Thinking alone and thinking together*

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Abstract The central observation of this paper is that belief ascriptions with plural subjects can be interpreted non-distributively, so that beliefs can be truthfully attributed to a plurality that cannot be attributed to any of the individuals that it comprises. Moreover, the beliefs of a plurality appear to be predictable from the beliefs of its associated individuals. Two potential analyses are offered for the relationship between the beliefs of individuals and those of pluralities. Both of these analyses, which are meant to negotiate the agreements and disagreements between individual experiencers, run into issues differentiating between relevant and irrelevant disagreement. To resolve these issues I invoke a notion of “aboutness”, which filters out contextually irrelevant beliefs.

Keywords: attitudes, belief ascriptions, plurals, event semantics, mereology, aboutness

1 Introduction

Attitude ascriptions with plural subjects are typically interpreted distributively. For example, consider the sentences in (1):

(1) a. Alexis and Brian think that Cass left.
b. Alexis and Brian want Cass to leave.
c. Alexis and Brian wish that Cass had left.
d. Alexis and Brian regret that Cass didn’t leave.

On a default interpretation of (1a), there is an entailment that Alexis believes that Cass left, and that Brian does, too. Similar inferences hold for want, wish, and regret

* Much (though not all) of the material in this paper originates from work in my Stony Brook University dissertation (Pasternak 2018). Thus, all of the acknowledgments stated there extend to this work. Special thanks to my dissertation committee—Richard Larson, Thomas Graf, Jiwon Yun, and Angelika Kratzer—as well as to Viola Schmitt, with whom I discussed this material in detail. Many thanks also to Paola Cepeda, Daniel Finer, So Young Lee, Lei Liu, Nazila Shafiei, Ildikó Emese Szabó, Hongchen Wu, and audiences at SALT 28 and the Leibniz-Center for General Linguistics (ZAS) for their helpful questions and comments.

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in (1b), (1c), and (1d), respectively. This is of course in contrast to non-distributive interpretations of plurals as in (2), which entails neither (3a) nor (3b):

(2) Alexis and Brian ate the whole pie.
(3) a. Alexis ate the whole pie.
   b. Brian ate the whole pie.

This paper is dedicated to demonstrating the possibility of, and subsequently analyzing, non-distributive attitude ascriptions, and more specifically, non-distributive belief ascriptions. In Section 2 I set forth the data to be accounted for, a variety of context-sentence pairs in which beliefs are truthfully attributed to pluralities that cannot be attributed to the individuals that those pluralities comprise. Once the possibility of non-distributive belief ascription is established, the problem then becomes one of elucidating the relationship between the beliefs of pluralities and the beliefs of their corresponding individuals, as well as the effects this relationship has on semantic interpretation. The rest of the paper thus straddles the line between semantics proper—that is, concern over the denotations of lexical items and their semantic composition—and what Bach (1986) famously refers to as natural language metaphysics, i.e., the study of the model used for semantic interpretation.

In Section 3 I offer two analyses of the data presented in Section 2, abstaining from any decision as to which is the superior proposal. The first theory puts the bulk of the work in the natural language ontology, positing that the beliefs of pluralities are related to the beliefs of individuals in a manner formally similar to premise-semantic approaches to modals and conditionals along the lines notably discussed by Kratzer (1977, 1981, 1991, 2012) and Lewis (1981). The second theory opts for a simpler metaphysics by transferring a certain amount of the first theory’s work to the semantic interpretation of plurals.

I show in Section 4 that there is a problem for both of these analyses: while each is successful at negotiating relevant (dis)agreements when going from individual to plural experiencers, neither allows for the important distinction between relevant and irrelevant disagreement between individuals. I resolve this issue by invoking a notion of “aboutness”: beliefs are about situations, and it is by means of pragmatically fixing the “about-situations” to which we make reference that we filter out those beliefs that are contextually irrelevant. Section 5 offers some concluding remarks, as well as areas for future inquiry.

2 A tour through the data

Our first example of a non-distributive belief ascription can be seen in (4), which is interpretable as true in the context provided:
Sam owns a construction company and has six clients, none of whom know of the others’ existence. She has convinced each client that she would build a house for him. In reality, she is a con artist and built no houses at all.

(4) (In total,) Sam’s six clients think she built six houses for them.

