a} & := \text{A}\ e
\end{align*}

Closure operators are easily ported over from \(\text{S}'s\) \text{not} is in (58), and the automatic upgrade of if (given the new definitions of \(\eta, \star,\) and \text{not}) is reported in (59).

\begin{align*}
\text{not} \ m & := \lambda_g \{ \forall \not\in \ m\ g \} & \text{not} & := \text{A}\ t \rightarrow \text{A}\ t \\
\text{if} \ l \ r & := \text{not} (l \star \lambda_p (\text{not} r) \star \lambda_q (p \land q)) & \text{if} & := \text{A}\ t \rightarrow \text{A}\ t \rightarrow \text{A}\ t \\
& = \lambda_g \{ \forall \ t \in \ l\ g \Rightarrow \ t \in \ r\ g \}
\end{align*}

Continuing the motif, I include semantic clauses for indefinite and universal determiners, with the latter value again drawing on the first-order equivalence \(\forall \equiv \neg \exists \neg c:\)

\begin{align*}
\text{ac} := \lambda_x \{ x \mid \ T \in \ c \ x\ g \} & \quad \text{a} := (e \rightarrow \text{A}\ t) \rightarrow \text{A}\ e \\
\text{every} \ k & := \text{not} (\text{ac} \star \lambda_x \text{not} (k\ x)) & \text{every} & := (e \rightarrow \text{A}\ t) \rightarrow (e \rightarrow \text{A}\ t) \rightarrow \text{A}\ t \\
& = \lambda_g \{ \forall \ t \in \ c \ x\ g \Rightarrow \ t \in \ k\ x\ g \}
\end{align*}

Because \(\text{A}\) is just a modestly richer monad than \(\text{S} -\) with functionality for static anaphora added to alternatives — we don’t lose anything in the transition from \(\text{S}\) to \(\text{A}\). For example, the basic derivations in Figure 3 can be imported wholesale: to do so we need only replace each occurrence of \(\text{S}\) with \(\text{A}\), and prepend the derived meanings with a vacuous \(\lambda_g\). In addition, since \(\text{A}\) enjoys an \text{associative} \(\star\), exceptional scope is still predicted: the exceptional scope derivation from Figure 4, and the equivalence between its left-hand side (with only local scope of the indefinite) and right-hand side (with non-local scope), remain valid in \(\text{A}\). Again, importing this result only requires changing \(\text{S}\) to \(\text{A}\), and prepending \(\lambda_g\) to the result in (49). (And as before, this route to exceptional scope is unavailable to genuine quantifiers like \text{every}.)

Along with covering \(\text{S}'s\) greatest hits, \(\text{A}'s\) extra functionality allows us to compositionally integrate and do things with pronouns (statically). A derivation of \text{a linguist saw her} \(n\) \text{cat} parallel to Figure 3, right, is given in (62). The derived meaning inherits the nondeterminism of \text{a.ling} and the assignment-dependence of \text{her} \(n\) (in other words, the pronoun remains unbound).\footnote{This is why I was cagey about scope back in footnote 7. Standardly, scope is effected syntactically, using movement and variable traces \(\text{e.g.}\) \(\text{Heim} \& \text{Kratzer 1998}\). But if scope leaves behind a variable, that variable (if its type is \(\text{A}\ e\)) will, like pronouns, need to take scope via \(\star\) — in which case the variable leaves behind another variable, which itself needs to take scope, ad infinitum. Fortunately, there are many possibilities for avoiding this issue. We can use a non-syntactic approach to scope \(\text{Hendriks 1993, Barker 2002, Barker} \& \text{Shan 2014}\), treat traces of quantifier raising differently from pronouns \(\text{Büring 2004, 2005}\), or adopt the in situ approach to monadic composition outlined in Appendix A.}

\begin{align*}
\text{a.ling} \star \lambda_x \text{her}_{a} \star \lambda_y \eta (\text{saw} (\text{cat} \ y \ x)) = \lambda_g \{ \text{saw} (\text{cat} \ g_{n}) \ x \mid \text{ling} \ x \} & \quad \text{type: A t}
\end{align*}
use Büring’s (2005) $\beta$-binding operation, defined in (63). $\beta^n$ effects binding in an assignment-dependent function $f$ by anchoring the index $n$ to $f$’s outer argument $x$.

\[
\beta^n f := \lambda_x \lambda_g f \times g^{m-x}
\]

\[
\beta^n : (a \rightarrow g \rightarrow b) \rightarrow a \rightarrow g \rightarrow b
\]

An example of $\beta^n$ in action is in (64). Here, $\beta^n$ modifies the function created by scoping some expression (an indefinite, definite, or quantifier) out of the left-most $\star$ in (62). The resulting function covalues the seer and the cat-haver, binding the pronoun, and can be combined with a definite, quantifier, or $\star$-shifted indefinite. For example, combining (64) with $\star$-shifted a.\text{ling} yields $\lambda_g \{ \text{saw} (\text{cat} x) x \mid \text{ling} x \}$.

\[
\beta^n (\lambda_x \text{her}_n \star \lambda_y \eta (\text{saw} (\text{cat} y) x)) = \beta^n (\lambda_x \lambda_y \{ \text{saw} (\text{cat} g_n) x \})
\]

\[
type: e \rightarrow A t
\]

\[
= \lambda_x \lambda_y \{ \text{saw} (\text{cat} x) x \}
\]

Note that while binding transpires here, the assignment modification introduced by $\beta^n$ disappears in the final result, and is unavailable to anchor the evaluation of any subsequent pronouns. This is the sense in which $\alpha$’s treatment of anaphora is static. Pronouns inside $f$ can only be bound by an argument of $f$, which is just another way of saying that binding in $\alpha$ requires scope.

I briefly mention two significant results pertaining to the interaction of indefinite scope and binding. First, because we achieve exceptional scope by scoping the island, it may appear that exceptional scope of an indefinite over some operator $O$ is inconsistent with $O$ binding any pronouns on the indefinite’s island (since in that event, the entire island scopes over $O$). This would be contrary to fact: everyone’s would be appalled if [they$_n$ met a rich relative of mine] easily allows a specific-relative reading. Happily, such readings are derivable. Parallel to (51), we may generate a higher-order meaning for the island, with indefiniteness represented in the outer layer, and pronominality sequestered in the inner layer, as in (65). Parallel to (52), this allows the inner $\lambda t$ to semantically reconstruct into the scope of $O$’s $\beta^n$, as set out schematically in (66).

\[
\beta^n (\lambda_x \text{her}_n \star \lambda_y \eta (\text{met} x y)) = \lambda_x \lambda_h \{ \text{met} x h_n \} \mid \text{rel} x
\]

\[
type: A (\lambda t)
\]

\[
M \star \lambda_m O (\beta^n \ldots m \ldots)
\]

Second, though indefinites scope freely, they can’t scope over anything that binds into their restrictor: e.g., everyone’s submitted a paper they$_n$ wrote can’t be interpreted with the indefinite outside the universal (Geurts 2000, Schwarz 2001, Brasoveanu & Farkas 2011). This, again, is as predicted: scoping the $\lambda e$ indefinite over $\text{eo}$ (via $\star$) keeps the pronoun from being bound, as (67) demonstrates ($\beta^n$ thus ends up idle). Semantic reconstruction of the pronoun under $\text{eo}$, à la (65) and (66), is impossible here. Because the pronoun helps restrict the indefinite, there is no way to derive a higher-order meaning with indefiniteness in the outer layer and pronominality inside.

\[
(\lambda_g \{ x \mid \text{paper} x, \text{wrote} x g_n \}) \star \lambda_x \text{eo} (\beta^n (\lambda_y \eta (\text{submit} x y)))
\]

\[
type: A t
\]

\[
= \lambda_g \{ \forall y : \text{submit} x y \mid \text{paper} x, \text{wrote} x g_n \}
\]

This result is ultimately due to the fact that the theory treats indefinite scope using bona fide scope mechanisms. It distinguishes the present account of exceptional quantificational scope from virtually all extant approaches. See Section 7.1 for more on these points.
4.2 Dynamics (adding output)

A augments S's nondeterminism with a notion of input, allowing pronouns to be valued and in-scope binding to be modeled. The empirical considerations and dynamic technical machinery developed in Section 2.2 suggest that one more enrichment is called for: a notion of output.

We begin, analogously to S and A, by characterizing dynamicity as an enrichment, in (68).

\[
D \alpha \coloneqq g \rightarrow S (a \times g)
\]

As I noted in Section 2.4, D bears the two key hallmarks of dynamicity: nondeterminism and output. Indeed \(D \alpha\) is closely related to the dynamic propositional type \(T \coloneqq g \rightarrow S g\), differing only in that it represents an actual a value, along with context change.

To assemble a complete compositional interface for D, we only need (as with S/A) to find a \(\eta\) recipe for wrapping values in trivial dynamic structure, and a \(\star\) operation for characterizing how complex dynamic values are assembled. These are given in (69) and (70). D's \(\eta\) turns \(x\) into trivially dynamic value in the obvious way, by returning, relative to an assignment \(g\), a singleton set containing a pair of \(x\) and \(g\). The redefinition of \(\star\) differs minimally from the corresponding mapping in A. Here, however, we expect \(mg\) to be a set of pairs of a value \(x\) and an output \(h\), which are then fed in succession to \(f\). The difference from A's \(\star\) is simply that \(m\) can return an updated output assignment \(h\), which is passed to \(f\) in lieu of the input \(g\).

\[
\begin{align*}
\eta x & \coloneqq \lambda g \{(x, g)\} & \eta & \colon a \rightarrow D \alpha \\
\star m f & \coloneqq \lambda g \bigcup_{(x, h) \in mg} f x h & \star & \colon D \alpha \rightarrow (a \rightarrow D b) \rightarrow D b
\end{align*}
\]

Like S and A, D is monadic. Its \(\eta\) and \(\star\) functions have types with the right shape, and they obey Left and Right Identity, along with Associativity. These are straightforward to check; e.g., for Left Identity, \(\eta x \star f = \lambda x \bigcup_{(y, h) \in f} f y h = \lambda x f x g = f x\).

We conclude this subsection with some values for indefinites and pronouns, once again minimal upgrades of the corresponding A \(\alpha\) entries in (56) and (57). The only difference from those earlier values is that these values retain an assignment function (unmodified) as output.

\[
\begin{align*}
\text{a.ling} & \coloneqq \lambda x \{ (x, g) \mid \text{ling } x \} & \text{a.ling} & \colon D e \\
\text{she}_n & \coloneqq \lambda g \{ (gn, g) \} & \text{she}_n & \colon D e
\end{align*}
\]

Figure 5 summarizes the relationships between the various compositional regimes considered in this paper, and the various meanings they associate with indefinites and pronouns.

