Connectedness as a constraint on exhaustification*

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Abstract

'Scalar implicatures' is a phrase used to refer to some inferences arising from the competition between alternatives: typically, 'Mary read some of the books' ends up conveying that Mary did not read *all* books, because one could have said 'Mary read all books'. The so-called grammatical theory argues that these inferences obtain from the application of a covert operator *exh*, which not only has the capability to negate alternative sentences, but also the capability to be embedded *within* sentences under other linguistic operators, i.e. *exh* has the potential to add to the meaning of expressions (not necessarily full sentences), the negation of their alternatives. This view typically seeks support from the existence of readings that could not be explained without the extra-capability of *exh* to occur in embedded positions. However, if some embedded positions seem to be accessible to *exh*, not all conceivable positions that *exh* could occupy yield sensible results. In short: the *exh* approach is powerful, maybe too powerful.

Various approaches based on logical strength and monotonicity have been proposed to justify on principled grounds the limited distribution of *exh*; these approaches are mostly based on a comparison between possible parses, and considerations of monotonicity (e.g., the Strongest Meaning Hypothesis). We propose a new constraint based instead on "connectedness", ruling out parses because of inherent problems their outcome may raise. Connectedness is a sister notion of monotonicity, which has been recruited to explain certain lexical restrictions on nouns, adjectives and more recently quantifiers; we propose here that connectedness could play a similar role at the level of propositional meanings.

1 The distribution of exhaustification

1.1 The *exh* operator

In many occasions, (1) is understood to imply that John didn't do all of his homework. This inference is not assumed to follow from the literal meaning of (1), which basically only says positive things about John and his homework. Instead, this inference is traditionally viewed as an enrichment, which can be explained if one assumes that (1) has (2) as a natural alternative, and that, in appropriate circumstances, alternatives are inferred to be false. Concretely then, the result is an *enriched* or *exhaustified* meaning for (1), which not only implies that the literal meaning is true, but also that its alternative is false. The inference that (2) is false from hearing (1) may be called a *scalar implicature*.

(1) John did some of his homework.

(2) John did all of his homework.

To formalize this, one may define an exhaustification function, *exh*, that takes as input a sentence ϕ and its set of alternatives A (indicated as a subscript), and outputs an enriched meaning which adds to the literal meaning the relevant negations of alternatives: The meaning of

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$exh_A[\phi]$ then is the meaning of ϕ conjoined with the negations of elements in A.[1] In the so-called Gricean view, scalar implicatures arise from pragmatic reasoning over speakers' intentions, and this function *exh* may thus be seen as a way to formally package the result of such a reasoning (cf. for instance van Rooij and Schulz 2004 or Schulz and van Rooij 2006). In this view, this operation most naturally applies to whole sentences, and (1) gets its enriched meaning as the result of the following application of *exh*:

(3) $exh_{\{\text{John did all of his homework.}\}}$ [John did some of his homework.]

In contrast, Chierchia, Fox, and Spector (2011) develop a view of implicatures as derived from the grammar, in which such a function *exh* would be a grammatical operator. As such, the *exh* operator would be capable of taking a more diverse types of inputs. It would still be able to take whole sentences as inputs, and to explain the above facts *ceteris paribus*, but additionally it may occur in embedded positions, within full sentences. One of the initial motivations for this grammatical view was indeed the attempt to account for cases in which meaning is modified in place, that is *some* seems to acquire a *some but not all* meaning from within the scope of linguistic operators. Chierchia, Fox, and Spector (2011) summarize a wealth of evidence for such embedded occurrences of *exh*. They argue for instance that (4a) has a reading where an embedded item like "most" is understood as "most but not all". This can be explained if *exh* can appear above "most", as in the parse given in (4b). Without embedded exhaustification, (4a) would be incoherent: professors who fail all of their students are said to both be fired and receive a new assignment.

(4) a. Every professor who fails most of the students will be assigned a new class, and every professor who fails all of the students will be fired.
(adapted from Chierchia, Fox, and Spector 2011)

b. Alleged parse: Every professor who fails *exh* [most of the students] will be assigned a new class, and every professor who fails all of the students will be fired.

Other cases seem to call for more flexibility than just global applications of *exh*, typical examples include cases in which a scalar item like 'some' appears in the scope of quantifiers. This is illustrated in (5) and (6) below. There it seems that the output predicted by a global application of *exh* is accessible, but the output predicted by the local application of *exh* within the scope of the quantifier also seems to be accessible (see Chemla and Spector 2011 and Potts et al. 2016 for discussions about the exact relevance of these examples, as well as quantitative data about the availability of the alleged readings):

(5) Every employee saw some of the criminals.

a. Possible parse: Every employee saw *exh* [some of the criminals.]
Resulting meaning: Every employee saw some of the criminals, but none saw all the criminals.

b. Possible parse: *exh* [Every employee saw some of the criminals.]
Resulting meaning: Every employee saw some of the criminals, but not all of them saw all the criminals.

(6) Exactly 3 employees saw some of the criminals.

a. Possible parse: Exactly 3 employees saw *exh* [some of the criminals.]
Resulting meaning: Exactly 3 employees saw some but not all of the criminals.

[1] For simplicity, we will consider that *exh* negates all alternatives a such that $\phi \wedge \neg a$ is not a contradiction; this is essentially the lexical entry for "only" in Krifka (1993). See Fox (2007) and Spector (2016) among others for necessary refinements.

b. Possible parse: *exh* [Exactly 3 employees saw some of the criminals.]
Resulting meaning: Exactly 3 employees saw some of the criminals, less than that saw all the criminals.

Another class of examples, also described by Chierchia, Fox, and Spector (2011), is that of so-called Hurford disjunctions. Here the argument is of a different nature: if we do not postulate the occurrence of an embedded *exh*, then the sentence may violate independent linguistic principles. For instance, in (7), without *exh*, the second disjunct is redundant: "all" is strictly stronger than "some," and if A^+ is stronger than A, then A or A^+ is equivalent to A. With *exh* however, the first disjunct of the form *exh*[some] has a meaning akin to 'some but not all', which makes the two disjuncts logically independent. If A^+ and A' are logically independent, then $A' or A^+$ isn't equivalent to either A^+ or A', and there is no redundancy. Thus, postulating the possibility of embedding *exh* in (7) can help explain why it does not violate a general ban against redundancy.[2]

(7) a. John did some or all of the homework.
b. Possible parse: John did *exh* [some] or all of the homework.

1.2 Empirical restriction on the distribution of exh

At this point, the grammatical approach is formally more expressive than the neo-Gricean view, in that it predicts strictly more possible meanings. We have also shown cases in which its extra expressive power was welcome (though see Chemla 2009 for a case where it may miss the mark). We will now review cases in which this additional expressivity seems to be *too* powerful, in that it may allow for parses that produce unattested meanings.

Most famously, there is no reason at this point in the grammatical view why *exh* could not appear under negation. But it is well-known that the readings this would lead to are at best highly dispreferred (see (15) however for a qualification). For instance, if sentence (8a) could have a parse as in (8b), then it should have the meaning paraphrased in (8c), corresponding to 'some' being interpreted as 'some but not all'. Under negation, we obtain a meaning akin to *all or none*, which does not seem to be readily available.

(8) a. It's not true that John did some of his homework.
b. Potential parse: *It's not true that John did *exh* [some of his homework].
c. Unattested meaning: John did all or none of his homework.

Another classic example is that of three-way disjunctions, as in (9a). Assume that the sentence has the structure A or (B or C), and that "or" has the alternative "and". Then we should be able to obtain a meaning for it as in (9c), if we allow a parse as in (9b). This reading, as before, is unattested, suggesting that the corresponding types of embedded occurrences of *exh* are banned.

(9) a. John will meet Ann, Bill or Carol.
b. Potential parse: $*exh(A \,\text{or}\, exh(B \,\text{or}\, C))$
c. Unattested meaning: John will meet exactly one or exactly three of those people.

We may also consider (10), a variation of (9). If parsed with an embedded *exh* as in (10b), it ought to mean that John will either visit his mother, or do some but not all of his homework. Such a reading would make (10a) false when John doesn't visit his mother, and does all of

[2]The analysis sketched here is not without challenges: see Katzir and Singh (2013b) for a more thorough discussion of Hurford disjunctions, and Mayr and Romoli (2016) for redundancy constraints in general.

his homework, but true if he does all of his homework and visits his mother. This reading, paraphrased in (10c), doesn't seem available.

(10) a. This afternoon, John will visit his mother or do some of his homework.
b. Potential parse: *This afternoon, John will visit his mother or do *exh* [some of his homework.]
c. Unattested meaning: If John doesn't visit his mother, he won't do all of his homework.

Embedding of the *exh* operator under existential quantifiers also seems to be difficult. In (11a), having an embedded *exh*, as in (11b) again essentially turns "some" into "some but not all", resulting in the meaning paraphrased in (11c). Such an interpretation paraphrased in (11c) is true in a scenario where some employees saw just some criminals, and some other employees saw all of them, but it is false if some employees saw all criminals and the rest saw none. Such a reading seems quite marked, if it exists at all.

(11) a. There are employees who saw some of the criminals.
b. Potential parse: *There are employees who saw *exh* [some of the criminals.]
c. Unattested meaning: There are employees who saw some but not all of the criminals.