Given the context, it is clear that no individual client thinks that Sam built six houses, meaning that the interpretation that makes (4) true cannot be a distributive one. Moreover, (4) cannot be accounted for via a de re construal of six houses in conjunction with some mereological wizardry in the matrix clause, as could conceivably be done for a sentence like (5).

(5) There are six houses that Sam’s six clients think she built for them.

After all, the nature of Sam’s dishonesty is such that there are no six houses to serve as witnesses for such a de re construal, hence why (5) is false in spite of (4) being true. It seems, then, that the best analysis of (4) is simply that it serves as an account of the cumulative beliefs of six individuals: each client thinks he got a house, so their conjoined belief is that they got six houses.

As another example, consider (6):

Paul just got married, and his cousins Arnie and Beatrice, who have never met, just caught wind of it. Arnie suspects that Paul’s husband is rich, and has no other relevant opinions. Beatrice thinks he’s a New Yorker, and has no other relevant opinions.

(6) Paul’s cousins think he married a rich New Yorker.

A distributive reading of (6) is once again false, since neither Arnie nor Beatrice believes that Paul’s husband is a rich New Yorker. What’s more, in this case there are no obvious alternative mereological tricks available, since there is nothing in the embedded clause for any mereological operator to actually operate on. It thus appears that the best explanation is the simplest one: we conjoin the beliefs of the individuals to get the beliefs of the plurality, so that Arnie’s belief that Paul’s husband is rich is conjoined with Beatrice’s belief that he is a New Yorker, leading to a belief of Arnie-plus-Beatrice that he is a rich New Yorker.

Notice that in the cases considered thus far, the individual experiencers’ relevant beliefs have been mutually compatible. In (4), each client is agnostic about the existence of other clients, so that Client 1’s beliefs are compatible with, but don’t entail the existence of a house built for Client 2 (for example). Similarly, while Arnie’s beliefs in (6) don’t entail that Paul married a New Yorker, they don’t contradict it either, and likewise for Beatrice and Paul’s husband’s wealth. The result seems to be that the beliefs of the plurality are a conjunction of the beliefs of the individuals: six house-beliefs are conjoined to make one six-house belief, and a wealth belief and a New Yorker belief are conjoined to make a rich New Yorker belief.
But what happens when the beliefs of the individual experiencers don’t play together so nicely? In the context below, which is a revision of the original wedding scenario, Arnie and Beatrice’s beliefs are mutually contradictory. This context renders the original (6) false, while the disjunctive sentence in (7) is true:

\[ \text{Arnie thinks that Paul married a rich Marylander, while Beatrice thinks he married a poor New Yorker.} \]

(7) Paul’s cousins think he married either a rich Marylander or a poor New Yorker.

From this one might be tempted to conclude that while mutual compatibility leads to conjunction of beliefs (rich + New Yorker = rich and New Yorker), incompatibility leads to disjunction of beliefs (rich Marylander + poor New Yorker = rich Marylander or poor New Yorker). But such a conclusion is not yet warranted: the sentence in (7) is also true on a distributive reading, since each cousin believes the weak proposition denoted by the clausal complement in (7). This in turn means that the truth of (7) cannot be used to bolster a particular view about the relationship between the beliefs of individuals and the beliefs of pluralities, since there is a reading of (7) that is true regardless of what that relationship is.

However, with some slight tweaking we can devise a context-sentence pair such that a distributive reading is false, but the prior observation about conjunction vs. disjunction is still observed. Consider the sentence (8) in the context provided:

\[ \text{Paul has three cousins, Arnie, Beatrice, and Kate. Arnie and Beatrice’s beliefs are as in the original wedding scenario: rich and New Yorker, respectively, and otherwise agnostic. Kate, like Beatrice, is unopinionated about Paul’s husband’s wealth, but she thinks he’s from Iowa, not New York.} \]

(8) Paul’s cousins think he married a rich man from either New York or Iowa.

A distributive reading of (8) is false: Arnie is not committed to Paul’s husband’s being from New York or Iowa, while Beatrice and Kate are not committed to Paul’s husband’s being rich. Nonetheless, (8) is true. Moreover, we see the informally stated generalization in action: where there is incompatibility, there is disjunction (New York or Iowa), and where there is compatibility there is conjunction (rich and [New York or Iowa]).