4.3 Composition and referent introduction

Let's run through a couple derivations to get a sense of how D works. In Figure 6, I give two derivations of a linguist walked in the park. On the left, we simply scope the indefinite via \(\star\),

\[\text{D} a \text{ is isomorphic to } S (g \times a \times g), \text{ and thus to } a \rightarrow S (g \times g) \text{ and to } a \rightarrow T. \text{ Thus, any } D a \text{ can be regarded as a dynamic property of } a \text{'s, analogously to how alternative semantics treats meanings as static properties of } a \text{'s, type } S a \text{ (or } A a)!\]
<table>
<thead>
<tr>
<th>$\eta x$</th>
<th>$m \star f$</th>
<th>$a.\text{ling}$</th>
<th>$\text{she}_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Sa$</td>
<td>${x}$</td>
<td>$\bigcup_{x \in m} fx$</td>
<td>${x \mid \text{ling} x}$</td>
</tr>
<tr>
<td>$g \rightarrow Sa$</td>
<td>$\lambda_g {x}$</td>
<td>$\lambda_g \bigcup_{x \in mg} fxg$</td>
<td>$\lambda_g {x \mid \text{ling} x}$</td>
</tr>
<tr>
<td>$g \rightarrow S(a \times g)$</td>
<td>$\lambda_g {(x, g)}$</td>
<td>$\lambda_g \bigcup_{(x, h) \in mg} f(x, h)$</td>
<td>$\lambda_g {(x, g) \mid \text{ling} x}$</td>
</tr>
</tbody>
</table>

**Figure 5:** Three monads for indefiniteness, progressively incorporating alternatives, alternatives alongside assignment-sensitivity, and alternatives alongside the ability to update an assignment.

$$= \lambda_g \{(\text{witp} x, g) \mid \text{ling} x\}$$

$$= \lambda_g \{(\text{witp} x, g^{m+7}) \mid \text{ling} x\}$$

**Figure 6:** Two derivations of *a linguist walked in the park*, with and without an application of $\beta^n$. In a dynamic setting, with composition oriented around $D$, the assignment modification induced by $\beta^n$ is stored for later use. This contrasts with static settings; with composition oriented around $A$, modified assignments are only passed down the tree, and only in-scope binding is possible.

Applying $\eta$ as usual to conjure up an initial dynamic meaning over which the indefinite can scope — exactly as in Figure 3. This derives an assignment-relative set of pairs, each containing a proposition and an unchanged assignment. Sentences with pronouns like *she* whistled can be derived similarly, scoping out the pronoun via $\star$, and applying $\eta$ in its wake. The result, relative to an assignment $g$, is a pair of the proposition whistle$_{g_n}$ and the unchanged assignment $g$:

$$(73) \quad \text{she}_n \star \lambda_y \eta (\text{whistle} y) = \lambda_g \{(\text{whistle} g_n, g)\}$$

Type: $Dt$

On the right of Figure 6, we carry out the exact same derivation as on the left, with one small but consequential shift: we apply $\beta^n$ to $a.\text{ling}$’s scope ($\beta^n$’s definition is unchanged, and is repeated below). In the static semantics based on $A$, this application of $\beta^n$ would be otiose: since there are no pronouns in $a.\text{ling}$’s scope, the modified assignment has no effect and flickers out of existence at the conclusion of the derivation. In a dynamic system, by contrast, the modified assignment is stored, paired with the sentence’s propositional content witp$x$ (with
x ranging over linguists). As in standard dynamic frameworks, the ability to store modified assignments is the key that unlocks binding without scope (Section 5).

\[(\beta^n f := \lambda x . \lambda y . f x g^{m+x}) \quad \beta^n : (a \rightarrow g \rightarrow b) \rightarrow a \rightarrow g \rightarrow b\]

We can carry out a derivation analogous to Figure 6, right, but with a definite \[\text{Polly} = p : e\] replacing the \(\star\)-shifted indefinite. The result, \(\hat{\beta}^n (\lambda x . \lambda y . \eta (\text{witp} x)) = \lambda x . \{ (\text{witp} p, g^{m+p}) \} : D t\), deterministically yields a proposition and modified assignment. These results are \(D t\) analogs of the pure context change versions associated with \(T\) depicted in Figure 1.

Dynamic referent introduction via \(\hat{\beta}^n\) can be effected highly locally in \(D\). An expression may take scope over itself, with \(\hat{\beta}^n\) and \(\eta\) sprinkled as in (74). The result is a dynamic individual with an anaphoric charge, with 'binding built in' (notice that without the invocation of \(\beta^n\) this would be an instance of Right Identity, and therefore equivalent to \(a.ling\)). Again, we may replace the \(\star\)-shifted indefinite with a scoped-out proper name, deriving \(\lambda g . \{ (p, g^{m+p}) \} : D e\).

\[a.ling \star \hat{\beta}^n (\lambda x . \eta x) = \lambda g . \{ (x, g^{m+x}) \mid \text{ling} x \} \quad \text{type: } D e\]

It is illuminating to consider what happens if we apply \(\star\) directly to these anaphorically charged dynamic individuals, as in (75) and (76). The terms that result have precisely the same form as the standard dynamic denotations for proper names and indefinites, defined back in (23) and (24). (The type of these terms, of course, is different: it is monadic and polymorphic, in that it expects to scope over and return a monadically dynamic something-or-other, type \(D b\).)

\[(\lambda g . \{ (p, g^{m+p}) \})^* = \lambda f . \lambda x . f p g^{m+p} \quad \text{type: } (e \rightarrow D b) \rightarrow D b\]
\[(\lambda g . \{ (x, g^{m+x}) \mid \text{ling} x \})^* = \lambda f . \lambda g \cup \text{ling} x f x g^{m+x} \quad \text{type: } (e \rightarrow D b) \rightarrow D b\]

It is reassuring to notice the close correspondences between the meanings derivable within our compositional theory and the meanings provided in standard dynamic systems.

5 Dynamic exceptional scope

5.1 Cross-sentential anaphora

Because \(D\) is a monadic extension of \(A\) (and, therefore, of \(S\)), we retain the beneficial empirical results associated with those earlier models of indefiniteness. Given the Associativity of \(D\)'s \(\star\), the left-hand side of Figure 4, with local scope of the indefinite, remains (mutatis mutandis) equivalent to its right-hand side, with non-local scope. Of course, we've not given an upgraded, \(D\)-style meaning for if just yet (see Section 5.2); but this does not affect the point: Associativity of \(\star\) alone is sufficient for exceptional quantificational scope, come what may (Section 3).\footnote{The two results on the interaction of binding and indefinite scope discussed at the end of Section 4.1 also hold in \(D\): an indefinite on an island with a pronoun can take exceptional scope over an operator that binds the pronoun; and an indefinite’s upward scope is delimited by any operators that bind into its restrictor. The explanation of these results in \(A\) relied, respectively, on the ability to build higher-order meanings for islands, and the fact that indefinites acquire scope via true scope mechanisms. These properties still hold in \(D\).}

\(D\), though, brings something new to the table: output. With output, the account of exceptional quantificational scope generalizes into a parallel account of exceptional binding scope.
Figure 7: Deriving a linguist walked in the park; she whistled. The conjuncts scope in their surface order at the conjunction’s edge. The assignment modification triggered by \( \beta^n \) is stored in the left conjunct and passed to the right by the highlighted instance of \( \star \). The resulting meaning identifies the walker and whistler (and retains modified assignments \( g^{7\rightarrow x} \) for later use).

I demonstrate this by using \( \Delta \) to derive cross-sentential anaphora, analyzing a linguist walked in the park; she whistled. We require just one additional piece, the common assumption that discourse sequencing or parataxis is interpreted conjunctively. Figure 7 provides the derivation, whose main constituents are already familiar. The left conjunct is derived exactly as in Figure 6, right, with an application of \( \beta^n \) generating a modified assignment. The right conjunct is derived exactly as in (73). Parallel to the derivation of exceptional scope in conditionals (cf. Figure 4), the individual conjuncts themselves take scope via \( \star \). They end up in their surface order, and \( \eta \) applies to the statically interpreted conjunction \( p \land q :: t \) left in their wake.\(^{14}\) As Figure 7 states, the resulting \( \Delta t \) covalues the walker and the whistler, as desired.

To see how binding is effected in the absence of scope, (77) unpacks the highlighted step in Figure 7, the result of applying \( \star \) to the meaning of the left conjunct. The effect, as the second line bears out, is to pass \( f \) a proposition \( \text{witp} \, x \), along with a modified assignment \( g^{7\rightarrow x} \), once

\[^{14}\text{This derivation does not require scope out of a coordinate structure (cf. Ross 1967; Fox 1995 on the Coordinate Structure Constraint). Similar to exceptional scope derivation in Figure 4, left, both clauses move to the coordinate structure’s edge, but neither moves out of the coordinate structure. In particular, this movement correctly does not allow anything external to the coordinate structure to scopally intervene between the two conjuncts (or any of their sub-parts).}\]
per linguist x (and then to union the resulting sets of pairs). This ultimately results in the modified assignments $g^{m+x}$ being sent 'down the tree' into $f$ and thereby to $\text{she}_n$. Binding succeeds, even as the pronoun's indefinite antecedent scopes within its minimal tensed clause.

\[(\lambda_g \{ (\text{witp} x, g^{m+x}) \mid \text{ling} x \}) \star f = \lambda_g \bigcup_{(l,h) \in \{(\text{witp} x, g^{m+x}) \mid \text{ling} x \}} f l h \]

A remarkable feature of the way in which exceptional binding scope is derived (Figure 7) is how similar it is to the derivation of exceptional quantificational scope (Figure 4). In both cases, the indefinite scopes at the edge of the island, which then itself takes scope. Exceptional quantificational scope results when $\star$ nondeterministically propagates values; exceptional binding scope results when $\star$ nondeterministically propagates modified assignments. These similarities are to be expected. Like exceptional quantificational scope in $S, A$, and $D$, exceptional binding scope in $D$ comes down to the Associativity of $D$'s $\star$, which guarantees that hierarchy is irrelevant for the propagation of dynamic effects when iteratively scoping via $\star$.$^{15}

### 5.2 On commutative conjunction

I should immediately forestall one possible objection to this account of exceptional binding scope. The treatment of conjunction in Figure 7 is static, and therefore commutative. Standard dynamic conjunction (defined in (68)), though associative, is non-commutative, or asymmetric: in general $(m; n) \alpha = m; (n \circ \alpha)$, but $m; n \neq n; m$. (Dynamic conjunction is relation composition, a generalization of the associative, non-commutative operation of function composition.)