1.3 Possible constraints on the distribution of *exh*: Strongest Meaning Hypothesis (SMH) and Economy Constraint (EC)

In the defense of the grammatical view, one thus needs to preserve its extra-expressiveness that helps account for attested cases of embedded exhaustification (Section 1.1), but also avoid over-generalization and limit its power through constraints that explain the limited distribution of *exh* (Section 1.2). These constraints should thus ban embedding of *exh* under negation as in (8), within disjunctions as in (9) or (10), or under existential quantifiers as in (11). One candidate for such a constraint is the Strongest Meaning Hypothesis or SMH (Dalrymple et al. 1998). It has been stated and studied in the context of embedded *exh* by Sauerland (2012) in the following form:

(12) **Strongest Meaning Hypothesis (SMH)**
A parse is dispreferred if there is another parse that differs only in terms of placement of *exh*, and that leads to a stronger meaning.

The SMH can explain several of the above examples. Consider the example with negation in (8) first. Generally, *exh*(A) is stronger than A, hence under negation ¬A is stronger than ¬ *exh*(A). As a result, the SMH systematically blocks the application of *exh* under negation, and therefore bans the parse in (8b). Similar arguments hold for the cases with disjunctions, (9) or (10). Focussing on (9), the unavailable parse is of the form *exh*(A or *exh*(B or C)). As explained, this parse would yield the interpretation 'exactly one or exactly 3 of A, B and C hold'. This parse is blocked by the SMH because dropping the embedded *exh*, as in *exh*(A or (B or C)), yields the strictly stronger reading: 'exactly one of A, B and C holds'. [3] The case with existential quantification (11) would work similarly. In short, the parse with a matrix-level *exh* results in the meaning that no employee saw all criminals; this is stronger than (11c), so the SMH predicts (11b) to be inaccessible.

[3] The computation of scalar implicatures arising from disjuncts, as in (9) and (10), involves some intricacies; the result we give is what the procedure of Fox (2007) predicts.

The SMH is thus able to block some of the parses generated by the possibility to embed *exh*, but yielding unattested readings. Does it annihilate the power of *exh* completely however, in banning desirable parses too? Here the results are more complicated. Consider (6) first, with a scalar item embedded under the non-monotonic quantifier ‘Exactly 3’. The two parses, with *exh* embedded or not embedded, were found to yield available readings, and indeed the readings they produce (6a) and (6b) are logically independent and therefore survive the SMH. However, in other cases, the SMH makes unfortunate predictions and bans too many parses. Consider the similar case (5) where a scalar item is embedded under a universal quantifier. There the two parses, with embedded and unembedded *exh* are in an entailment relation, (5a) entails (5b), and therefore the latter does not survive the SMH, although it is an attested reading.

So, a more permissive variant of the SMH is needed, and one has been formalized by Fox and Spector (2018), as an Economy Constraint (EC). Simplifying to a degree, they propose that:

(13) **Economy Constraint (EC)**
Parses are banned if they contain an occurrence of *exh* that is *weakening*, in the sense that removing it would yield a stronger meaning.[4]

Going back to (5), with a scalar item embedded under a universal quantifier, the two parses (5a) and (5b) survive the EC: they are both stronger than other versions of them with fewer *exh* operators. Many other cases will remain unchanged, because the reasonings above were about finding stronger meanings by removing embedded *exh* operators (that is the case for embedding under negation (8) and disjunctions (9)). However, this EC constraint doesn’t rule out (10b) nor (11b) anymore: the original version of the SMH was able to rule these out by comparison with parses with *exh* operators in other creative positions, but simply removing the embedded *exh* actually results in a weaker meaning, compliant then with the EC.

Finally, note than both the SMH and the EC predict a complete ban on exhaustification embedded below negation, as *exh* could only play a weakening role in this position. It isn’t entirely clear whether this is correct in at least one case, specifically that of *free choice* inferences. The phrase “free choice” (FC) refers to patterns of inferences like that in (14a), as schematically presented in (14b). Taken literally, the initial utterance should not entail that the addressee is allowed to choose; the fact that it is usually understood that way has been thought to derive from some exhaustification mechanism (e.g., Kratzer and Shimoyama 2002 and Fox 2007).

(14) a. Sue may take an apple or a banana.
$\rightsquigarrow$ Sue may take an apple, Sue may take a banana.
b. $\Diamond(A \vee B) \rightsquigarrow \Diamond A \wedge \Diamond B$

Importantly, the same analysis may also predict an additional, optional inference, that Sue may not take an apple and a banana *at the same time*. This is indeed a possible inference from (14a). We will follow Fox (2007) in calling it the anti-conjunctive (AC) inference and distinguishing it from the FC inference proper, outlined in (14b).

If the FC and AC inferences may be derived below negation, then (15) should have a variety of readings besides its literal reading (16a). First, as paraphrased in (16b), it should have a reading in which FC is derived below negation; a reading true if, for instance, you may take an apple, but you may not take a banana. We think that this reading is available, in the sense that compared to other cases discussed here such as that of (8) with a regular scalar item under negation, we can access it without special intonation or other marking.[5] On the other hand, the

[4]They in fact make a distinction between *incrementally weakening* and *globally weakening*, but this is not relevant to our discussion.

[5]Fox (2007, fn. 16) classifies the weak reading as an instance of an “intrusive” local implicature, and con-

reading in (16c), which one would obtain by computing the anti-conjunctive inference below negation, is not accessible at all. Finally, the reading paraphrased in (16d) takes into account both inferences at once, it seems inaccessible as well.

(15) It is not the case that you may take an apple or a banana.

(16) a. Literal reading: you may neither take an apple nor take a banana.
b. Neg{FC}: taking an apple and taking a banana are not both allowed.
c. Neg{AC} (unattested): you may neither take an apple nor take a banana, or you may take both at once.
d. Neg{FC+AC} (unattested): taking an apple is not allowed, or taking a banana is not allowed, or taking both at once is allowed.

Crucially, all three non-literal readings are constructed by adding inferences below negation. They are therefore all weaker than the literal reading, and should all be equally ruled out by the SMH or the EC, if one accepts an exhaustification analysis for these inferences. If real, the (even relative) availability of the Neg{FC} reading (16b), is thus a challenge for the SMH and the EC which do not seem to deliver the right constraints on the distribution of *exh*.

To sum up, the predictions of each constraint are given in Table 1. The SMH and EC have different strengths and weaknesses, but they are both based on the idea that parses with and without *exh* compete with one another. Instead, we are going to propose a different constraint, the Connected Meaning Hypothesis (CMH), which excludes possible parses on the basis of their output value only. The CMH is thus of a different nature, and it will have a different motivation, as such it could even co-exist with the SMH or EC. Beyond its conceptual motivation, it is a good contender also because, as we will now present, its coverage of the empirical landscape is just as good as for the other options currently available.

Example, description and schematic parse			Fact	SMH	EC	CMH
(5a)	Universal quantifier	$\forall$ *exh*	YES	YES	YES	YES
(5b)	Universal quantifier	*exh* $\forall$	YES	NO	YES	YES
(6a)	Non-monotonic quantifier	Q *exh*	YES	YES	YES	NO
(6b)	Non-monotonic quantifier	*exh* Q	YES	YES	YES	YES
(8)	Negation (basic case)	$\neg$ *exh*	NO	NO	NO	NO
(9)	3-way disjunction	$exh(A \vee exh(B \vee C))$	NO	NO	NO	NO
(10)	Scalar item in disjunct	$A \vee$ *exh*	NO	NO	YES	NO
(11)	Existential quantifier	$\exists$ *exh*	NO	NO	YES	NO
(15)	Negation Free Choice	$\neg$ *exh*FC	YES	NO	NO	YES

Table 1: Availability of inserting *exh* in various environments, as observed and as predicted by various constraints. SMH: Strongest Meaning Hypothesis (Sauerland 2012); EC: Economy Condition (Fox and Spector 2018); CMH: Connected Meaning Hypothesis (current proposal).

2 Proposal: the Connected Meaning Hypothesis

In this section we propose a different principle that may contribute to restrict the distribution of *exh*, by ruling out parses unless they produce 'good' outputs. A good output will be one that is 'connected', where connectedness is a formal property that has been proposed first to

siders that it is marked, but also remarks that this judgement is less clear than in similar cases of embedded exhaustification.

account for constraints on the meaning of content words, which was then extended to generalized quantifiers, and that we here propose to generalize to full sentential meanings. We will present this constraint and its origins in this section informally, showing how it intuitively covers the cases discussed so far. In the next section, we set up a formal fragment for which we derive theorems closely mimicking the empirical generalizations as they are known so far.

2.1 Constraints on words

A body of literature in semantics and related fields has been devoted to finding invariants or constraints that lexical items uphold or mostly uphold. The observation is that many lexical items share some abstract property, and that many other conceivable concepts which miss that property are lexicalized only in few or no languages, thus leading to the conjecture that the space of possible words is constrained by this property (see von Fintel and Matthewson 2008 for a survey of such conjectures, among other things). Horn (1973) makes this observation in the specific case of English logical words, such as connectives and quantifiers. Barwise and Cooper (1981) identify several mathematical properties often verified by lexicalized quantifiers (envisioned as logical functions) across languages. Among those is *monotonicity*. Monotonicity can work in two directions: if a set is part of the denotation of an *increasing monotonic* quantifier, then any larger set also is; if a set is part of the denotation of a *decreasing monotonic* quantifier, then any smaller set also is. For instance, the denotation of "having a pet" is a superset of that of "having a cat". Because "everybody" is an increasing monotonic quantifier, (17a) entails (17b). Because "nobody" is a decreasing monotonic quantifier, (18b) entails (18a). Neither (19a) nor (19b) entails the other, showing that "exactly two people" is not monotonic.