Now that the evidence for non-distributive belief ascriptions is on the table, in the next section we will explore how semantics and ontology might work together to produce the judgments discussed in this section. I will offer two possible analyses, each of which derives our informal observation that compatibility leads to conjunction, and incompatibility to disjunction.
3 Two analyses

3.1 Basic assumptions

Before introducing the two analyses, let’s go over some basic assumptions. I assume a theory of attitude semantics that is both (i) (neo-)Davidsonian, in that the denotations of sentences involve an existentially quantified-over variable over eventualities introduced by the verb \( \text{Davidson} \ 1967 \); and (ii) Hintikkan, in that the denotations of attitudes like \text{believe} (and \text{think}) universally quantify over accessible worlds \( \text{Hintikka} \ 1969 \). I also follow \text{Kratzer} \ 1996 in treating the external argument as “severed” from the verb and introduced separately by a voice head, though this assumption is not crucial to the theory espoused here. As a result of these assumptions, the denotation of \text{believe} (and \text{think}) will be as in (9), where Dox\((e) \) is the set of doxastically accessible worlds; this definition will undergo some minor revisions in Section 4. The ensuing denotation for a simple belief ascription can be seen in (10), where Exp\((e) \) is the experincer of \( e \), and \( a \) is Arnie.

\[
\text{believe}_{\text{take } 1}^c = \lambda \, p \lambda \, e. \ \forall w \in \text{Dox}(e)[p(w)]
\]

(9) \[
\text{Arnie believes that Paul married a rich man}^c = 1 \iff \\
\exists e[\text{Exp}(e) = a \land \forall w \in \text{Dox}(e)[\text{rich-man}(w)]]
\]

(10) Temporarily adopting the simplifying view that each individual has only one belief state, suppose that Arnie’s belief state is \( e_a \), and Beatrice’s is \( e_b \). Thus, if a sentence like (10) is true, it is because all of the worlds in Dox\((e_a) \) are rich man worlds, meaning that \( e_a \) satisfies the conditions in the scope of the existential quantifier over events. With this in mind, a question we must address in making the leap to non-distributive belief ascriptions is what the belief state is of \( a \sqcup b \), the plurality whose members are Arnie and Beatrice. While a variety of possibilities are on a technical level compatible with my own proposal, I will adopt the seemingly plausible view that it is \( e_a \sqcup e_b \), the sum of Arnie and Beatrice’s belief states. So for a non-distributive belief ascription like (11), the witness to the existential quantification over eventualities will be \( e_a \sqcup e_b \) (if the sentence is true).

\[
\text{Paul’s cousins believe that } p^c = 1 \iff \\
\exists e[\text{Exp}(e) = a \sqcup b \land \forall w \in \text{Dox}(e)[p(w)]]
\]

(11) With our basic assumptions now in place, we can state the problem at hand in more formal terms. The original framing of the wedding scenario provided information only about Arnie and Beatrice’s individual beliefs. That is, we were only given information pertaining to Dox\((e_a) \) and Dox\((e_b) \). But this information was somehow sufficient to determine the relevant properties of Dox\((e_a \sqcup e_b) \) that made (6) true, meaning that there must be some predictable relationship between Dox\((e_a) \), Dox\((e_b) \), and Dox\((e_a \sqcup e_b) \). More generally, given a plurality-experienced belief
state $e$, there is some function $J$ that makes (12) true, where $\text{Atm}(e)$ is the set of atomic (i.e., non-plural) belief states that make up $e$:

\begin{equation}
\text{Dox}(e) = J(\{\text{Dox}(e') \mid e' \in \text{Atm}(e)\})
\end{equation}

We now turn to the task of defining $J$.

3.2 Analysis I: Premise negotiation

For those acquainted with Kratzer’s (1977, 1981, 1991, 2012) work on modality, the facts discussed in the previous section might have a ring of familiarity to them. This is because the types of inferences that arose in Section 2 are of the same sort that motivated Kratzer’s well-known premise-semantic approach to modals. While a full discussion of Kratzer’s theory and the evidence she marshals for it is beyond the scope of this paper, a brief overview of a particular aspect of her theory will help to highlight the parallels with our own data.