Asymmetric conjunction is a fundamental part of standard dynamic treatments of anaphora. Indeed, such failures of commutativity are sometimes taken to be constitutive of dynamicity more generally (e.g., Dekker 1993, Rothchild & Yalcin 2017). Less abstractly, non-commutativity is argued to be empirically justified, since cross-sentential anaphora is difficult-to-impossible if the indefinite occurs in the right conjunct, and the pronoun in the left:

\[(\text{She}_n \text{ whistled. A linguist}^n \text{ walked in the park.}) \]

The commutativity of our conjunction may appear to be problematically in tension with (78). Nothing prevents us from giving a derivation like Figure 7, but in which (78)'s right conjunct scopes over its left, in which case binding is derived.

I do not, however, think that this is is a real cause for concern (and I believe that an examination of this point is revealing about the empirical upshots of standard dynamic conjunction as well). For one, we are free to adopt the non-commutative characterization of conjunction in (79). As with T's conjunction, this definition passes the assignments output by the left conjunct to the right conjunct, and not vice versa.

\[(\lambda_r \star \lambda_p \star \lambda_q \eta (p \land q) = \lambda_g \{(p \land q, i) \mid (p, h) \in l g, (q, i) \in r h\}]^{15}

Note that applying Associativity directly to Figure 7 is not so illuminating due to the way we've operationalized binding, i.e., via $\beta^\circ$. One may, however, replace a ling with (79), the version of the indefinite with 'binding built in', in which case Associativity guarantees that Figure 7 is equivalent to a logical form in which (79) directly scopes over the pronoun. More directly applicable to Figure 7 as given is a law we might dub $\beta$-Associativity: $(m \star \beta^\circ (\lambda_x f s)) \star e = m \star (\beta^\circ (\lambda_x f x \star e))$. This law does not hold in $\lambda$; it is a characteristically dynamic fact about $\lambda$.}
Lexically asymmetric entries such as this are, of course, not incompatible with the present account (in fact, we will adopt asymmetric lexical entries for if and every parallel to the entries we’ve given for T, S, and A in Section 5.3). Yet positing (79) seems less than desirable, since it stipulates a fairly complex lexical meaning which is independently derivable given \( \eta, \star \), and scope — indeed, the structure of (79) precisely mirrors that of the derivation in Figure 7!

What’s more, the asymmetry of (79) does not actually rule out right-to-left binding relationships like (78) (though it forces us to get creative)! Even though \( l \) out-scopes \( r \) in (79), we can once again scope \( r \) up and out over \( l \) (via \( \star \)), in which case binding is possible after all. Nor are standard dynamic accounts safe from these sorts of scope-based machinations: scope can, in principle, carry an indefinite in a right conjunct up and over the left conjunct, to a scope position from which it can bind a pronoun. Given that indefinites take virtually unbounded quantificational scope, and given that quantificational scope feeds binding (Barker 2012), this is a real concern for standard dynamic treatments, one which asymmetric conjunction alone does not solve. (See Section 7.2 for a more detailed version of this argument, once we have an idea of how exceptional quantificational scope can be added to standard dynamic systems.)

What this highlights is that, independently of asymmetric conjunction, scope-taking must be restricted in order to prevent arguably impossible right-to-left binding relationships like (79). One obvious analogy to draw is to crossover. Though the object quantifiers in (80) can scope over the pronouns in principle, all four potential binding relationships are ungrammatical. The putative right-to-left binding derivations in \( D \) are closely analogous to so-called secondary weak crossover, the instance of (80) with a possessive subject and possessive object.

\[
(80) \quad \begin{cases}
  \text{"He}_n \\
  \text{His}_n \text{ mom}
\end{cases}
\text{ licked } \begin{cases}
  \text{every tomcat}
  \\
  \text{every tomcat}'s paw
\end{cases}
\]

Recent work on crossover and related phenomena argues that binders must linearly precede the expressions they bind (very roughly; see, e.g., Shan & Barker 2006, Barker & Shan 2008, Bruening 2014). The existence of other binding phenomena mediated by linearity is strong evidence that dynamic conjunction need not shoulder the explanatory burden of linear asymmetries in binding. Developing a general account of these asymmetries is beyond the scope of the present work (especially given that standard dynamic accounts lack such a treatment themselves). However, see Appendix A for a sketch of how to build a general left-to-right asymmetry into \( D \)-style composition, which may represent a promising basis for future work.\(^6\)

5.3 Closure and feeding binding

In this subsection, I flesh out the \( D \)-style dynamic system with closure operators. This proceeds exactly as in S and A (and analogously to T): we’ll define a notion of negation appropriate to \( D \) t meanings, and then use it to automatically upgrade our definitions of if and every. I’ll briefly review how these entries, in concert with our treatment of indefiniteness and binding, predict donkey anaphora (in conditionals). I’ll then consider an important prediction of the resulting system: that exceptional quantificational scope feeds exceptional binding scope.

\(^6\) There is one puzzle here: weak crossover is known to be especially weak when the binder is a ‘specific’ indefinite (e.g., Wasow 1972, Heim 1982). Why this should be unable to rescue (78) is mysterious to me.
$$D t$$ is somewhat richer than $$T$$. Because $$D t$$ outputs pairs of $$t$$’s and assignments, it potentially represents ‘false’ outputs in addition to ‘true’ ones, whereas $$T$$ only yields ‘true’ outputs (i.e., $$T$$ identifies truth with $$having$$ outputs).\(^7\) It’s useful here (and cuts down on notational overhead) to define an auxiliary mapping that forgets this extra structure:

\[(81) \quad g \langle m \rangle h \coloneqq (T, h) \in mg\]

Using this shorthand, we can define $$D$$-negation as in (82). As with our prior alternative-aware definitions of negation, this entry requires that its prejacent is wholly false, i.e., that there is no way of making it true. Like $$T$$’s negation in (25), the definition in (82) is externally static: the output assignment is the unchanged input (any modified assignments output by $$m$$ are ignored).

\[(82) \quad \text{not } m \coloneqq \lambda_g \{ (-\exists h : g \langle m \rangle h, g) \} \quad \text{not }: D t \to D t\]

\[(83) \quad \text{if } l r \coloneqq \text{not } (l \star \lambda_p (\text{not } r) \star \lambda_q (p \land q)) \quad \text{if }: D t \to D t \to D t\]

\[= \lambda_g \{ (\forall h : g (l) h \Rightarrow \exists i : h (r) i, g) \}\]

The automatically upgraded definition of $$\text{if}$$ is given in (83). It requires that every way of making the antecedent $$l$$ true is also a way of making the consequent $$r$$ true, and is externally static due to the outermost occurrence of $$\text{not}$$. Definitions for indefinite and universal determiners are provided in (84) and (85). (As in Sections 3.3 and 4.1, ‘genuinely’ quantificational expressions such as universals are predicted not to be exceptional scope-takers. Once they take scope, they yield a singleton output, and so further scope-taking via $$\star$$ will be semantically idle.)

\[(84) \quad a c \coloneqq \lambda_g \{ (x, h) \mid g (c x) h \} \quad a : (e \to D t) \to D e\]

\[(85) \quad \text{every } k \coloneqq \text{not } (a c \star \lambda_x, \text{not } (k x)) \quad \text{every } : (e \to D t) \to (e \to D t) \to D t\]

\[= \lambda_g \{ (\forall x, h : g (c x) h \Rightarrow \exists i : h (k x) i, g) \}\]

As previewed in Section 5.2, our entries for $$\text{if}$$ and $$\text{every}$$ are asymmetric. Ultimately, it would be preferable to not need to hard-wire this behavior (see Chatain 2018 for discussion of how to leverage local contexts to the same end; see Champollion, Bumford & Henderson 2019 for a modular treatment of dynamic quantification that leaves the lexical semantics of quantifiers uncomplicated). Flagging this as a potentially fruitful area for further inquiry, I set it aside.

Given the characterization of $$\text{if}$$ in (83), donkey anaphora is immediate. The conditional universally quantifies over the alternative assignments output by its antecedent, passing them to the consequent. This derives the so-called ‘strong’ donkey reading of If a linguist\(^n\) walked in the park, she\(_n\) whistled, which requires every linguist who walked in the park to have whistled:

\[(86) \quad \text{if } (a.\text{ling} \star \beta^n (\lambda_x, \eta (\text{witp } x))) (\text{she}_n \star \lambda_x, \eta (\text{whistle } y)) \quad \text{type: } D t\]

\[= \text{if } (\lambda_g \{(\text{witp } x, g^{\text{witp}_x}) \mid \text{ling } x\}) (\lambda_g \{(\text{whistle}_g, g)\})\]

\[= \lambda_g \{(\forall x \in \text{ling} : \text{witp } x \Rightarrow \text{whistle } x, g)\}\]

Generating donkey readings is the least a dynamic semantics can do. More noteworthy is that $$D$$’s $$\eta$$ and $$\star$$ predict an attested interaction between exceptional quantificational and

\(^7\) However, see footnote 4 for a perspective on which $$D t$$ meanings are not meaningfully richer than $$T$$ meanings. Some further, preliminary considerations of the utility of the extra structure in $$D t$$ are outlined in Section 8.
Figure 8: Exceptional quantificational scope feeds exceptional binding scope: \( \text{if } [\text{a rich relative of mine}^{\text{n}} \text{ dies}], \text{I'll inherit a house} \). The assignment modification generated by \( \beta^{\text{n}} \) evades capture by \( \text{if} \).

binding scope in the presence of closure operators. As noted in Section 2.2, if \( [\text{a rich relative of mine}^{\text{n}} \text{ dies}], \text{I'll inherit a house}; \text{she is an oil baron} \) forces the exceptional wide-scope reading of the indefinite. That the narrow-scope reading should be inconsistent with binding is as predicted, given that the conditional externally static. That exceptional quantificational scope of the indefinite should allow binding is in a sense unsurprising; yet it is by no means trivial to predict within standard dynamic theories, as we will see in Section 7.2.

Monadic dynamic semantics predicts that exceptional scope feeds binding. See Figure 8. This derivation is virtually identical to the exceptional quantificational scope derivation in Figure 4, left, but is carried out in \( \mathcal{D} \), and with the addition of \( \beta^{\text{n}} \). The indefinite takes scope at the edge of the island, and the island in turn takes scope, up and over the externally static conditional. This takes the assignment modification induced by \( \beta^{\text{n}} \) out from under \( \text{if} \), ultimately allowing referent introduction to evade closure. The resulting \( \mathcal{D} t \) meaning gives the indefinite exceptional quantificational scope, and preserves the modified assignments for future pronouns, as desired. It can be slotted directly into a cross-sentential anaphora derivation such as Figure 7.