(17) a. Everybody has a cat.
b. Everybody has a pet.

(18) a. Nobody has a cat.
b. Nobody has a pet.

(19) a. Exactly two people have a cat.
b. Exactly two people have a pet.

Even though non-monotonic quantifiers are easily conceivable, Barwise and Cooper (1981) identify a bias in favour of monotonic quantifiers in the lexicon.[6] Katzir and Singh (2013a), without using the word, propose that the meaning of logical words is represented in terms of monotonic primitives, and that non-monotonic items require a complicated representation and are therefore marked.

Chemla, Buccola, and Dautriche (in press) investigated this issue and proposed to study a sister notion of monotonicity, connectedness. They first show that for a quantifier q, monotonicity of q is equivalent to connectedness of q and of its negation $\neg q$. Then, they present observation of lexicons and results from learning experiments arguing for a bias towards connectedness for quantifiers in humans, and even in baboons in Chemla, Dautriche, et al. (2018). One further interest of this notion of connectedness is that it actually originated not as a constraint on quantifiers, but in fact as a constraint on the denotation of content words, like nouns, as championed by Gärdenfors (2004) (see also Gärdenfors 2014 for an extension to other types of words). Formally, the denotation of a word is connected if for any two elements in it, any elements that are *in-between* those two are also in the denotation. For instance, if one knows that the word "animal" denotes a connected set of species, and that the duck and the badger

[6] To be precise, they identify a bias in favour of quantifying determiners that produce monotonic quantifiers, such as "every".

are instances of animals, one can deduce that the platypus is an animal. This is, of course, if one thinks that the platypus is in-between ducks and badgers. Hence, in general, the notion of connectedness depends crucially on that of in-betweenness. In the case of content words, one needs an in-betweenness relations over objects, which is often intuitive but certainly difficult to formalize in general; in the case of quantifiers, one needs an in-betweenness relation over sets of objects, and the formal, canonical order of set inclusion has been argued to be productive. We will engage with similar issues once moving to the current enterprise about plain sentential meanings.

2.2 Principle: connectedness beyond words

Our proposal builds upon the work on the lexicon: we are going to propose a constraint enforcing connectedness at the level of full sentence meanings. Since we are concerned with the distribution of *exh*, the objects of interest to us are not single words, whether content words or logical words, but entire propositions. We claim that there is a straightforward way to apply the notion of connectedness to propositions: for any two worlds where a connected proposition is true, it is also true in every world *in-between* those two, for some intuitive notion of in-betweenness among worlds (see discussion in Section 4.2). And we claim that non-connected meanings are problematic. Consider (20), for instance; it is not an easy sentence to cope with, which we claim could be because it is not connected, in the following sense: take three worlds w_{10}, w_{30} and w_{50} which are most similar to one another except that there are 10, 30 and 50 people attending my class. The world w_{30} intuitively counts as in-between the worlds w_{10} and w_{50}. Yet, the sentence (20) is true at w_{10} and w_{50}, and false in between, at w_{30}. So, (20) is not connected, and indeed it is quite a weird sentence, suggesting that there is a dispreference towards propositions whose meaning isn't connected.

(20) ? Fewer than 20 or more than 40 people attended my class.

Of course, (20) is not an impossible sentence. It is certainly not a plainly deviant sentence *per se* and non-connectedness may not be problematic in general (more on this below). But we propose that *exh*, which is optional, should not be recruited if it ends up creating non-connectedness:

(21) **Connected Meaning Hypothesis (CMH)**
Among the parses of a sentence (with or without *exh*), those that result in non-connected meanings are dispreferred/marked.

This first statement of the principle remains at a non-formal, intuitive level. We will resolve this in backward order. Only Section 4 will make all of the details explicit: it will offer a proposal to derive an in-betweenness relation and its associated connectedness property that account for the examples to be reviewed here, and it will include a formal fragment with theorems as to what types of parses are penalized by this principle (e.g., exhaustification under negation will be generally penalized). In Section 3, the status of this constraint will be discussed, for at this point it may be implemented as a strict impossibility constraint, a contextual constraint, or an entirely soft constraint. But for now, in the upcoming Section 2.3, we want to show how the notion applies intuitively and how it captures the essential facts about the distribution of *exh* presented earlier. In short, we are going to argue that the unattested parses typically have weird paraphrases, weird in some intuitive sense related to non-connectedness.

2.3 Application

Consider first the paradigmatic example of blocking *exh* from under negation, (8), repeated below: the parse in (22), and the meaning it ought to result in, are intuitively unavailable. The CMH proposes to account for this by showing that this meaning is problematic. To put it simply, there is something complicated about the idea that John did either none or all of the homework. Concretely, this meaning is non-connected, in the same way as the meaning of (20) is non-connected. To see this, consider the sequence of worlds w_a, w_b and w_c, which are as similar as possible except that: in world w_a, John spent the day watching series, in world w_b, he took the time to do half of his homework, in world w_c, he worked hard to complete his homework. Intuitively, w_b is in-between w_a and w_c, but (22) is true at both w_a and w_c, and false at w_b. This is schematized in (23), where we see an F in between two Ts.

(22) Sentence: It's not true that John did some of his homework.
Unattested parse: *It's not true that John did *exh* [some of his homework].
Intended meaning: It isn't true that John did some but not all of his homework.

(23) Offending sequence of worlds:

	$\neg$ *exh*(A)	*exh*($\neg$A)
World w_a: John did no homework	T	T
World w_b: John did half of his homework	F	F
World w_c: John did all of his homework	T	F

As schematized in (23) as well, we see that this sequence is not a problem for the parse without *exh*, which in fact is fully connected: the sentence is true whenever John did none of his homework, and a world in-between two worlds where he didn't work ought to be one where he didn't work either. Hence the CMH predicts the interpretation without *exh* to always be favoured over the one with *exh* under negation.

Consider now (9), repeated here as (24). It has the structure $A \vee B \vee C$. (24) can be parsed in several ways, including as (24a), and as (24b). As before, we claim that the meaning of (24b) is hard to reason about, even based on the explicit paraphrase. It is also non-connected in the following intuitive sense: take a world where John meets only Ann, a world where he meets Ann and Carol, and a world where he meets all three people. (24b) is true at the first and third world, but not at the second one, which is intuitively in-between the other two. This is summarized in (25).

(24) John will meet Ann, Bill or Carol.

a. Attested parse: $exh[A \vee B \vee C]$
Meaning: John will meet exactly one person within Ann, Bill and Carol.

b. Unattested parse: $*exh[A \vee exh[B \vee C]]$.
Intended meaning: John will meet exactly one person within Ann, Bill and Carol, or all of them.

(25) Offending sequence of worlds:

	$exh(A \vee exh(B \vee C))$	$exh(A \vee B \vee C)$
World a: A	T	T
World b: A and B	F	F
World c: A and B and C	T	F

As before, we see that this sequence of worlds is not a counter-example to the connectedness of the attested parse (24a), and in fact, we argue that (24a) is connected. First, for any two worlds where John meets just one person, the same in both, he certainly also meets them in any world in-between. For a world where he meets just Ann and a world where he meets just Carol, one could imagine that in some in-between worlds he meets both or neither, but we adopt a notion of in-betweenness where there is no world in-between those two, because there is no obvious way to relate one to the other (cf. Appendix A). This makes the meaning of (24a) connected, explaining why it is felicitous while (24b) isn't.

Let's turn to (11), repeated here as (26a). The structure of (26a) can be represented by the formula in (26b).

(26) a. There are employees who saw some of the criminals.
b. $\exists e, \exists c, S(e, c)$

We will assume that both elements with existential force ('there are' and 'some') have a universal quantifier ("all of the...") as an alternative. Several parses can be considered. In (27) we represent some of them, starting with those in which *exh* always take into account all alternatives potentially triggered from its scope, as signalled by the indices that match with the set of scalar items in the scope of *exh* in each case. For completeness we also present there two other options that can be treated similarly.

(27) a. * $\exists_1 e, exh_2\, \exists_2 c, S(e, c)$
Some employees saw some but not all criminals.
b. $exh_{1,2}\, \exists_1 e, \exists_2 c, S(e, c)$
Some but not all employees saw some criminals, and no employee saw all criminals.
c. * $exh_1\, \exists_1 e, exh_2\, \exists_2 c, S(e, c)$
Some but not all employees saw some but not all criminals.
d. $exh_2\, \exists_1 e, \exists_2 c, S(e, c)$
Some employees saw some criminals, but no employee saw all criminals.

In (28), we use the same method as before to show that (27a) (as well as (27c)) are not connected. This counter-example is compatible with the connectedness of (27b) (and (27d)), however. Proving connectedness is more demanding: one needs to show that *no* counter-example could be found, which we submit is the case here for reasonably intuitive notions of in-betweenness (as is made explicit in Appendix A). Overall then, the CMH appropriately predicts that (27a) and (27c) should be unavailable, and that (27d) and (27b) are possible parses.