Suppose we have a set $Q$ of propositions, where each proposition corresponds to some obligation that our friend Linda is expected to fulfill. These obligations can be whatever we like; we can even include in $Q$ propositions that are logically inconsistent with each other. Given this set of tasks, one might reasonably ask what precisely it means for Linda to have maximally fulfilled her obligations, i.e., what the set $A$ of worlds is in which she has done as much as possible. If the obligations are mutually compatible, then this is an easy question: for Linda to maximally fulfill her obligations is for her to satisfy all of them. For example, if Linda is picking a card and her obligations are to pick a diamond and to pick a queen, then the set of worlds in which she maxes out on obligation fulfillment consists of those worlds in which she picks the queen of diamonds. But things get trickier in cases where $Q$ is inconsistent. Here Linda is forced to make the best of a bad situation, since she cannot fulfill all of the obligations in $Q$. Say for instance that in addition to the two previous obligations of picking a diamond and picking a queen, we also add a requirement that the card Linda picks be a king. Since she is picking only one card, she cannot fulfill all three requirements, as that card cannot be both a queen and a king. The best she can do is to satisfy two of the three requirements by picking either the queen of diamonds or the king of diamonds.

There are many conceivable ways in which one could formalize this idea of making the best of a bad situation. The most common way, following the work of Lewis (1981) and Kratzer (1981), is to induce an ordering over worlds as in (13), where $w_1 \preceq_Q w_2$ iff $w_1$ is at least as ideal as $w_2$ with respect to $Q$:

\begin{equation}
w_1 \preceq_Q w_2 \text{ iff } \{p \in Q \mid p(w_1)\} \supseteq \{p \in Q \mid p(w_2)\}
\end{equation}

Put simply, $w_1 \preceq_Q w_2$ iff every proposition in $Q$ that is true in $w_2$ is also true in $w_1$. 
Thus, $w_1$ will be at least as ideal as $w_2$ with respect to Linda’s obligation fulfillment if each task that is accomplished in $w_2$ is also accomplished in $w_1$.

As a toy example, say that $Q = \{ p, q_1, q_2 \}$, where $p$ is consistent with each of $q_1$ and $q_2$ (e.g., $p \cap q_1 \neq \emptyset$), but $q_1$ and $q_2$ are mutually incompatible ($q_1 \cap q_2 = \emptyset$). The world-ordering $\preceq_Q$ for $Q$ will then be as in Figure 1, where worlds are divided into equivalence classes based on which propositions in $Q$ hold in them. In this case, the two clusterings of ideal worlds are those in which $p$ and $q_1$ both hold, and those in which $p$ and $q_2$ both hold. Thus, the set of worlds in which obligations are maximally satisfied is $(p \cap q_1) \cup (p \cap q_2)$, or equivalently, $p \cap (q_1 \cup q_2)$. Going back to our example, if $p$ is the set of worlds in which Linda chooses a diamond, $q_1$ those in which she chooses a queen, and $q_2$ those in which she chooses a king, then we predict that those worlds in which Linda maximally fulfills her obligations are those in which she picks the queen or king of diamonds.

A world-ordering algorithm such as (13) determines a particular relationship between the propositions in $Q$ and the set of worlds deemed ideal. As it turns out, this relationship bears a strong prima facie resemblance to the relationship between the beliefs of individuals and the beliefs of pluralities as discussed in the previous section. In our toy premise-semantic example, $q_1$ and $q_2$ were incompatible with each other, but compatible with $p$. As a result, the ideal worlds consisted of the intersection of $p$ with the union of $q_1$ and $q_2$, i.e., $p \cap (q_1 \cup q_2)$. Meanwhile, in (8), Beatrice and Kate’s beliefs about Paul’s husband’s hometown were incompatible with each other, but compatible with Arnie’s belief about Paul’s husband’s wealth. As a result, the plurality’s belief was the conjunction of Arnie’s belief with the disjunction of Beatrice and Kate’s beliefs (rich and [New York or Iowa]).

With this parallelism in mind, our first analysis uses the algorithm in (13) to define the function $J$ in (12). This is formally stated in (14):

(14) **Belief summing, Analysis I:**

$$\text{Dox}(e) = \text{BEST}(\sim \{\text{Dox}(e') \mid e' \in \text{Atm}(e)\})$$

(where $\text{BEST}(\preceq) \equiv \{ w \mid \exists w' [w' \prec w] \}$)

Notice that strictly speaking, (14) is only an indirect reflection of the parallel between premise semantics and belief negotiation as discussed in the previous para-
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graph. When this parallel was discussed above, it was as if the believed propositions were serving as the premises, with the ideal worlds being those worlds that best accorded with those beliefs. But that’s not what’s happening in (14): because we are operating within a Hintikkan framework, the propositions serving as premises are actually sets of belief worlds, rather than individually believed propositions. Nonetheless, we will see that the same results are generated, though as discussed in Section 4 some minor tweaks will eventually be needed.