6 Connecting static and dynamic scope

Our theory yokes exceptional quantificational and binding scope together tightly: it predicts that an expression can take exceptional quantificational scope if it can take exceptional binding scope. This section explicates and supports this prediction, using data involving disjunction,
bare and non-bare plurals, and dynamic generalized quantifiers. I end by considering anaphora
to definites. Though quantificationally scopeless, I argue that they take exceptional binding
scope, a prediction made by the present account.

Given these points, the theory here can be argued to conceptually improve on static theories
of indefinite’s exceptional quantificational scope, and dynamic theories of their exceptional
binding scope. Both treat indefinites differently from ‘genuine’ quantifiers: static theories
treat indefinites as choice-functional or as generators of alternatives (Section 2.1), and dynamic
theories treat indefinites as externally dynamic (Section 2.2). But within these more narrowly
construed empirical domains, there is little reason to assign indefinites these exceptional
semantic features, except in order to model the exceptionality in question. The present account
does not eliminate this explanatory issue, but it does give greater purchase on it, by linking
exceptional quantificational and binding scope (see Reinhart 1997: 385ff for related observations).

6.1 Bootstrapping disjunction

The same multifaceted exceptionality seen with indefinites is observed with disjunction. Dis-
junction freely takes exceptional scope out of islands: (87) (modeled after examples given in
Schlenker 2006) allows a reading, forced by the parenthetical sluice, with disjunction taking
scope out of the relative clause island, paraphrasable as every student who studied Greek failed
the exam, or every student who studied Latin did (see also Rooth & Partee 1982, Brasoveanu &
Farkas 2011, Charlow 2014, Dawson 2019). Disjunctions also bind pronouns in cross-sentential
and donkey anaphora configurations, as seen in (88) and (89) (Rooth & Partee 1982, Groenendijk
& Stokhof 1991a, Stone 1992, Simons 1996, Schlenker 2011, Charlow 2014; the examples here are
modeled after ones given by Simons and Stone, respectively).

(87) Every student [who studied Greek or Latin] (I can’t remember which) failed the exam.
(88) George sang [an aria or a ballad]”. Itn had German lyrics.
(89) If I see [Al or Bill]”, I’ll wave to himn.

Cross-sentential anaphora to disjunction generally seems less acceptable than donkey anaphora
to disjunction. I have no account of this fact, and I will abstract away from it in what follows.

Like indefinites, disjunction takes exceptional quantificational and binding scope. This two-
sided exceptionality is captured in one fell swoop, by treating disjunction as a (polymorphic)
generator of (dynamic) alternatives, as in (90). Given an input assignment, m, n :: Da return two
sets of pairs. Disjunction unions these sets, yielding a meaning that is indeterminately m or n.18

(90) m ∘ n := λg mg ∪ ng

Like the alternative-semantic disjunction of, e.g., Alonso-Ovalle (2006: 11) or the dynamic
’Program disjunction’ of Groenendijk & Stokhof (1991a: 88), this definition treats disjunction as

18 Two points on this definition. First, it can be assigned a much more general type than the type I have given in (90). The
form of this definition requires only that m, n :: g → 5a. Indeed, given that g → 5a is isomorphic to 5 (g × a) (footnote 12),
disjunction can be viewed simply as set union (cf. Partee & Rooth 1983). Second, the definition does not allow anaphora
from one disjunct into the other, i.e., it is internally static. The data motivating internally dynamic disjunction is
difficult to model in the simple sort of dynamic system we’re using. See Krahmer & Muskens 1995 for discussion.
set union. But while those treatments can offer piecemeal accounts of disjunction's exceptional quantificational or binding scope, our definition underwrites a unified account of both, treating them as different aspects of the same underlying phenomenon.

For example, disjoining two indefinites, as in (91), yields a new indefinite meaning that inherits the nondeterminism of the individual disjuncts. Two definite expressions (e.g., proper names) can be disjoined as well, by first lifting them into $\mathcal{D}$ via $\eta$, and then applying disjunction; as shown in (92), this results in an indefinite meaning intermediate between $x$ and $y$.

\begin{align*}
(91) \quad a.\text{ling} \circ a.\text{phil} &= \lambda_x \{ (x, g) \mid x \in \text{ling} \cup \text{phil} \} \quad \text{type: } \mathcal{D} e \\
(92) \quad \eta (x :: e) \circ \eta (y :: e) &= \lambda_z \{ (z, g) \mid z \in \{ x, y \} \} \quad \text{type: } \mathcal{D} e
\end{align*}

These values have the same type and semantic form as garden-variety indefinite meanings, and they can therefore be slotted directly into the previous derivations of exceptional quantificational and binding scope.

It is interesting to consider in more detail what happens when disjoined indefinites are slotted into an exceptional quantificational scope context, as in if [a rich aunt of mine or a rich uncle of mine dies], I'll inherit a house. Directly disjoining the indefinites as in (91) and carrying out a derivation along the lines of Figures 4 or 8 generates a reading paraphrasable as: a rich aunt of mine or a rich uncle of mine is such that if they die, I'll inherit a house, i.e., with disjunction and indefiniteness both interpreted outside the scope of if. This reading is certainly available, but so is a reading in which the disjunction takes exceptional wide scope, with the indefinite disjuncts taking narrow scope: it's either the case that if any rich aunt of mine dies, I'll inherit a house, or... This meaning can be generated by applying $\eta$ to the individual disjuncts before disjoining them, to yield a higher-order meaning (cf. the account of selectivity in Section 3.3):

\begin{align*}
(93) \quad \eta \ a.\text{ling} \circ \eta \ a.\text{phil} &= \lambda_x \{ (m, g) \mid m \in \{ a.\text{ling}, a.\text{phil} \} \} \quad \text{type: } \mathcal{D} (\mathcal{D} e)
\end{align*}

This distinguishes the nondeterminism of disjunction from indefiniteness, allowing disjunction to scope separately. This is a general technique for dissociating disjunction scope from the scope of the disjuncts (Rooth & Partee 1982, Charlow 2014, Dawson 2019). For example, it allows disjunction to scope over operators that bind into its disjuncts (cf. Section 4.1).

### 6.2 (Bare) plurals

Plural indefinites headed by some, or by cardinal determiners like two, take exceptional quantificational scope, just as singular indefinites do. The existential force of bare plurals, by contrast, is not subject to scopal displacement of any sort (exceptional or otherwise).

\begin{align*}
(94) \quad \text{If } \left\{ \begin{array}{c} \text{two} \\ \text{some} \end{array} \right\} \text{ rich relatives of mine die, I'll inherit a house.}
\end{align*}

As discussed by Ruys (1992) and Reinhart (1997), the existential force of non-bare plural indefinites is unbounded, but they are associated with distributive force that is tethered to scope islands (see also Szabolcsi 2010: Ch. 7 on existential and distributive scope). For example, the
exceptional scope reading of (94) with two relatives of mine entails the existence of a plurality of two relatives such that, if they each die, I’ll inherit a house. The distributive force cannot scope over the conditional, in which case either of two relatives’ deaths would be sufficient. The bare plural’s existential force scopes only in situ; the determiner-less version of (/nine.onum/four.onum) is unambiguous and means that any deaths would be sufficient.

A parallel between exceptional quantificational and binding scope is again observed, as set out in (95). Exceptional anaphora to two and some friends of mine (the exceptional quantificational scopers) is grammatical. Exceptional anaphora to the bare plural, though acceptable, is obligatorily associated with a maximal interpretation, in that it requires all the apples Mary ate to have been delicious. Though two and some are typically associated with maximality, this inference is defeasible: unlike the bare plural, the two and some examples can be continued with she also ate some apples that weren’t without contradiction (see Sells 1985, Kadmon 1990, Cohen & Erteschik-Shir 2002). This suggests that anaphora to the bare plural is effected via a distinct, maximality-driven mechanism, similarly to cases of ‘genuine’ quantification (see Section 6.3).

(95) Mary ate \{\begin{align*}
\text{two} & \quad \text{some} \\
\emptyset &
\end{align*}\} apples”. They were delicious.

Treating exceptional quantificational and binding scope of plural and cardinal indefinites is as simple as adding basic tools for plurality. We make the standard assumption that type e includes plural as well as atomic entities (e.g., Link 1983, Schwarzschild 1996). Then two.rels is assigned the meaning in (96), where X ranges over pluralities of two relatives of mine. To model distributive predication and distributive scope-taking more generally, we may assume silent distributivity operators that universally quantify over the atomic parts of a plurality, as in (97).

(96) \text{two.rels} := \lambda_x \{ (X, g) \mid \text{two.rels}X \}

(97) \delta f := \lambda x \lambda g \{ (\forall X \leq_{at} X : \exists h : g(f x) h, g) \}

As with disjunctions, plural indefinites like two.rels can be slotted into our prior derivations of exceptional quantificational and binding scope, inserting \delta’s as needed. For example, the exceptional-existential-scope/narrow-distributive-scope reading of (94) is derived as follows (inserting \delta” above \delta will result in output assignments that associate n with X):

(98) (\text{two.rels} \ast \delta (\lambda x \eta (\text{dies x}))) \ast \text{if} (\eta p) (\eta \text{house})

As this demonstrates, because \delta is ‘genuinely’ quantificational (analogous to every), it is correctly predicted that the distributive scope of plural expressions is confined to scope islands.

Brasoveanu (2007, 2008) agrees with Kadmon (1990) and Cohen & Erteschik-Shir (2002) that anaphora to the bare plural is maximal, but also judges that cross-sentential (and donkey) anaphora to plural some is maximal. There is something to this intuition; the maximal interpretation of plural some strikes me as especially salient. Nevertheless, I agree with Kadmon and Cohen & Erteschik-Shir that exceptional anaphora to plural some is not obligatorily maximal. I do not treat bare plurals here, but the neo-Carlsonian (1977) treatment of Chierchia (1998) can be naturally adapted to the present framework. Given the considerations in this subsection, such an adaptation should treat the existential quantifier associated with Derived Kind Predication as an instance of ‘genuine’ quantification, and therefore model its (maximal) anaphoric properties along the lines sketched in Section 6.3. See Dayal 2013 for arguments, consonant with the approach here, that bare plurals do not have existential force at all, but are more akin to scopeless indefinites.
6.3 Dynamic generalized quantifiers

Non-indefinite quantifiers display an exceptional scope duality that's by now familiar. On the one hand, they don't take exceptional quantificational scope. The examples in (99) only allow readings with local scope of the quantifier (e.g., the exactly two example can't be understood as claiming that there's exactly two rich relatives of mine each such that, if they die, I get a house).