(28) Offending sequence of worlds:
Consider that there were a couple of criminals (so, seeing one criminal counts as seeing some but not all), and that among the employees none has seen any criminal, except for A and B, for which the situation is as follows:

	(27a)	(27b)	(27c)	(27d)
World a: *A* saw one criminal, *B* saw none	T	T	T	T
World b: *A* saw all criminals, *B* saw none	F	F	F	F
World c: *A* saw all criminals, *B* saw one	T	F	T	F

Consider now (5), repeated as (29a). Again, we give it a formula representation in (29b). There are two ways to parse (29a) with *exh*, listed in (30).[7]

(29) a. Every student did some of the homework.
b. $\forall s, \exists h, D(s, h)$

[7] We ignore the possibility of obtaining alternatives by replacing the universal quantifier at the top of the sentence with an existential quantifier.

(30) a. $\forall s, exh\, \exists h, D(s,h)$
Each student did a proper part of their homework.
b. $exh\, \forall s, \exists h, D(s,h)$
Every student did some of the homework, but some students didn't do all of it.

We think both of those have a connected meaning. The fact that (30a) is connected is quite intuitive: in-between two worlds where (30a) is true, the extent of their homework each student did ought to be in-between the extent of their homework they did in the extreme worlds. If it is a proper part on both sides, it is a proper part in-between. We can also give a sense as to why (30b) is connected: consider two such worlds w_a and w_c where (30b) is true, that is, in both of these worlds, all students did part of their homework. For a world w_b to be in between w_a and w_c, under our definition, all students would have to have done more homework in w_b than in one of these worlds (say, w_a), and less than in the other world (w_c) (cf. Appendix A). Necessarily then, because not all students have done part of their homework in w_c, this would also hold in w_b, making (30b) true at w_b. Overall, then, both (30a) and (30b) are connected, and the CMH therefore does not exclude these parses.

As the next example, we will look at (6), repeated here as (31), and consider the two parses in (32).

(31) Exactly three employees saw some of the criminals.

(32) a. *exh* [Exactly three employees saw some of the criminals.]
Meaning: Exactly three employees saw some of the criminals, less than that saw all of them.
b. Exactly three employees saw *exh* [some of the criminals.]
Meaning: Exactly three employees saw some but not all of the criminals.

As seen in the sequence of worlds in (33), (32b) doesn't have a connected meaning. Meanwhile, it can be shown that (32a) does under identifiable assumptions (cf. Appendix A). Thus, the CMH predicts (32b) to be unavailable. While (32a) does seem to be the more accessible reading, it isn't clear that (32b) is as inaccessible, and there is experimental evidence for its existence. We will discuss limitations of the CMH and potential reasons for its violation below.

(33) Offending sequence of worlds:
Again, assume that there were a couple of criminals (so, seeing one criminal counts as seeing some but not all). Among the employees none has seen any criminal, except for A, B, C, D, E and F for which the situation is as follows:

	(32a)	(32b)
World a: A, B, C saw one criminal, D, E, F saw none	T	T
World b: A, B, C saw all criminals, D, E, F saw none	T	F
World c: A, B, C saw all criminals, D, E, F saw one criminal	F	T

Our final example is that of Free Choice under negation, as in (15), repeated here as (34), with its schematic structure also given there. As we already discussed, some analyses predict that $\Diamond(A \vee B)$, when subject to exhaustification, may become $\Diamond A \wedge \Diamond B$. Thus, if (34) is parsed with an embedded *exh* as in (35a), it ought to be equivalent to (35b) (this is the Neg{FC} reading from (16b)).

(34) It is not the case that you may take an apple or a banana.

(35) a. Possible parse: $\neg\, exh\, \Diamond(A \vee B)$
b. Meaning: $(\neg\Diamond A) \vee (\neg\Diamond B)$

(You may not take an apple or you may not take a banana.)

Despite the fact that it requires an *exh* operator under negation, the meaning in (35b) is connected, and therefore complies with the CMH. To get an intuition as to why this is true, take two worlds where (35b) is true. There are three possibilities: if in both worlds you may not take a banana, then it is true in any in-between world; if in both worlds you may not take an apple, then again it should hold in any in-between world; finally, if in one world you may take an apple but not a banana, and in the other it is the opposite, then there is no obvious way to relate the two worlds, and by our definition there is no world in-between. Thus, the CMH tolerates Free Choice inferences under negation, what we have called the Neg{FC} above in (16).

One can go further and investigate the other options mentioned in (16), involving the anti-conjunctive inference: neither the Neg{AC} ((16c)) nor the Neg{FC+AC} ((16d)) readings are connected, and they are therefore appropriately banned by the CMH. We leave the details to the reader and to Table (36), which provides a counter-example valid for both cases.

(36) Offending sequence of worlds:

		(16c)/(16d)
World a:	There is nothing you may take.	T
World b:	You may take an apple, you may take a banana, but not both at once.	F
World c:	You may take both (at once).	T

3 Discussion

Having established the potential of the CMH to be at the source of a range of empirical facts, we would now like to discuss the conceptual motivations of the CMH, and explore some of the parameters we have currently left unset: is the CMH a strict or a soft constraint? what is its status as a linguistic constraint and how does it interact with other linguistic constraints?

3.1 Status and origin of the CMH

To get started, one may ask whether the CMH is a hard constraint, as in: non-connected meanings are ungrammatical. This is not the spirit of the CMH, nor does it seem to comply with the introspective status of the relevant sentences. Looking back at the case with no *exh* operator, it would be strange to declare a sentence like (20) ungrammatical; intuitively, it is merely an unlikely thing to say, something hard to think about even, which is what the CMH tries to capture. The idea then is that the CMH prevents us from using roundabout ways, the application of *exh*, to derive meanings we will end up not being confortable with.

One may expect from this general approach that the CMH should not be a strict constraint: even if we may have difficulties manipulating non-connected objects, we ought to have ways to do it, in case this happens to be useful. So, even if non-connectedness will be restricted, general reasoning abilities and language should find a way to allow us to manipulate non-connectedness. In fact, language surely does provide the means to express non-connected meanings, often times through disjunction: after all, (20) is hard, but we can construct it and work with it. Other sources of non-connectedness in natural languages involve embedding under non-monotonic constituents, as in (37) or (38); again, these examples are hard and may require a second thought, but they are possible sentences.

(37) It isn't the case that John did exactly two exercises.[8]

(38) Some students did exactly two exercises.

So, the CMH is best viewed as a the result of a cognitive dislike for non-connected meanings, which results in the (soft) blocking of some parses that would unnecessarily create such non-connected meanings. Ideally, one should try to explain *why* we are not comfortable with non-connected meanings, and show that this cognitive distaste for non-connectedness generalizes beyond the linguistic domain (as attempted to in Chemla, Buccola, and Dautriche in press and Chemla, Dautriche, et al. 2018). For now, it is sufficient to take away that the CMH is not to be thought about as a hard constraint. We will thus now discuss several potential sources for CMH violations.

3.2 Contextually salient non-connectedness

If we take the CMH to derive from a cognitive bias against non-connectedness, we might expect that context could help alleviate the difficulty, making some non-connectedness proposition cognitively easy or salient, to the point that violations of the CMH could be saved. This view is somewhat risky, in that it could deprive the CMH of much of its predictive power. However, if one had an independent measure of the difficulty of a particular non-connected meaning in a particular context, the predictive power could be recovered, with the expectation that CMH violations would be higher, for difficult non-connected meanings. At the very least, we can start investigating the issue by trying very clear cases of easy non-connectedness.

Consider for instance (39a). By default, it seems to have a connected meaning, whereby some students did two or more exercises. But in a context as the one given in (39b), which makes salient and important the possibility that a student does exactly two exercises, it may receive a non-connected interpretation, with the numeral receiving an 'exact' reading under the existential quantifier, therefore obtaining the same non-connectedness as (38) (or (27a)).

(39) a. Some students did two exercises.
b. Supportive context: if a student does exactly two exercises, the grading database crashes.

Thus, the availability of the non-connected meaning in (39) could be taken to indicate that the CMH's strictness is only relative to the difficulty of conceiving of non-connectedness in the context.[9] But it is most important to act with caution here because it does not seem to be so easy to allow violations of the CMH by simple manipulations of the contexts in other cases. For instance, embedded exhaustification isn't as clearly accessible in (40) as in the very

[8] This is one of several cases in this article where finding examples involving matrix negation is made difficult by structural ambiguities. Here, matrix negation makes very salient the reading that there are exactly two exercises John didn't do.

[9] A similar, but more abstract effect can perhaps be found in the experimental results of Chemla and Spector (2011). As we already mentioned, they report that participants in a judgement task seemed to access parses similar to (6a), repeated here as (i), where *exh* is embedded below a non-monotonic quantifier. This is problematic for the CMH, as this parse doesn't result in a connected meaning. Interestingly, they also report that in a second judgement task, the same participants seemed to access parses where *exh* occurs below negation more easily than other participants whose first task did not involve non-connected meanings.

(i) Exactly three employees saw *exh* [some of the criminals.]