Let’s go through each of the examples in Section 2 and see how our proposal generates the right results. Starting with (4), let $k_1$ through $k_6$ be Sam’s six clients, and let $e_1$ through $e_6$ be their corresponding belief states. I will use $k_1 \sqcup k_2 \sqcup \ldots \sqcup k_6$ and likewise for $e_1 \sqcup e_2 \sqcup \ldots \sqcup e_6$. In this case, (4) will have the denotation in (15):

$$[(4)]^c = 1 \text{ iff } \exists e [ \text{Exp}(e) = k_1 \sqcup \ldots \sqcup k_6 \wedge \forall w \in \text{Dox}(e)[\text{six-houses}(w)]]$$

As per the discussion above, $k_1 \sqcup \ldots \sqcup k_6$’s belief state will be $e_1 \sqcup \ldots \sqcup e_6$. In order to determine whether (4) is true, then, we must find out what $\text{Dox}(e_1 \sqcup \ldots \sqcup e_6)$ is. By (14) this should be the set of ideal worlds as determined by the premise set $\{\text{Dox}(e) \mid e \in \text{Atm}(e_1 \sqcup \ldots \sqcup e_6)\}$, i.e., $\{\text{Dox}(e_i) \mid 1 \leq i \leq 6\}$. Importantly, the context for (4) dictates that each client is agnostic about whether anyone else gets a house, so while every world in $\text{Dox}(e_i)$ is such that $k_i$ gets a house, for every other client $k_j$ there will be some worlds in $\text{Dox}(e_i)$ in which they get a house, and some worlds in which they don’t. Because of this general agnosticism, there are some worlds that are compatible with the beliefs of all six clients: namely, those worlds in which all six have a house built for them. It is easily proven that given a premise set $Q$, if $\bigcap Q$ is non-empty (meaning the premises are all mutually compatible), then the set of ideal worlds as determined by $\preceq Q$ will be $\bigcap Q$. Since the belief worlds of the six clients are mutually compatible, the ideal worlds in this case will simply be $\bigcap\{\text{Dox}(e_i) \mid 1 \leq i \leq 6\}$. Because these ideal worlds—the worlds compatible with the beliefs of all six clients—are all worlds in which all six clients get a house from Sam, and this set of worlds has been determined to be $\text{Dox}(e_1 \sqcup \ldots \sqcup e_6)$, we rightly predict (4) to be true, since $e_1 \sqcup \ldots \sqcup e_6$ is a successful witness to the existential quantification over events seen in (15).

For (6), we get essentially the same result, for essentially the same reason. We predict (6) to have the denotation in (16):

$$[(6)]^c = 1 \text{ iff } \exists e [ \text{Exp}(e) = a \sqcup b \wedge \forall w \in \text{Dox}(e)[\text{rich-NYer}(w)]]$$

Much like in the house-building scenario, Arnie and Beatrice are agnostic in just the right ways to allow conjunction of their beliefs. While all worlds in $\text{Dox}(e_a)$ (Arnie’s belief worlds) are worlds in which Paul’s husband is wealthy, because of Arnie’s agnosticism about the husband’s hometown there will be some worlds in which Paul’s husband is from New York, and some worlds in which he is not. Similarly, there will be some worlds in $\text{Dox}(e_b)$ in which Paul’s husband is rich, and
some in which he is not, but all of the worlds will be worlds in which Paul’s husband is a New Yorker. Thus, \( \text{Dox}(e_a) \) and \( \text{Dox}(e_b) \) will have a non-empty intersection, meaning that \( \text{Dox}(e_a \sqcup e_b) \), which is the set of ideal worlds given the premise set \{ \text{Dox}(e_a), \text{Dox}(e_b) \}, will be \( \text{Dox}(e_a) \cap \text{Dox}(e_b) \). Since all \( \text{Dox}(e_a) \) worlds are rich man worlds, and all \( \text{Dox}(e_b) \) worlds are New Yorker worlds, all \( \text{Dox}(e_a \sqcup e_b) \) worlds are rich New Yorker worlds, meaning that \( e_a \sqcup e_b \) serves as a witness to the event-quantification in (16).