(99) If \{exactly two few at least six\} rich relatives of mine die, I'll inherit a house.

On the other hand, though quantifiers participate in exceptional binding processes, anaphora in such cases is invariably to a maximal entity (e.g., Kamp & Reyle, Kanazawa, Chierchia, van den Berg, Krifka, Nouwen), in contradistinction to the non-maximal character of exceptional anaphora to indefinites (Section 6.2). The pronoun in (100) refers to the refset — roughly, the intersection of the quantifier's restriction and scope, here the maximal plurality of linguists who walked in the park. Anaphora to other maximal entities is possible (e.g., 'maxset' anaphora to the restriction), but we'll focus on refset anaphora here.

(100) \{Exactly two At least six Few\} linguists\" walked in the park. They\" whistled.

We've so far only considered externally static quantification. Data like (100) shows (as is well known) that a more nuanced approach is called for. We'll first see how this standardly goes in T, adapt the solution to D, and then explore and defend the associated predictions. A schematic lexical semantics for T-style dynamic generalized quantification is given in (101) (following Kanazawa, Chierchia). \$Q\$ is the relation on sets associated with the quantificational determiner \$Q\$ (Barwise & Cooper), and \$\oplus X\$ is the mereological fusion of \$X\$, the set of (atomic) linguists who \$f\$, i.e., the refset. The meaning is deterministic: it outputs at most a single modified assignment pointing \$n\$ to \$\oplus X\$, conditional on \$Q\$-many linguists being linguists who \$f\$.

(101) \[$Q$ linguists$\]\(\) := \lambda g \{g^{\oplus X} \mid X = \{x \in \text{ling} \mid \exists h \in f x g\}, Q (\text{ling}, X)\}

\text{type: } (e \to T) \to T

The insights of dynamic generalized quantification translate straightforwardly to a system built on D, as shown in (102). As in (101), this meaning schema is fully deterministic. The single pair that results has two parts, a truth condition that \$Q\$-many linguists are linguists who \$f\$, and a modified assignment storing in \$n\$ the mereological fusion of the linguists who \$f\$.

(102) \(Q \text{.lings}\) := \lambda g \{\{g (\text{ling}, X), g^{\oplus X}\} \mid X = \{x \in \text{ling} \mid \exists h : g \langle f x h \rangle\}\}

\text{type: } (e \to D) \to D

Notice that \$f x\$ is interpreted relative to the unmodified input assignment \$g\$. If in-scope binding is desired, we'll need to apply a \$\beta\$-operator to the nuclear scope \$f\$. This entails, correctly, that pronouns bound in-scope by dynamic generalized quantifiers range over atomic entities, and
are interpreted non-maximally. A subset of these atoms eventually constitutes the maximal discourse referent $\oplus X$, available to out-of-scope pronouns (see, e.g., Kamp & Reyle 1993).

The fact that quantifiers are externally dynamic crucially does not entail that they take exceptional scope — at least not in the usual sense of that claim. Like all potential scope-takers, including indefinites and disjunction, dynamic quantifiers can scope as far as the nearest delimiting scope island. At this point, the quantifier is evaluated: its scope $f$ is determined once and for all, generating a pair of a truth condition and a modified assignment hosting the newly constructed maximal refset. Though the island may continue taking scope via $\star$, this has no effect on the size or shape of the dynamic quantifier’s scope argument $f$.

While dynamic quantifiers are thus correctly predicted not to take exceptional quantificational scope, a kind of exceptional binding scope is predicted: scope of a quantifier-hosting island out from under a closure operator such as negation may make the quantifier’s refset anaphorically accessible. Provisionally, this prediction looks correct. Consider the interpretation of (103) with negation scoping over more than ten. Perhaps surprisingly (given that the meaning in question has the quantifier in the scope of externally static negation), it’s natural to anaphorically refer back to the quantifier’s refset (cf. van den Berg 1996, Nouwen 2003). This is as predicted, as (104) demonstrates: the dynamic quantifier takes scope over witp, yielding a prejacent $D \eta$ meaning which takes scope via $\star$ over not. Though the truth conditions are as if the prejacent had remained in situ, scoping the island spares the refset from externally static not.21

(103) More than ten linguists’ didn’t walk in the park. They numbered nine, tops!

\[
(104) \quad \text{(more than ten lings’)} (\lambda x, \eta (\text{witp } x)) \star \lambda g \text{ not } (\eta g) = \lambda g \{ (\neg \text{more than ten (ling, witp)}, g^{\langle x \in \text{ling} | \text{witp } x \rangle}) \}
\]

My aim in this subsection has been modest. I only wished to demonstrate that the observed behavior of dynamic quantifiers is consistent with the link between exceptional quantificational and binding scope predicted in D.22 I certainly don’t mean to suggest that (102) constitutes a complete treatment of dynamic quantification. Important phenomena like quantificational subordination (every linguist drank a martini; many linguists drank it quickly) remain unaccounted for, and will likely require us to eventually adopt a dynamic plural logic that allows us to dynamically construct and refer to dependency (e.g., van den Berg 1996, Krifka 1996, Nouwen 2003, Brasoveanu 2007, 2008, Henderson 2014, Kuhn 2017, Law 2018). I leave elaborations such as these for future work (see Section 8 for a bit more on these points).

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21 It’s less than straightforward to test this prediction with examples like (99). When a dynamic quantifier occurs in an intensional context (e.g., the antecedent of a conditional), its refset ends up modally subordinated (just as, e.g., wolves are subordinated to modal possibilities in a wolf might come in (e.g., Roberts 1989, Brasoveanu 2010). Subordination affects referent accessibility in a way that is well modeled by plural dynamic logics, and less so within the simple ‘first-generation’ approach to dynamic interpretation used here.

22 Hungarian quantification may offer fairly dramatic confirmation of this link. Szabolcsi (1997: 13ff) notes that Hungarian quantificational DPs in a position she terms ‘HDistP’ are associated with non-maximal discourse referents, and can be interpreted as independent of quantifiers in superordinate clauses. By contrast, DPs in the ‘PredOP’ position are associated with obligatorily maximal discourse referents, and cannot be construed as independent of superordinate quantifiers. Again, we find exceptional quantificational and (non-maximal) binding scope yoked together. The twist is that in Hungarian, the same DPs can occur in both HDistP and PredOP, with their interpretations modulated accordingly. I leave a more detailed consideration of these matters for future work.
6.4 Exceptional scope of definites

A striking piece of support for the approach here comes from anaphora to definite expressions. For concreteness and simplicity, we'll focus on proper names (though the data and arguments in this section apply to definite descriptions and pronouns). Like indefinites, proper names can antecede pronouns from deeply embedded positions. Unlike indefinites, proper names are not associated with quantificational force — that is, they are scopeless.

Our theory predicts that proper names, like indefinites (and refsets), should take a kind of exceptional scope. Because proper names are quantificationally scopeless, this exceptional scope behavior cannot be detected by consulting intuitions about the proper name's 'quantificational force' (since it has none). However, it should be detectable anaphorically: exceptional scope of proper names, out from under externally static operators, should allow anaphoric relationships which would be precluded if the proper name's anaphoric charge remained within the operator's scope. This is analogous to the exceptional binding scope of the refset, explicated in (103) and (104): exceptional scope of proper names is not visible in judgments about relative scope, but may be detected anaphorically.

This prediction is confirmed, but showing this requires some setup. After all, it isn't the case that a proper name that intuitively antecedes a pronoun necessarily binds it; the two may merely, 'accidentally' corefer. Evidence for binding can, however, be adduced by considering sloppy readings in ellipsis. There are many treatments of sloppy readings, but virtually all of them agree that sloppy pronouns must be bound by an expression in the elliptical clause (see Rooth, Fox, Takahashi & Fox, and references therein). We can therefore use sloppy pronouns whose antecedents would otherwise be anaphorically inaccessible — but for exceptional binding scope — to argue that the empirical situation is as predicted.

Consider first example (105), where greyed-out text represents elided material, and caps represents focus. It allows the indicated, sloppy reading, in which the interpretation of the elided pronoun differs from its correlate in the antecedent VP. Following Rooth (1992), Fox (1999) (and simplifying a great deal), ellipsis is licensed only if the antecedent and elliptical clauses express the same proposition modulo FOCUSED material — if there's a way to replace focused material in the elliptical clause that generates something with the same propositional content as the antecedent clause. In (105) this condition is met if BILL binds him, BILL can be replaced with John, and DIDN'T with nothing. If BILL binds the pronoun, replacing BILL with John means that the pronoun will evaluate to John. If, on the other hand, BILL and the pronoun are merely accidentally coreferential, replacing BILL with John has no effect on the pronoun's meaning, in which case ellipsis can't be licensed (since Bill # John).

\[(105) \quad \text{John's mom likes him,} \]
\[\text{But BILL's mom DOESN'T like him,!} \]

Achieving binding is straightforward in (105): though the proper name doesn’t c-command the pronoun, it can easily scope over it (notice, e.g., that the pronoun in exactly one boy's mom hugged him; is interpreted non-maximally). Achieving binding is a bit less straightforward in (106) (Wescoat 1989), but ultimately unproblematic: though the proper name can't scope over the pronoun (e.g., if BILL is replaced with exactly one man, the pronoun is interpreted
maximally), this is just a case of donkey anaphora out of DP, and is thus generable given an internally dynamic semantics for the (cf. the internally dynamic semantics for every in (8); see Gardent 1991, Tomioka 1999, Hardt 1999).

(106)  The cop [who arrested John'] insulted him.,  
But the cop [who arrested BILL'] DIDN'T insult him.!

Here’s the punchline. Example (107) is grammatical on the indicated sloppy reading (it’s most natural to also elide be promoted as indicated, but this is orthogonal to the point). This shows that the sloppy elided pronoun him, is bound by the focused antecedent BILL'. However, the antecedent is inside a scope island, its minimal tensed clause; outside the scope island is the externally static nobody. In sum, the sloppy reading requires the pronoun to be bound, and binding the pronoun requires its antecedent to take exceptional scope, out of its scope island and above nobody. The facts do not appear to change with additional layers of embedding: the sloppy reading remains available if we replace John/BILL with anybody [who likes John/BILL].