Chemla and Spector (2011) speculate that the first category of participants were somehow primed on embedded exhaustification, because something about their first task made such parses more salient. We may venture that, while we cannot explain why these participants were able to access non-connected readings in the first place, their first task accustomed them to non-connectedness and let them relax the CMH and access non-connected meanings in the second task too.

similar (39a), that is, with ‘some’ as a scalar item, rather than a numeral.

(40) Context: students who did nothing get a C, those who did part of the work get a B, those who did all of it get an A. If a student gets a B the database crashes.
Some students did some of the exercises (? and therefore the professor couldn’t enter their grades).

Hence, the previous result about (39) may be specific to numerals since, as already seen from (37) or (38), non-connected propositions are particularly easy to obtain from numerical expressions. There would be several ways to explain such a fact. First, (39) is only a violation of the CMH if one assumes that the non-monotonic reading of “two” derives from exhaustification. If that isn’t the case, we do not have such a clear case for the sensitivity of the CMH to contextual salience in the first place. Second, it could be easier to reason about non-monotonic or non-connected things in numeric or mathematical contexts. Intuitively, the concept of “exactly two” is more accessible than that of “a proper part”. As a result, *exh* remains very attractive despite the threat of the CMH. In this latter case, then, the CMH does show sensitivity to content, rather than to context. The resulting view would be highly compatible with the idea that the distribution of *exh* is governed by the shape of its output, complemented with the idea that the mathematical domain has a pragmatics of its own.

3.3 Contextually connected non-connectedness

Another way in which context could interact with the CMH is through the common ground, considering that it could transform non-connected meanings into contextually connected meanings. So, a particular proposition may be non-connected, while the intersection of it with the common ground may be connected. Assume then a notion of logical connectedness as well as a notion of contextual connectedness. If the CMH is a principle that bans meanings that are hard to think about, and if the common ground can be incorporated without efforts into the decoding of linguistic meanings, then we may find that contextual connectedness is the appropriate notion, to the effect that the common ground is sufficient to relieve the burden of non-connectedness.

Unfortunately, the current evidence is hard to judge. Consider for instance (41). In this context, the sentence is either trivially true (a contextual tautology) with its normal connected meaning, or it is contextually connected (but not logically connected). This is thus an ideal situation to see the otherwise unattested meaning rise. It is unclear however what the facts really are: the sentence may well receive that interpretation, but it still sounds a bit deviant.

(41) Context: You have to do all exercises to get an A. All students did part of the homework.
Some students did some of the exercises (? and therefore didn’t get an A).

Thus, there is no evidence that contextual connectedness, rather than bare connectedness, has much to offer.[10]

[10] Interestingly, an effect of the CMH may be seen in the interaction between the common ground and *overt* strategies for exhaustification. Both unusual focus marking and ‘only’ adverbs have effects similar to covert exhaustification: the following two sentences both unambiguously imply that John didn’t do all the relevant exercises. This may be analysed by taking “only” to have similar semantics to *exh*, and focus marking to co-occur with *exh*.

(i) a. John only did some of the exercises.
b. John did SOME of the exercises.

As a result, just like unmarked occurrences of *exh*, these constructs introduce the possibility of non-connectedness when embedded in some environments. However, a closer look suggests that non-connectedness does not strike in then. Consider what happens when these overt exhaustification strategies are embedded in the

3.4 Interaction of the CMH with Hurford's constraint

The CMH is meant to block some occurrences of *exh*. One may ask what happens when the satisfaction of other constraints may force the occurrence of *exh*. Chierchia, Fox, and Spector (2011) present a variety of examples where Hurford's constraint has precisely this effect; those examples are (specific cases of) *Hurford disjunctions*.

Recall that in a Hurford disjunction, the exhaustification of a disjunct is necessary to avoid that disjunct to be redundant. This derives from a principle called Hurford's constraint (HC): HC states that disjunction where the first disjunct entails the second are infelicitous (see e.g., Katzir and Singh 2013b, Mayr and Romoli 2016). We can construct an example where HC forces local exhaustification, but where this results in a non-connected meaning. This is the case for instance of (42). Hurford's constraint pushes in favour of the insertion of *exh* in the first disjunct, but the resulting meaning would be non-connected as long as there are more than 3 problems, and we allow 'three problems' to be an available alternative available for the exhaustification mechanism. The non-connectedness of the resulting meaning can be established from (43). We observe that the sentence is acceptable despite the violation of the CMH.

(42) Peter either solved both the first and the second problem or all of the problems. (Chierchia, Fox, and Spector 2011)

(43) Offending sequence of worlds:
Assume there are 5 problems.

	(42)
World a: Peter solved problems 1 and 2.	T
World b: Peter solved problems 1 through 3.	F
World c: Peter solved problems 1 through 5.	T

Overall, then, it seems that the CMH is liable to being overridden by other principles that force the presence of *exh* in certain positions, such as Hurford's constraint. The situation can be

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problematic negative or existential environments:

(ii) a. It's not the case that John only did some of the exercises.
b. It's not the case that John did SOME of the exercises.

(iii) a. Some students only did some of the exercises.
b. Some students did SOME of the exercises.

The overall meanings obtained actually qualify as connected: in (ii), the sentences' overall meaning is that John did all of the exercises (and not that he did either none or all of the exercises) and in (iii), the sentences' overall meaning seems to be that all students did some of the exercises, but not all students did all of the exercises. A common analysis of these 'overall meanings' states that they involve not only assertive components, but also presuppositions or some form of non at-issue content (see Coppock and Beaver 2014 or Roberts 2006 for discussion about 'only', Schwarzschild 1999 about focus marking), projecting out of the environments (through universal projection in the case of the existential environments *à la* Heim 1983, or more recently Schlenker 2008, *pace* Beaver 2001). These special parts of meanings turn out to block non-connectedness. To put it simply, one may say that the presuppositions of 'only' and of unusual focus marking protect them from introducing non-connectedness when embedded in these environments.

We could thus argue that connectedness constraints have a role to play not only in the selection of the meanings of lexical items, but also in the selection of their presuppositions. One may then ask why *exh* does not grow a presupposition that would have the same protective effect: a possible answer is that interpreting a sentence with an unmarked *exh* is always optional, and a parse without *exh* can always be selected instead if non-connectedness would occur otherwise. Another question is why certain other items that, like "only", introduce non-monotonicity (such as non-monotonic quantifiers) do not acquire a similar protective presupposition. A language with a certain level of expressive power would certainly need to include items capable of resisting all sorts of non-connectedness constraints, but it is an open question to tell which items may be elected to do so.

described by saying that one should not create connectedness with embedded *exh* for no reason, but Hurford's Constraint is actually a good reason. Thus, the CMH appears to be a relatively soft constraint.

3.5 The CMH on unembedded exhaustification?

Finally, while we have been focussing on the consequences of the principle on the occurrence of *exh* in embedded positions, it is conceivable that even regular, unembedded exhaustification could be affected by the CMH. Indeed, certain choices of non-monotonic alternatives could lead to non-connectedness from global exhaustification alone. In this respect the CMH differs from the SMH and related hypotheses: global occurrences of *exh* may only result in strengthening by definition, and will never be banned by the SMH.

As an attempt to find relevant examples, consider (44). In principle, the first statement could be used as an alternative to the second statement, which would then be exhaustified as described at the bottom of the example. This does not seem to happen, however, and the CMH predicts this impossibility since it would result in a non-connected meaning. However, such type of exhaustification may not be that easy and could be blocked for independent reasons: a similar type of exhaustification also does not happen in (45), even though there the final meaning would be connected. Now, there is an independent reason why this reading does not emerge in (45), which is that there was a much simpler way to get at it, not relying on exhaustification, and simply putting in the right number in the sentence in the first place; thus simply replacing 30 with 20 in the target sentence would do the trick. If this is the reason why that reading is not available below, then we need an explanation as to why the reading discussed in (44) is not available, and the CMH could provide one.

(44) Yesterday, there were between 20 and 30 people.
Today, there are between 10 and 40 people.
$*\rightsquigarrow$ There are between 10 and 20 or between 30 and 40 people.

(45) Yesterday, there were between 20 and 30 people.
Today, there are between 10 and 30 people.
$*\rightsquigarrow$ There are between 10 and 20 people.

4 The CMH in an explicit fragment

In this section, we present the CMH in a more formal and systematic manner, by investigating definitions and generalizations in a well-defined fragments roughly corresponding to predicate logic, with exhaustification. In Section 4.1 we define an explicit fragment capable to mimic almost all of the cases reviewed so far. In Sections 4.2 and 4.3 we define monotonicity and connectedness as formal properties of linguistic objects, and prove various theorems about which structures lead to monotonic or connected meanings. Section 4.4 connect these results to the empirical generalizations presented above.

4.1 Fragment

Our fragment is essentially that of first-order logic, with constants, variables, predicates, negation, disjunction and conjunction, existential and universal quantifiers, all with their usual semantics. We need to add the operator *exh* to such a fragment. We do so first by adding the straightforward syntactic inductive rule:

(46) If ϕ is a sentence, $exh\,\phi$ is a sentence.

The semantics of *exh* is as follows, where we assume the existence of an operator alt which applies to every sentence to provide a set of alternative sentences (we do not give further compositional structure to the formation of alternatives here, see Katzir 2007):

(47) $[\![exh\,\phi]\!] = [\![\phi]\!] \wedge \left(\bigwedge_{\phi' \in \mathrm{alt}(\phi)} \neg [\![\phi']\!]\right)$

Of importance to us is the fact the semantic value of a sentence is a function from situations to truth value (i.e., it has type $\langle s, t\rangle$). A situation is an interpretation function for the variables and predicates: it maps variables to individuals in some domain and predicates to predicates with the appropriate arity over the same domain. In what follows we will use two notational shortcuts: (i) we will drop double brackets when it is possible to do so without creating ambiguities, (ii) we will write down semantic objects of type $\langle \alpha, t\rangle$ in set notation.

4.2 Assumptions and formal definition of connectedness

To continue, the semantic objects must be equipped with more structure. In particular, to define connectedness we need in-between relations for input types and orders for output types — here we will derive the in-betweenness relation for input types from an order, in the same way as Chemla, Buccola, and Dautriche (in press) do.[11]

First, we assume that truth-values are ordered in the usual, conventional sense: $1 > 0$, truth is ranked higher than falsity. From there, we can derive an order for functions of any type ending in t.

Second, we also assume that objects of type s are ordered. Although this would be desirable (see for instance discussion in the literature on counterfactuals, as in Kratzer 1979, 1981; Lewis 1973, 1981; Stalnaker 1968), there is no universal formal way to define an order on situations or possible worlds. However, we claim that a consensual order on worlds can be found in every example of relevance, on a case-by-case basis. Furthermore, we note that orders can be derived from the more familiar, although maybe not more deterministically given, notion of a Question Under Discussion (QUD, see Roberts 1996). A QUD is, in short, a question to which the sentence is understood to be an answer to. It is typically meant to be inferred from contextual information as well as from the utterance itself. Assuming that a QUD is given then, we can map each possible world onto the set of answers to the QUD that are true at this world (the Karttunen denotation of the QUD at this world). A pre-order can then be derived over worlds: any two worlds can be pre-ordered in the same way as the way their corresponding sets of answers are ordered by inclusion. If the situations correspond to the classes of worlds which behave similarly with respect to the QUD, then the pre-order over worlds defines an order over situations. We hope that this connection with the existing literature makes the relevant notion of an order more palpable.[12]

Thus, sentences are assumed to denote functions from an ordered type to another one, and for any such function over ordered types, we can define monotonicity and connectedness (see Appendix B for alternative ways to define connectedness):

(48) **Definition:** Monotonicity and connectedness.
Consider a sentence ϕ of type $\langle \alpha, \beta\rangle$, where α and β are ordered.

a. ϕ is *increasing monotonic* iff for all x, x' of type α, if $x \leq x'$, then $\phi[x] \leq \phi[x']$.

[11] For discussion of potential alternatives to our formal definition of connectedness, see Appendix B.

[12] As pointed out by Benjamin Spector (p.c.), the order on worlds one would derive that way would be essentially the same one that certain theories of exhaustification already require; see for instance Spector (2006), van Rooij and Schulz (2004) and Schulz and van Rooij (2006). Instead of defining it in terms of sets of true answers to the QUD, they use sets of true alternatives to the utterance.

b. ϕ is *decreasing monotonic* iff for all x, x' of type α, if $x \leq x'$, then $\phi[x'] \leq \phi[x']$.
c. ϕ is *monotonic* iff it is increasing monotonic or decreasing monotonic.
d. ϕ is *connected* iff for all x, x', x'' of type α, if they form a *chain*, i.e. $x \leq x' \leq x''$, then:

$$\phi[x'] \leq \phi[x] \wedge \phi[x''] \tag{$*$}$$

To tie this together with the informal notion of connectedness we have been using so far, let's define in-betweenness as follows: x' is in between x and x'' iff x, x', x'' form either a chain ($x \leq x' \leq x''$) or an antichain ($x \geq x' \geq x''$). If x' is in-between x and x'' either (x, x', x'') or (x'', x', x) form a chain; in both cases, ($*$) holds if ϕ is connected. Given that in the specific case where $\beta = t$, ($*$) can be read as "if $\phi[x]$ and $\phi[x'']$ are both true, so is $\phi[x']$," our old informal definition is just a special case of the new formal one for $\beta = t$.

We can finally mention the following relation between monotonicity and connectedness:

(49) **Result:** Monotonicity entails connectedness.
If ϕ is monotonic, ϕ is connected.

Proof. Let ϕ be a monotonic function, which we will assume without loss of generality to be increasing, and let $x \leq x' \leq x''$ be a chain. Then: $\phi[x''] \geq \phi[x'] \geq \phi[x]$, and: $\phi[x] \wedge \phi[x''] = \phi[x]$. Thus ($*$) holds, and since this is true for any chain, ϕ is connected. □

For concreteness, Figure 1 provides a visual illustration of the possible types of predicates obtained when $\beta = t$, and α is a domain of 4 elements ordered (partially) in a particular way.

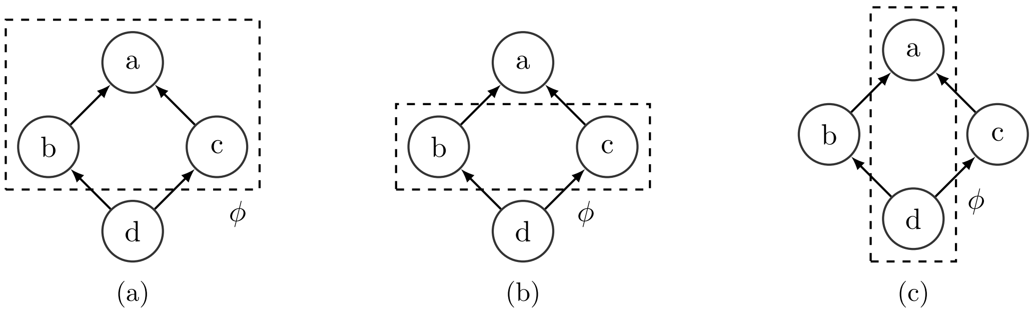

Figure 1: Consider that the domain α contains four elements, ordered as in the diagrams above: a is above b and c, which are incomparable with one another, but both being above d. The dotted boxes provide examples of (a) an upward monotonic and connected predicate (b) a non-monotonic but connected predicate, and (c) a non-connected predicate.

4.3 Structures leading to connected meanings

In this section, we ask for several types of structures whether we can, or cannot, be sure that the result will be connected. We inspect the cases following the syntactic construction of our fragment, asking the same question for sentences headed by a negation, a conjunction, a disjunction, a quantifier or, crucially, an *exh* operator. In each case, we ask whether the obtained sentence is connected or monotonic, depending on whether the elementary pieces it is made of are monotonic (in the (a) parts of each result) or connected (in the (b) parts of each result). Depending on the cases, we may obtain two types of generalizations: theorems showing that certain structures will necessarily output a monotonic or a connected meaning, and theorems showing that some constructions will not necessarily output a monotonic or a connected meaning.

(50) **Result:** Negative sentences

a. If ϕ is monotonic, $\neg\phi$ is connected.
b. If ϕ is connected, $\neg\phi$ isn't necessarily connected.

Proof. The result in (a) follows as a corollary of result (49): if ϕ is monotonic, $\neg\phi$ is also monotonic, and therefore it is connected.

The following counter-example proves (b): Suppose that three elements form a chain $x_1 < x_2 < x_3$. Define ϕ as the function over these three elements, which is true for x_2 and no other element. Then ϕ is connected, but $\neg\phi$ isn't connected, since its denotation includes x_1 and x_3 but not the in-between point x_2. □

(51) **Result:** Conjunctive sentences

a. If f_i is a collection of monotonic functions, all increasing or all decreasing, $\bigwedge_i f_i$ is monotonic in the same direction.
b. If f_i is a collection of connected functions, $\bigwedge_i f_i$ is connected.

(52) **Corollary:** Universally quantified sentences

a. If ϕ has a free variable y, and if for all individuals e_0, $\phi[y \leftarrow e_0]$ is monotonic in the same direction, then $\forall y, \phi$ is monotonic in this direction.[13]
b. If ϕ has a free variable y, and if for all individuals e_0, $\phi[y \leftarrow e_0]$ is connected, then $\forall y, \phi$ is connected.

Proof. [Proof of (51)] (a) is an immediate consequence of the fact that conjunction preserves the order, i.e. that it is itself an upward monotonic operation. Focussing on (b) then, let us define $g := \bigwedge_i f_i$. Take a chain $x \leq x' \leq x''$. We have: $g[x'] = \bigwedge_i f_i[x'] \geq \bigwedge_i (f_i[x] \wedge f_i[x''])$ (because each f_i is connected). Therefore $g[x'] \geq \bigwedge_{i,j} (f_i[x] \wedge f_j[x''])$ (a conjunction with more terms is smaller). Therefore $g[x'] \geq \left(\bigwedge_i f_i[x]\right) \wedge \left(\bigwedge_i f_i[x'']\right)$. And so $g[x'] \geq g[x] \wedge g[x'']$. □

(53) **Result:** Disjunctive sentences

a. If f_i is a collection of monotonic functions, all increasing or all decreasing, $\bigvee_i f_i$ is monotonic in the same direction.
b. If ϕ and ψ are two elements that are both connected, $\phi \vee \psi$ isn't necessarily connected.