(107)  If nobody thinks [John should be promoted], I’ll feel bad for him.  
But if nobody thinks [BILL should be promoted] I WON’T feel bad for him.!

So definite expressions, though quantificationally scopeless, may nevertheless take exceptional binding scope. This is as predicted: the proper name and associated $\beta^\eta$ may scope within their minimal tensed clause, which takes scope up and over the externally static nobody:

(108)  \[ \text{if } (\beta^\eta (\lambda x \ldots) b) \star \lambda_y \text{nobody} (\lambda_y \ldots) (\text{him}_{\eta} \star \lambda_z \ldots) = \text{type: D t} \]

Note that this scope position, above nobody but under if, allows binding of the pronoun given the internally dynamic, externally static, definition for if. Further exceptional scope-taking over if allows anaphoric reference to BILL beyond the scope of the conditional (as in Figure 8).

### 6.5 Taking stock

Our dynamic compositional interface, based on $\mathcal{D}$ and its associated $\eta$ and $\star$, gives a unified account of indefinites’ exceptional quantificational and binding scope. This unification entails that exceptional quantificational and binding scope will be yoked together in general. This section substantiated this prediction with evidence from disjunction, indefinite and bare plurals, and dynamic quantification: exceptional quantificational scope was argued to be available if non-maximal exceptional binding scope was available, as well. Definite expressions were likewise shown to take exceptional scope, though (as expected given their quantificational scopeless-ness) this is detectable only via anaphoric processes.

It is worth dwelling for a moment on the picture of dynamic binding that has emerged. The theory here straightforwardly predicts that indefinite and definite expressions alike should

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23 I have purposely chosen the negative quantifier nobody in lieu of, e.g., everybody. It is quite plausible that quantifiers of the latter sort are externally dynamic, as insightfully modeled in plural dynamic systems (e.g., van den Berg 1996, Krifka 1996, Nouwen 2003, Brasoveanu 2007, 2008). Standard plural-dynamic definitions for nobody, while formulated analogously to (e.g.) everybody, have truth-conditional content which renders them externally static in practice.
be able to bind pronouns, no matter how deeply embedded. With indefinites, the upward scope-taking that feeds exceptional binding generates wide quantificational scope for the indefinite (Figure 8). With definites, this exceptional upward mobility can be appreciated only anaphorically. Ultimately, on our theory binding is far less constrained by the syntax than is usually assumed, even within (standard) dynamic theories: surface-structural relations between pronouns and (in)definites have little to do with whether the pronoun can come to referentially depend on the (in)definite, given enough exceptional scope-taking.24 (Dependency relations between dynamic quantifiers and pronouns, in comparison, are more sensitive to structural relations. Quantifiers can scope to the nearest scope island edge, at which point they generate a maximal refset, on which out-of-scope pronouns may come to depend.)

7 Comparing theories

This section compares the present account with prior static treatments of exceptional quantificational scope, and with prior dynamic treatments of exceptional binding scope. In Section 7.1, I argue that the monadic theory improves on static treatments, even if exceptional binding scope is set aside. In Section 7.2, I consider how the present account relates to standard dynamic theories. I argue that, while standard dynamic theories can be combined with choice-functional treatments of indefinite scope, this extended theory does not offer a natural account of how exceptional scope feeds binding, and must supplemented with exogenous, empirically fraught mechanisms of descriptive enrichment (and inherits the problematic features of choice-functional accounts of indefinite scope).

Finally, in Section 7.3, I consider how the monadic grammar based on A relates to the monadic grammar based on D. I’ll show how to decompose A and D in a way that highlights their common alternative-semantic core, S. This decomposition reveals that, while A is a monadic operationalization of standard frameworks for alternatives and static binding (Kratzer & Shimoyama 2002), D combines alternatives with an approach to binding that is essentially novel in the formal semantics literature, but which bears a close correspondence to the way in which state is modeled in functional programming languages like Haskell (e.g., Wadler 1992).

7.1 Static

Static theories based on choice functions and (standard) alternative semantics, overviewed briefly in Section 2.1, offer insight into indefinites’ exceptional quantificational scope, but do not explain why indefinites and disjunction are systematically associated with exceptional binding scope (let alone explain the varieties of exceptional scope reviewed in Section 6). Setting aside issues of exceptional binding scope (which are taken up in Section 7.2), this subsection argues that the present account improves on static approaches to exceptional quantificational scope, even with respect to this more narrowly construed empirical domain.

24 This view is certainly not unprecedented. First, as is well known, the standard DRS construction algorithm assigns proper names widest scope (e.g., Kamp & Reyle 1993). Second, Safir (2004) argues that anaphoric dependency relations are basically unconstrained by syntax (modulo Condition C effects). The considerations here can be seen as offering independent, semantic vindication of these views of grammar, coupled with a formally explicit proposal that makes a range of detailed predictions about scope-taking and anaphoric relations.
We have already seen one way in which this is so: our account predicts that it is impossible for an operator $O$ to bind into an indefinite that scopes over $O$ (this was originally demonstrated for $A$ in Section 4.1, and holds in $D$ as well). Accounts based on choice functions, by contrast, leave indefinites in situ. The ‘scope’ of the indefinite, such as it is, is ultimately determined by the existential closure operator, which may be inserted above $O$, even as $O$ binds into the indefinite. As pointed out by Schwarz (2001) (see also Winter 1997, Geurts 2000) this yields unattested predictions. For example, no candidate° submitted a paper she, had written may be assigned the meaning in (109). This says, roughly, that there’s a way of picking papers, such that no candidate submitted that paper of hers — in other words, no candidate submitted every paper she had written — not a possible reading.

(109) $\exists f \in CF : \text{no.candidate} (\lambda x \text{submit} (\{ y \in \text{paper} \mid \text{wrote}yx\}) x)$

This issue afflicts in situ theories, which divorce indefinites’ existential scope from the scope of their descriptive content, quite generally. For example, a version of this problem recurs in accounts of exceptional quantificational scope based on singleton domains (von Fintel 1999, Schwarzschild 2002). While the problem can be stipulated away, at least in certain frameworks (Brasoveanu & Farkas 2011; cf. Reinhart 1997: 393f), it’d be better not to have to do so.

I’ll linger a bit more on choice functions, since they’re the tool we use to add an account of exceptional quantificational scope to dynamic semantics (Section 7.2). First, as is well known, choice functions have an empty-set problem; the definition of $\text{CF}$ in (4) places no constraints on what a choice function returns when its argument is empty (though intuitively, when the indefinite in such cases occurs in an upward-entailing context, the sentence should simply be false). As with the binding issue just discussed, this problem isn’t insoluble, but dealing with it introduces complications that lessen the appeal of the resulting theory (Reinhart 1997, Winter 1997, Geurts 2000). Second, and less familiarly, choice functions have a novelty problem. A linguist saw a linguist doesn’t have any readings that entail that a linguist saw themself. But such a reading can be derived, if the two indefinite determiners are coindexed (and thereby interpreted via the same choice function). Because two indefinites can be arbitrarily structurally distant from each other, it seems that this cannot be ruled out with anything less than a global constraint that forces each occurrence of an indefinite to be assigned a novel index.²⁵

I turn to theories of exceptional quantificational scope based on alternatives. Again, we’ve already seen one respect in which the present theory improves on these accounts: it predicts full selectivity when multiple indefinites occur on an island, because it can generate higher-order meanings that can be used to distinguish multiple sources of indefiniteness in such cases (this was first demonstrated for $S$ in Section 3.3, and holds in $A$ and $D$ too). By contrast, the alternative-semantic grammar defined in (8) inflexibly associates a sentence with a type-$S t$ meaning, and therefore has difficulty distinguishing multiple indefinites in such cases.

In addition, like choice functions, standard alternative semantics has a binding problem. Suppose we wished to treat object-language abstraction, i.e., give an alternative-semantic characterization of the meaning of $\lambda^a \alpha$. In analogy with $\llbracket \cdot \rrbracket^e$, which associates this expression with a meaning of type $e \rightarrow a$ (where $a$ is whatever type $a$ gets), $\llbracket \cdot \rrbracket^e$ should associate this

²⁵ It’s sometimes thought dynamic theories must stipulate novelty for indefinites anyway, but this isn’t so (Charlow 2009c).
expression with a meaning of type $S (e \to a)$. An initial, failed, attempt is given in (110). This misses the mark in two (related) ways: the result is a singleton set (undesirably entailing that abstraction halts the propagation of $a$’s alternatives!), and it has the wrong type: $S (e \to S a)$.

\[(110) \quad \{\{\lambda^n a\}\}^f := \{\lambda x \{\{a\}\}^{x \to e} \mid f \in CF\}\]

Two sorts of fixes for this problem have been proposed in the literature. The first effectively uses choice functions to flatten the inner layer of alternatives, yielding \(\{\lambda x f \{\{a\}\}^{x \to e} \mid f \in CF\}\) (cf. Hagstrom 1998, Kratzer & Shimoyama 2002). The resulting account naturally inherits the problematic features of choice functions \textit{vis à vis} binding (in fact, this understates the problem; see Shan 2004, Charlow 2009b for discussion). The second fix replaces assignment-dependent alternative sets with sets of assignment-dependent meanings (Poesio 1996, Romero & Novel 2013). Though this solves the technical issue, it is a poor fit for indefinites whose restrictors host pronouns: the number and nature of the alternatives in such cases should depend on the pronoun meaning, and thus on the assignment (e.g., \textit{a paper she} wrote should denote different sets of papers, possibly with different cardinals, depending on who the assignment associates with $n$; again, the reader is referred to Shan 2004, Charlow 2009b for in-depth discussion).

Our compositional theory is standard, based on functional application and some mechanism for scope (while $\eta$ and $\ast$ grease the skids, composition is still ultimately via functional application). Since we don’t rely on $\{\{\}\}^e$ to propagate nondeterminism, we don’t need to define new versions of binding operations (whether $\lambda^n$ or $\beta^n$) that work in $\{\{\}\}^f$. The standard, unproblematic definitions will suffice (whether in the static $A$ or the dynamic $D$).

### 7.2 Dynamic

As for dynamic accounts of exceptional binding scope, we have already observed (at the end of Section 2.2) that they do not predict exceptional quantificational scope. But dynamic accounts of binding may, of course, be combined with theories of exceptional quantificational scope. Here we’ll consider one particularly straightforward such combination: adding choice-functional quantification to the dynamic system based on $T$ (cf. related proposals in von Heusinger 2000, 2002, Chierchia 2005, Brennan 2011). Clearly, such a system will inherit the issues associated with choice functions discussed in Section 7.1. I will argue here that it also fails to explain why exceptional scope feeds anaphora, and must be supplemented with exogenous, empirically problematic, mechanisms for descriptive enrichment.