(54) **Corollary:** Existentially quantified sentences

a. If ϕ has a free variable y, and if for all individuals e_0, $\phi[y \leftarrow e_0]$ is monotonic in the same direction, then $\exists y, \phi$ is monotonic.
b. If ϕ has a free variable y, and if for all individuals e_0, $\phi[y \leftarrow e_0]$ is connected, then $\exists y, \phi$ isn't necessarily connected.

Proof. [Proof of (53)] Result (a) is an immediate consequence of the fact that disjunction is an increasing monotonic operation.

(b) Choose three elements x_1, x_2, x_3 such that $x_1 < x_2 < x_3$, and define the following functions over that input set and onto truth-values (so these are sets): $\phi = \{x_1\}$ and $\psi = \{x_3\}$. Both are monotonic (albeit in different directions), and therefore connected. Yet, $\phi \vee \psi = \{x_1, x_3\}$ is not connected. □

(55) **Result:** Exhaustified sentences

a. If ϕ and all of its alternatives are monotonic in the same direction, then $\mathit{exh}\,\phi$ is connected, but not necessarily monotonic.
b. If ϕ and all of its alternatives are connected, $\mathit{exh}\,\phi$ isn't necessarily connected.

[13] Note that this complicated condition is equivalent to "$\tilde{\phi}$ is monotonic", where: $\tilde{\phi} = \lambda w \cdot \{e_0 / [\![\phi[y \leftarrow e_0]]\!](w)\}$ (this is an object of type $\langle s, \langle e, t\rangle\rangle$). The same is true if "monotonic" is replaced by "connected".

Proof. For (a), assume without loss of generality that ϕ and its alternatives are upward monotonic. For any $\phi' \in \text{alt}(\phi)$, $\neg\phi'$ is decreasing monotonic, and therefore connected. Because conjunction preserves connectedness, it follows that $exh\,\phi$ is connected. The following counter example shows that $exh\,\phi$, however, may not be monotonic when ϕ is: assume that ϕ's domain is $\{x_1, x_2, x_3\}$ such that $x_1 < x_2 < x_3$. Take $\phi = \{x_2, x_3\}$, $\phi' = \{x_3\}$, and $\text{alt}(\psi) = \{\phi'\}$. Then $exh\,\phi = \{x_2\}$, which isn't monotonic.

For (b), we will use the following counter-example, take: $\phi = \{x_1, x_2, x_3\}$, $\phi' = \{x_2\}$, and $\text{alt}(\phi) = \{\phi'\}$. Then $exh\,\phi = \{x_1, x_3\}$ which is not connected. □

4.4 The CMH and the distribution of *exh*

Thus, we may relate the theorems of Section 4.3 to generalizations discussed in Sections 1 and 2, and summarized in Table 1.

To do so, first suppose that primitive predicates are monotonic.[14] Then note that *exh* transforms monotonic constituents into just connected constituents (see Result (55a)). Upon some further embedding then, the outcome may be non-connected at all, namely for those embeddings with a negative result about the preservation of connectedness (the b-part of the results in Section 4.3). This is the case for negation ((50b)), disjunction ((53b)) or existential contexts ((54b)). In other words, *exh* in these environments may produce non-connectedness and therefore violate the CMH. In conjunctions or in the scope of universals however, connectedness is preserved ((51b), (52b)), and this guarantees that *exh* will not by itself create non-connectedness, that is, it should comply with the CMH. Although these concern results about our idealized fragment, they correspond neatly with the corresponding generalizations from Table 1 about negation ((8)), disjunctions ((9), (10)), and existential environments ((11)).

Certain other examples in Table 1 are not immediately covered by this fragment. For instance, we did not include non-monotonic quantifiers, as in (6a) and (6b). This is essentially for the sake of brevity, as they could be treated in a similar fashion. Doing so would lead to the conclusion that *exh* may lead to non-connectedness when embedded in a non-monotonic environment (see (6a)). In other words, non-monotonic environments should pattern just like negation or existential quantification, which is certainly the one least satisfying prediction of the CMH.

The case of Free Choice inferences, in its traditional *exh* treatment, also escapes from the fragment above. To fully treat these cases, the semantics of *exh* would have to be made more precise and sophisticated (cf. for instance Fox 2007, and discussion in Spector 2016). That being said, let us simply accept that the resulting reading with a free choice inference is monotonic ($\Diamond A \wedge \Diamond B$ is monotonic, certainly if we consider the modal $\Diamond$ as an existential quantifier). Then the reason why it can be embedded under negation without creating non-connectedness is that it is a case where $exh\,\phi$ is not only connected, but also monotonic. Connected sentences may yield non-connected meanings under negation (Result (50b)), but monotonic ones may not (Result (50a)).

Hence, the current paradigm explains why certain occurrences of *exh* operators can appear under negation, and others may not, independently of the fact that the resulting reading is weaker than the reading obtained without *exh*. Overall, the generalizations from Table 1 correspond to the formal properties of connectedness, with the sole exception of the case of non-monotonic quantifiers.

[14] In particular, if we order situations based on what set of propositions from a certain class they make true, and if among the propositions that we consider, there are all statements of the kind $P(x)$ for an individual x, then the predicate P will always be monotonic. This is always the case in examples that we consider.

5 Conclusions

The need for a restriction on the distribution of embedded exhaustification is a well-known problem of the grammatical theory of implicatures. We have formalized the Connectedness Meaning Hypothesis. This principle relies on the premise that non-connectedness create interpretive difficulties. The principle then proposes that parses with *exh* which break connectedness are deviant. This proposal differs from previous approaches (referred to as the SMH or the EC) in two ways: first, these previous approaches rely on monotonicity and logical strengths considerations and, second, they rule out a given parse based on how it compares to another potential parse.

On the empirical side, the current proposal captures essentially the same generalizations as its predecessors, which is a good feature. We exhibited however a case where the proposals differ: previous accounts cannot tolerate the presence of *exh* under negation, while the CMH allows it when *exh* creates free choice inferences, and this seems to be an accurate result.

Constraints based on monotonicity and logical strength are natural in formal semantics, and the move to connectedness considerations may sound unwarranted, if not suspicious. It is therefore worth pointing out that connectedness can be well-thought about as a weak form of monotonicity. Importantly, in other domains, connectedness based filters had been proposed as natural, cognitive tendencies, and they have been argued to be capable to explain restrictions in the lexicons of content words (Gärdenfors 2004) and more recently in the lexicons of logical words (Chemla, Buccola, and Dautriche in press). Connectedness filters are therefore natural constraints to explore, and we here show how they could have applications at the level of plain propositional meanings.

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A Detailed treatment of some of the examples

In this section we go over some of the examples of Section 2.3 in more detail, showing why certain meanings are or aren't connected under the definition of connectedness in Section 4.2.

A.1 Example: basic negation

We begin by the basic example (56), repeated from (22).

(56) Sentence: It's not true that John did some of his homework.
Unattested parse: *It's not true that John did *exh* [some of his homework].
Intended meaning: It isn't true that John did some but not all of his homework.

Making the simplifying assumptions that John's homework consists in two exercises, that John never does part of an exercise without finishing it, and that there are no other relevant things John could have done (or that none of these are crucial), we suppose that the appropriate order on situations is the one in Figure 2, where the situations are as in (57). This could be derived in the way described in Section 4.2 if one takes the QUD to be something like “What did John do?”.

(57) $w_\varnothing$: John did neither exercise 1 nor exercise 2.
w_1 : John did exercise 1, but not exercise 2.
w_2 : John did exercise 2, but not exercise 1.
$w_{1,2}$: John did both exercise 1 and exercise 2.

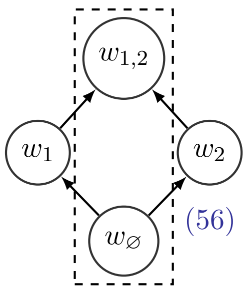

Figure 2: Representation of the order over the situations in (57) for a sentence such as (56). The meaning of (56) is represented with the dotted box (it contains all situations that make the sentence true).

Figure 2 also indicates as a box the meaning of (56): one can see that this meaning is non-connected because the chain $w_\varnothing \to w_1 \to w_{1,2}$ has its extremities in the set and its in-between element out of it (see also Figure 1). On the other hand, the meaning one would obtain by parsing the sentence without *exh* only contains $w_\varnothing$ and is therefore trivially connected.

A.2 Example: double disjunction

We now move on to (58), repeated here from (24). We take the order on worlds to be the one in Figure 3, which could derive from a question like "Who will John meet?". The labels of the situations correspond to the set of people that John meets. For the sake of simplicity we ignore people other than Ann, Bill and Carol (which does not affect the result).

(58) John will meet Ann, Bill or Carol.
  a. Attested parse: $exh[A \vee B \vee C]$
  Meaning: John will meet exactly one person within Ann, Bill and Carol.
  b. Unattested parse: $*exh[A \vee exh[B \vee C]]$.
  Intended meaning: John will meet exactly one person within Ann, Bill and Carol, or all of them.