Consider (111), where we’ve supplemented the $T$-style approach to referent introduction with dynamic quantification over choice functions (passing over the compositional niceties). The result here differs from prior meanings like (15) principally in that the output assignments store a way of choosing linguists who walked, along with the linguists thereby chosen.

\[(111) \quad [\exists^n [a_n \text{ linguist}]^m \text{ walked}] = \lambda^g \{g^{n \to f, m \to f} \mid f \in CF, \text{ walked} (f \ling)\} \quad \text{type: T}\]

\[26\] There are other possibilities for simultaneously treating exceptional quantificational and binding scope. One line essentially combines Heim 1982-style existential closure over type-e variables with sortal restrictions on these variables (see, e.g., Abusch 1994, Szabolcsi 2003, Jäger 2005, 2007, Onea 2013, 2015). Simplifying greatly, on such approaches, indefinite scope is analogized to presupposition projection (see Reinhart 1997: 360f, fn. 19 for critical remarks on treating indefinites presuppositionally). It isn’t immediately obvious how such accounts might be elaborated to explain the varieties of exceptional scope treated in Section 5, but I consider this a promising avenue for future research.
Though choice-functional existential closure is treated dynamically here, this doesn’t change much about how dynamic binding is effected. The simplest way for a subsequent pronoun to refer back to the indefinite is (still) for it to bear the index $m$. After all, a choice function isn’t an individual, but a way of choosing individuals from a set. Accordingly, the choice function stored in $n$ can’t fix the value of a pronoun unless it’s supplemented with a set to choose from.

Consider now how this sort of theory extends to a case of exceptional quantificational scope, as in (112). Like (111), the existential closure operator and the indefinite both induce assignment modifications. Unlike (111), however, the indefinite is trapped inside the externally static conditional (see the T-style meaning for if in (26)), and so only the choice-functional referent $f$ is available for the interpretation of subsequent pronouns. For these pronouns to refer to the same relative selected by $f$, they’ll need to be descriptively supplemented with $rel$, the same set passed to the choice function inside the conditional antecedent.

$\exists^n \text{ if } ([a_n \text{ relative}]^m \text{ dies} | I \text{ get a house}) = \lambda g \{ g^n \mapsto \text{h} | f \in \text{CF}, \text{dies} (f \text{ rel}) \Rightarrow \text{house} \}$

This doesn’t predict that exceptional quantificational scope feeds binding. Inserting an existential closure operator above an externally static operator like if or not makes a choice function anaphorically accessible, but that’s only half of what’s required for binding; the descriptive content anchoring choice-functional referent selection must still be somehow supplied.

One possibility for making sense of this is that the pronouns in question just are covert descriptions (Evans 1977, Cooper 1983, Heim 1990, Elbourne 2009). This line has been explored by Chierchia (2005) and Brennan (2011). It has two empirical issues. First, descriptively enriching exceptionally bound pronouns is difficult to square with the fact that such pronouns can be interpreted sloppily in elliptical constructions, as in (e.g.) our example (107) (this point was first articulated by Tomiooka (1999: 234f)). That is, the descriptive content required to interpret such pronouns creates an inherent mismatch between the sloppy pronoun and its correlate in the antecedent VP. This should prevent sloppy ellipsis, analogously to how accidental coreference in lieu of binding short-circuited licensing of sloppy readings; descriptive supplementation isn’t a binding mechanism, but binding is what’s required to license sloppy readings.

Second, if an independent mechanism of descriptive enrichment supplements dynamic choice-functional binding, we erroneously predict that referents will be accessible in discourse which have never been explicitly introduced! Consider (113), with two indefinites interpreted via two choice functions. Nothing said so far prevents a subsequent pronoun from being interpreted as $they_n \text{ noteworthy philosopher}$ — that is, with the first indefinite’s choice function selecting a referent on the basis of the second indefinite’s descriptive content! But then the conditional might be true in virtue of $x$ and $y$, while $they_n \text{ noteworthy philosopher}$ might refer to some random, heretofore unmentioned, philosopher $z$?

$$\exists^n \exists^n [\text{if } [a_n \text{ famous linguist sees } a_n \text{ noteworthy philosopher}] \ldots ]$$

---

27 A theory combining dynamic views of anaphora with choice-functional accounts of indefinites was first proposed by Peregrin & von Heusinger (1993) (see also Egli & von Heusinger 1995, von Heusinger 2004). Their approach, however, is ill-suited to exceptional quantificational scope (and was not intended to extend to such cases): it holds the indefinite responsible for quantifying over choice functions, rather than some arbitrarily distant existential closure operator.
Needless to say, this is a highly counterintuitive result. While there may be ways to avoid it, for example by ensuring that descriptive enrichment of exceptionally bound pronouns does not happen independently of the pronoun’s choice-functional index, such mechanisms would seem to greatly complicate the resulting theory. This compares unfavorably with the approach in this paper, which predicts that exceptional quantificational scope feeds binding (Section 5.3).

Finally, I wish to return to an argument sketched previously in relation to our commutative treatment of conjunction (Section 5.2). Though dynamic accounts treat conjunction as non-commutative, I suggested that right-to-left binding would be possible when dynamic treatments were enriched with apparatus for exceptional quantificational scope. This is shown in (14), in which we apply top-level existential closure over choice functions, and interpret the pronoun using the descriptive content of a subsequent indefinite.

(14) \(\exists^n [\text{she}_n \text{ linguist whistled, and a}_n \text{ linguist walked in the park.}].\)

My point isn’t that dynamic semantics with choice functions must over-generate right-to-left binding for such cases — it’s coherent to suppose that existential closure and/or descriptive supplementation are restricted in ways going beyond what I’ve set out here — but that whatever mechanisms rule out LFs like (14) are independent of the non-commutativity of conjunction.

7.3 Alternative and dynamic semantics

The last comparison I’d like to undertake is between the two systems for indefiniteness considered in this paper: \(A\) and \(D\). Let me begin by introducing you to two monads you haven’t seen in this paper, but which will probably look a bit familiar anyhow. On the left, we have \(I\), a monad for input; on the right is \(IO\), a monad for input, together with output.

\[
\begin{align*}
(15) & \quad I a := g \rightarrow a & 10 a := g \rightarrow (a \times g) \\
& \quad \eta x := \lambda g \cdot x & \quad \eta x := \lambda g \cdot (x, g) \\
& \quad m \star f := \lambda g \cdot f (m g) g & \quad m \star f := \lambda g \cdot (\lambda (x, h) f x h) (m g)
\end{align*}
\]

\(I\) and \(10\) are deterministic variants of \(A\) and \(D\) — they are monads for static and dynamic variable binding, without \(S\)’s nondeterminism. One way to make this precise is by considering the alternative definitions of \(A\) and \(D\) in (16) below. These definitions are equivalent to the definitions given earlier, but they characterize \(A\) and \(D\) explicitly in relation to an underlying \(S\) monad for nondeterminism. If \(\eta_S\) and \(\star_S\) are replaced with operations from the trivial ‘Identity’ monad, such that \(\eta_{1d} x := x\) and \(m \star_{1d} f := f x\), the definitions in (16) reduce to those in (15).

\[
(16) \quad \eta_A x := \lambda g \cdot \eta_S x & \quad \eta_D x := \lambda g \cdot (x, g) \\
& \quad m \star_A f := \lambda g \cdot mg \star_S \lambda x \cdot f x g & \quad m \star_D f := \lambda g \cdot mg \star_S (\lambda (x, h) f x h)
\]

In fact, we can swap out \(\eta_S\) and \(\star_S\) in (16) for any monadic \(\eta\) and \(\star\). The result is guaranteed to be a monad: (16) implies general recipes for transforming ‘base’ monads with added functionality for input, or input plus output (Liang, Hudak & Jones 1995, Shan 2001b, Charlow 2014).

\[\text{28 Not to be confused with Haskell’s built-in IO monad, which is nevertheless closely related to our definition of 10 (e.g., Launchbury & Peyton Jones 1999). The monads in (15) are known to functional programmers as Reader and State.}\]
As this highlights, the static compositional system for indefiniteness oriented around A, and the dynamic variant oriented around D, can both be viewed as proper enrichments of S. While D offers a more general and more powerful approach to binding than A, both monads possess a common core, the S monad. On this perspective, dynamic semantics just is a variant of alternative semantics, one with resources for treating exceptional binding scope alongside exceptional quantificational scope.

The I monad is in close correspondence with standard, static approaches to binding (Heim & Kratzer 1998, Charlow 2019a). The IO monad, though common in functional programming as a denotational reification of state (i.e., the accessing and manipulating of values in memory; see, e.g., Wadler 1992, 1994 on the use of IO to model state), is comparatively under-explored in linguistics (but see Giorgolo & Unger 2009, Unger 2012). There seems to be no deep reason for this, and so future work should map out the consequences of IO, independent of indefiniteness. Appendix B shows how IO can be decomposed into separate I and O actions, and demonstrates that this decomposition in fact determines the IO transformer in (16), with the aim of facilitating future research into the foundations of dynamicity in natural language.

8 Conclusion

This paper has argued that static and dynamic exceptional scope are intrinsically connected, and defined a compositional, monadic interface for indefiniteness that connects them. I showed how the requisite formal tools emerged naturally from the literature on type-shifting and compositional theories of alternatives, and predicted a range of attested exceptional scope behavior for a range of expressions, both indefinite and not. The resulting theory can be seen as a kind of alternative semantics, suggesting a fundamental connection between alternatives-based and dynamic approaches to indefiniteness and exceptional scope.

Aside from the treatment of dynamic generalized quantification, I’ve mostly stayed within the confines of ‘first-generation’ dynamic systems. I am, however, optimistic that the theory here can be fruitfully combined with the insights of modern, plural dynamic systems. For one, because our theory is so close in form and character to standard compositional dynamic systems, the usual techniques developed for extending them into plural dynamic systems can be straightforwardly applied (e.g., van den Berg 1996, Krißka 1996, Nouwen 2003, Brasoveanu 2007, 2008, Henderson 2014, Kuhn 2017, Law 2018). Of particular interest is that plural dynamic systems offer a range of options for modeling indefiniteness and referent introduction (see, e.g., the discussion of PCDRT referent introduction in Brasoveanu 2008: 141). The arguments developed here may offer a basis for deciding between these options, and the ‘functional’, dependency-oriented characterizations of exceptional binding scope developed within these theories may in turn provide a basis for investigating functional manifestations of exceptional quantificational scope (see, e.g., Schlenker 2006, Solomon 2011, and especially Bumford 2015).