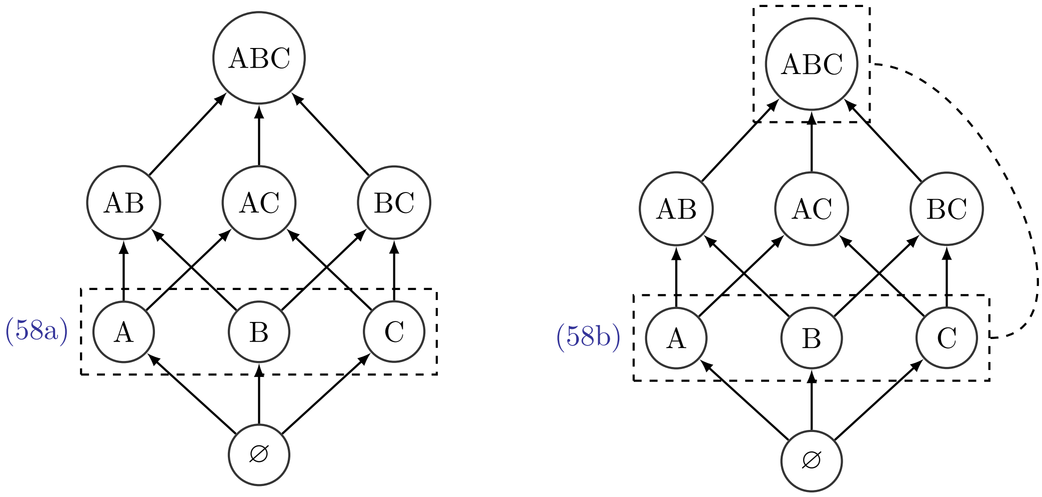

Figure 3: Order on situations for (58) and meanings of parses of interest.

It can be seen from Figure 3 that (58a) is connected, as there is no non-trivial chain with its extremities in the truth box. On the other hand, for (58b), a number of chains have extremal points in the truth box and have in-between points outside of it($A \to AB \to ABC$ is an example). Thus (58b) is not connected.

A.3 Example: existential sentences

As a third example, we consider (59), repeated from (26a). Recall that it could have all the interpretations in (60). Assuming that there are 2 employees A and B and 2 criminals 1 and 2, we take it that the appropriate order on situations is that of Figure 4, which could for instance derive from a QUD like "Which criminals did each employee see?". The labels of situations correspond to the extension of "see" (for example 'A1,A2,B1' corresponds to a situation in which Employee A saw Criminal 1 and Criminal 2, while Employee B only saw Criminal 2). One may check on Figure 4 that (60c) is non-connected, while (60b) is (the other two cases are omitted for the sake of clarity).

(59) a. There are employees who saw some of the criminals.
  b. $\exists e, \exists c, S(e, c)$

(60) a. $*\ \exists_1 e, exh_2\, \exists_2 c, S(e, c)$
  Some employees saw some but not all criminals.

b. $exh_{1,2}\ \exists_1 e, \exists_2 c, S(e,c)$
Some but not all employees saw some criminals, and no employee saw all criminals.

c. * $exh_1\ \exists_1 e, exh_2\ \exists_2 c, S(e,c)$
Some but not all employees saw some but not all criminals.

d. $exh_2\ \exists_1 e, \exists_2 c, S(e,c)$
Some employees saw some criminals, but no employee saw all criminals.

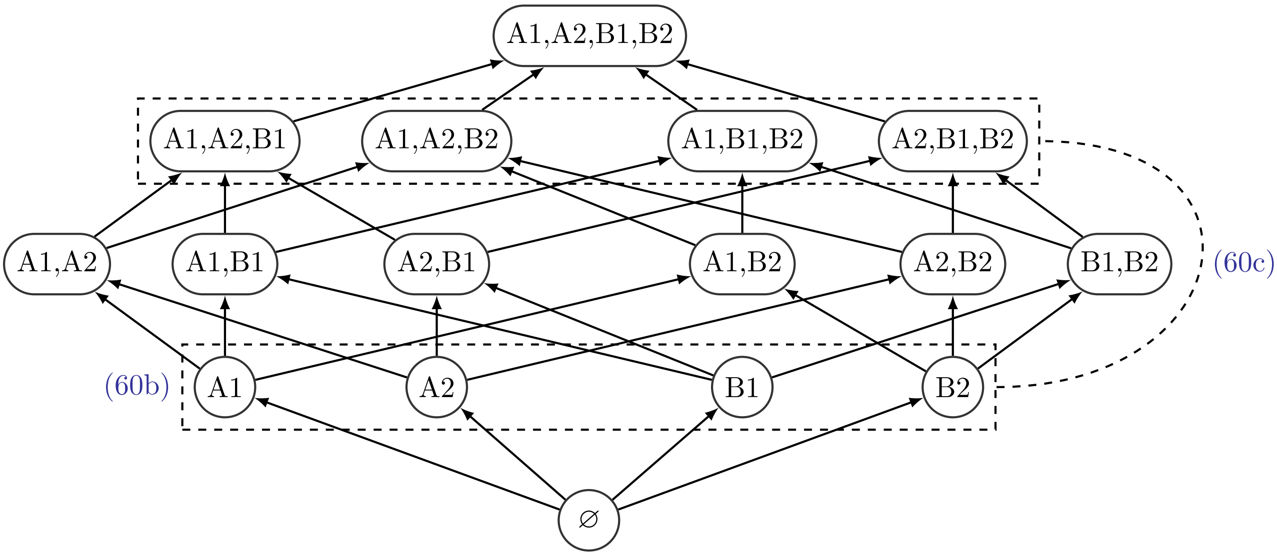

Figure 4: Order on situations and meanings for (59).

A.4 Other examples

The cases of universal quantification and non-monotonic quantification ((29a) and (31)) are not detailed here for the sake of brevity. We simply just note that we assume the relevant order for them is something similar to the one for (59).

B Other notions of in-betweenness and connectedness

In this section, we discuss three alternatives to our definitions of in-betweenness and connectedness, and show why they seem inappropriate for the purpose at hand. These alternatives seem to be the most natural other choices we could have made, but we make no claim of comprehensiveness.

B.1 In-betweenness based on minimal paths

A natural definition of in-betweenness in metric spaces is the following: x is in-between y and z if it is on a minimal-distance path from y to z.[15] This requires a notion of distance then. How can this apply to situations? Let us take the example of the four situations of (56):

[15]With this definition of in-betweenness, what we call connectedness corresponds to what is usually called convexity in mathematics.

(61) $w_\varnothing$: John did neither exercise 1 nor exercise 2.
w_1 : John did exercise 1, but not exercise 2.
w_2 : John did exercise 2, but not exercise 1.
$w_{1,2}$: John did both exercise 1 and exercise 2.

A natural notion of distance can be inferred from a diagram like Figure 5, where the distance between two nodes is given by the minimal number of edges that must be traversed to go from one to the other. Thus, w_1 and w_2 are 2 units apart; both paths joining them are minimal, so that both $w_\varnothing$ and $w_{1,2}$ are in-between them. As a result, even the most basic sentence, "John did some of his homework", is non-connected with or without its exhaustified meaning, because it is true in both w_1 and w_2, but not in $w_\varnothing$. Hence, this notion of connectedness surely is too stringent for our current purposes.[16]

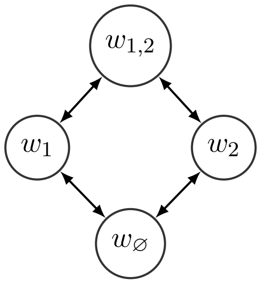

Figure 5: A possible way to map the possible worlds in (61) into a feature space with a notion of distance.

B.2 Standard notion of connectedness on undirected graphs

An alternative approach also not based on an order, would be to take the same graph structure as from B.1, and define connectedness as such: if there are two worlds where the proposition is true, there must be *some* path from one to the other throughout which the proposition stays true. Here the notion of in-betweenness does not play a direct role. We note that this is closer to some usual definitions of connectedness in analysis.

This notion is less stringent than the one above in B.1. Under this view, the non-exhaustified (and in fact monotonic) reading of "John did some of his homework" would be connected, but its exhaustified meaning, again, would not be. In fact, Result (55) would not hold, and none of the usual logical operations (conjunction, disjunction, etc.) preserves this notion of connectedness. Hence, again, this notion cannot plausibly constrain the distribution of *exh*.

B.3 Weak order-based connectedness

We may adopt our order-based definition of in-betweenness, but also a weaker notion of connectedness: instead of requiring that every world in-between two true worlds make the proposition true, we could require that *at least some* of them do. This notion is strictly weaker than ours. It has some merits: it doesn't change anything for basic examples such as (56), and it makes Result (55) (about how *exh* preserves monotonicity and connectedness) true. We haven't fully investigated whether this notion's empirical predictions would be better or worse than those of

[16]Adding an edge between w_1 and w_2 may solve the problem, but other issues would still arise in more complicated cases. For instance, one may verify that (27d) would not be connected in this sense. More generally, this notion does not seem to make a result like (55) true; i.e., it is not guaranteed that global exhaustification of monotonic components will be connected.

the definition we adopted; in general, as it is a weaker constraint, it ought to ban fewer parses, and our cursory look suggests it is too lax: for instance, it doesn't ban (62).

(62) Parse: *John didn't *exh* [do exercise 2.]
Intended meaning: it isn't the case that exercise 2 is the exact extent of what John did.