Finally and speculatively, as noted back in Section 5.3, monadic dynamic propositions (type 0=t) are somewhat richer than standard dynamic propositions (type T): whereas T’s only output ‘verifying’ assignments, 0=t’s report their verifiers as well as their falsifiers. The dissociation of truth from context change, inherent in the monadic perspective, opens the door to alternative definitions of negation — including, potentially, versions that are externally dynamic. The
possibility for alternative characterizations of negation consistent with its truth-conditional effects (and the possibility to achieve this without appealing to partiality à la van den Berg 1996) suggests an area of dynamic logical space that is ripe for future exploration.

A Composition without scope

This appendix shows an alternative way to define monads, by distributing the action of a monadic $\star$ across two functions, $\otimes :: T (a \to b) \to T a \to T b$ and $\mu :: T (T a) \to T a$. This approach is weakly equivalent to the definition in terms of $\star$ used in the main text, in the sense that the two operationalizations of monads are inter-derivable. However, $\otimes$ and $\mu$ allow for a more in situ (and perhaps more conservative) approach to composition. Taking $\otimes$ as a primitive in turn allows us to hard-wire linear asymmetries into dynamic composition.

We begin by defining $\otimes$ and $\mu$ for $D$. The definition of $\otimes$ evaluates the ‘argument’ $n$ in the anaphoric context established by the ‘function’ $m$; $\mu$ forgets higher-order structure.

\begin{align}
(117) \quad m \otimes n & := \lambda g \{ (f, x, i) \mid (f, h) \in m g, (x, i) \in n h \} \\
(118) \quad \mu M & := \lambda g \{ m h \mid (m, h) \in M g \}
\end{align}

Again, the dynamic features of this semantics can be appreciated by considering it in relation to a variant which handles assignment functions in a static manner (without storing outputs).

\begin{align}
(119) \quad m \otimes n & := \lambda g \{ f x \mid f \in mg, x \in n g \} \\
(120) \quad \mu M & := \lambda g \{ m g \mid m \in M g \}
\end{align}

If $T$ is monadic, its $\star$ can be derived from its $\otimes$ and $\mu$, and vice versa. Figure 9 shows the first half of this equivalence (in $D$), using $\otimes$ and $\mu$ to compose a $De$ with an $e \to Dt$ to yield a $Dt$ (just as $\star$ does). Considered in isolation, a monadic $T$’s $\eta$ and $\otimes$ (i.e., with $\mu$ excluded) are known as an applicative functor (e.g., McBride & Paterson 2008, Kiselyov 2015, Charlow 2019a).
Despite this inter-derivability, monadic composition in the applicative style can be done in a more in situ way than monadic composition using $\star$. For example, a linguist walked in the park can be derived without any scope, as in (121). In contrast, using $\star$ for this case requires us to create an $e \to D \tau$ input for $\star$, which requires $a_{\text{ling}}$ to take scope (with $\eta$ applied in its wake).

\begin{equation}
\eta\text{witp} \circ a_{\text{ling}} = \lambda g \{ (\text{witp} x, g) \mid \text{ling } x \} \quad \text{type: } D \tau
\end{equation}

Monadic derivations in applicative style are thus somewhat more 'conservative' (or at least, less scope-reliant) than monadic derivations using $\star$. Indeed, the definitions of $\circ$ for $D$ and $A$ are highly reminiscent of alternative-semantic composition, (8). Freeing monadic composition from scope in turn allows us to treat scope using standard syntactic mechanisms. Previously, this was somewhat problematic: syntactic approaches to scope assume that scope-taking leaves behind a pronominal trace, which (assuming $\star$) would itself need to take scope, leaving behind a pronominal trace, ad infinitum. Using $\circ$, pronominal elements (including traces of covert and overt movement) can be compositionally integrated without further scope-taking.

With the in situ, applicative approach, monadic composition can be given a general linear bias. The definition of $\circ$ in (177) evaluates the $D (a \to b)$ 'function' $m$ before the $D a$ 'argument' $n$. We can use directed types (e.g., within categorial or type-logical grammar) to ensure that $m$ always occurs to the left of $n$, and thus that things on the left are in general evaluated before things on the right (Shan & Barker 2006). This is summed up in (122) and (123). As (122) indicates, the undirected functional type $a \to b$ can be refined into a directed type that expects its $a$ to the left or to the right; e.g., $\cdot a \cdot b \cdot b'$ means that $a$ followed by $a \cdot b$ has type $b$. Then $\circ$ can be given the type/category in (123), which encodes that it expects $m$ on the left and $n$ on the right.

\begin{align}
(122) & \quad a \cdot a \cdot b \cdot b \quad b / a \cdot a \cdot b \\
(123) & \quad \circ :: (D (b / a)) \setminus (D b / D a) \\
(124) & \quad \text{LIFT} :: (b / (a \cdot b)) / a
\end{align}

In cases where a dynamic argument, type $D a$, ostensibly occurs on the left, with a dynamic function, type $D (a \cdot b)$, on the right, we can apply $\text{LIFT}$ to the argument (under the $D$), as in (124) (recall that $\text{LIFT} x := \lambda f \{ f x \}$). This turns the $D a$ into $D (b / (a \cdot b))$, and allows composition to proceed, delivering a dynamic output, type $D b$, and retaining the left-to-right bias in interpretation.

While this does not absolutely preclude right-to-left binding (which can still transpire, e.g., via scope), it does make left-to-right processing the grammatical default. This represents a promising avenue for making sense of linear asymmetries in dynamic binding (Section 5.2), fuller exploration of which I leave to future work.

\section*{B Input and Output}

The input-output monad $10$ (Section 7.3) can be decomposed into separate $I$ and $0$ actions (cf. Shan 2001a). See (125), which characterizes $1$ and $0$ as functors, where a functor $F$ is associated with a mapping $\circ :: (a \to b) \to Fa \to Fb$ satisfying certain conditions (e.g., Mac Lane 1971).

\begin{align}
(125) & \quad I a :: g \to a \quad 0 a :: a \times g \\
& \quad f \circ m := \lambda g f (mg) \quad f \circ (x, g) := (f x, g)
\end{align}
Every monad is a functor, with \( f \circ m = m \star \lambda_x \eta(f(x)) \), but the reverse does not generally hold: for example, \( I \) is monadic (Section 7.3), but \( 0 \) isn’t.\(^{29}\)

\( I \) and \( 0 \) are adjoint functors. A functor \( L \) is a left adjoint to a functor \( R \), \( L \dashv R \), iff \( La \to b \) is isomorphic to \( a \to Rh \). In the present case, \( 0 \) happens to be left adjoint to \( I \) in virtue of the curry/uncurry isomorphisms; that is, \( 0 + I \) since \( (a \times g) \to b \equiv a \to g \to b \).

Whenever \( L \dashv R \), the composite functor \( RL \), where \( RL a := R(La) \), is guaranteed to be a monad (e.g., Mac Lane 1971): \( L \dashv R \) implies the existence of \( \eta : a \to RL a \) and \( \varepsilon : LR a \to a \), which can be used to define \( RL \) as a monad, i.e., with associated \( \eta \) and \( \varepsilon \):

\[
\begin{align*}
(126) & \quad \eta x := \lambda_y (x, y) \\
(127) & \quad \varepsilon(f, g) := fg
\end{align*}
\]

\( I0 \)'s \( \eta \) is given by (126), and \( \varepsilon \) can be used to characterize \( I0 \)'s \( \star \). This is sketched in (128) below: we apply \( f : a \to 10b \) under \( m \)'s \( I0 \) (using \( I \) and \( 0 \)'s \( \circ \) operations to do so; blue highlights the portion of the type targeted by the operation above the arrow). This yields an intermediate, higher-order meaning, with an \( \Omega \) sequence that we may eliminate using \( \varepsilon \) (by applying \( \varepsilon \) under \( I \)). This delivers a \( \Omega b \), exactly as we would expect \( I0 \)'s \( \star \) to do. The reader can check that this derived \( \star \) is equivalent to \( I0 \)'s \( \star \), as given in definition (119).

\[
\begin{align*}
(128) & \quad I0a \xrightarrow{10f} I0(10b) \xrightarrow{1r} 10b
\end{align*}
\]

\( I \) and \( 0 \)'s adjoint-ness makes \( I0 \) monadic. It also means that we can transform any ‘base’ monad into an enriched monad with \( I0 \) functionality (cf. Liang, Hudak & Jones 1995), as shown for \( S \) in (129) and (130). Note that \( ISOa := I(S(0a)) \); I omit parentheses for readability. As (129) shows, \( ISO \)'s \( \eta \) is derived by applying \( I0 \)'s \( \eta \), and then applying \( S \)'s \( \eta \) under \( I \). As (130) shows, \( ISO \)'s \( \star \) can be derived analogously to \( I0 \)'s \( \star \). First, as in (128), we apply \( f : a \to ISOb \) under \( ISO \) to yield a higher-order meaning, and eliminate the resulting \( \Omega \) sequence by applying \( \varepsilon \) under \( I \). This finally yields an \( SS \) sequence, which is collapsed using \( S \)'s \( \mu \) (Appendix A).

\[
\begin{align*}
(129) & \quad a \xrightarrow{\gamma} I0a \xrightarrow{1\gamma} ISOa \\
(130) & \quad ISOa \xrightarrow{ISO\cdot ISOb} ISOb \xrightarrow{IP} ISOb
\end{align*}
\]

The reader can check that these derived mappings are equivalent to \( D \)'s \( \eta \) and \( \star \). The ability to decompose \( D \) into adjoint input and output functors, along with a monad for nondeterminism, suggests a perspective on which \( \eta \) and \( \star \) are not the compositional primitives I have taken them to be throughout this paper. This in turn suggests that separate study of input, output, and nondeterminism, along with their lexical manifestations, may yield insight into the foundations of dynamicity in natural language (this is essentially the approach adopted in Shan 2001a).

\(^{29}\) More generally, \( F a := a \times b \) is monadic only when \( b \) is a monoid. A type is a monoid if it has associative operation with an identity element (e.g., sets are monoidal given \( \cup \) with \( 0 \) as the identity element). Assignments aren’t monoidal: though they can be unioned, the result of doing so is not necessarily another assignment function. See Giorgolo & Asudeh 2012 for use of monadic \( a \times b \) to model supplemental content and projection.
References


Chatain, Keny. 2008. Local contexts for anaphora. Talk presented at Institut Jean Nicod, ENS.