Next up is (7), our first case of incompatibility. Our denotation is in (17):

\[
(17) \ [ (7) ]^c = 1 \text{ iff } \exists e \left[ \text{Exp}(e) = a \sqcup b \land \
 \forall w \in \text{Dox}(e) [\text{rich-MDer}(w) \lor \text{poor-NYer}(w)] \right]
\]

As was the case for (16), \( e_a \sqcup e_b \) will be the witness to our quantification over events. Once again, \( \text{Dox}(e_a \sqcup e_b) \) will be the set of ideal worlds as determined by the premise set \{ \text{Dox}(e_a), \text{Dox}(e_b) \}. But for the new context, \( \text{Dox}(e_a) \) and \( \text{Dox}(e_b) \) are incompatible: all of the worlds in \( \text{Dox}(e_a) \) are worlds in which Paul’s husband is a rich Marylander, and all of the worlds in \( \text{Dox}(e_b) \) are worlds in which he is a poor New Yorker. Since Paul’s husband cannot be both a rich Marylander and a poor New Yorker, \( \text{Dox}(e_a) \cap \text{Dox}(e_b) = \emptyset \). As a result of this incompatibility, the world-ordering for premise set \{ \text{Dox}(e_a), \text{Dox}(e_b) \} will be as in Figure 2, where worlds are once again lumped into equivalence classes according to which propositions in \{ \text{Dox}(e_a), \text{Dox}(e_b) \} are true in them.

![Figure 2](image-url)

Figure 2  World-ordering in the rich Marylander/poor New Yorker scenario

Given this ordering, the set of ideal worlds—and thus \( \text{Dox}(e_a \sqcup e_b) \)—will be \( \text{Dox}(e_a) \cup \text{Dox}(e_b) \). Since all of the worlds in \( \text{Dox}(e_a) \) are rich Marylander worlds, while all of the worlds in \( \text{Dox}(e_b) \) are poor New Yorker worlds, this entails that all of the worlds in \( \text{Dox}(e_a \sqcup e_b) \) are either rich Marylander worlds or poor New Yorker worlds, meaning that we rightly predict (7) to be true. Moreover, we predict (6) to be false here: not only are not all worlds in \( \text{Dox}(e_a \sqcup e_b) \) rich New Yorker worlds, but none of them are.

Finally, we have the most complex case, namely (8). Since this scenario involves three cousins, we predict (8) to have the denotation in (18):

\[
(18) \ [ (8) ]^c = 1 \text{ iff } \exists e \left[ \text{Exp}(e) = a \cup b \cup k \land \
 \forall w \in \text{Dox}(e) [\text{rich}(w) \land (\text{NYer}(w) \lor \text{Iowan}(w))] \right]
\]
In order to know whether (8) is true, we must determine what \( \text{Dox}(e_a \sqcup e_b \sqcup e_k) \) is. Because Arnie is agnostic about Paul’s husband’s hometown, and Beatrice and Kate are both agnostic about Paul’s husband’s wealth, \( \text{Dox}(e_a) \) shares some worlds in common with each of \( \text{Dox}(e_b) \) and \( \text{Dox}(e_k) \) (rich New Yorker worlds and rich Iowan worlds, respectively). However, there are no worlds shared between \( \text{Dox}(e_b) \) and \( \text{Dox}(e_k) \), since Paul’s husband cannot be from both New York and Iowa. As a result, the world-ordering given the premise set \{\text{Dox}(e_a), \text{Dox}(e_b), \text{Dox}(e_k)\} \) will be as in Figure 3, very much parallel to the toy example given in Figure 1.

\[
\begin{array}{ccc}
\{\text{Dox}(e_a), \text{Dox}(e_b)\} & \{\text{Dox}(e_a), \text{Dox}(e_k)\} \\
\{\text{Dox}(e_b)\} & \{\text{Dox}(e_a)\} & \{\text{Dox}(e_k)\} \\
\emptyset & & \\
\end{array}
\]

**Figure 3** World-ordering in the three-cousin scenario

As can be seen in Figure 3, we now have two clusters of ideal worlds: those worlds compatible with both Arnie and Beatrice’s beliefs (\( \text{Dox}(e_a) \cap \text{Dox}(e_b) \)), and those compatible with both Arnie and Kate’s beliefs (\( \text{Dox}(e_a) \cap \text{Dox}(e_k) \)). Since the former are all rich New Yorker worlds, while the latter are all rich Iowan worlds, all of the ideal worlds are worlds in which Paul’s husband is a rich man from either New York or Iowa. Since these ideal worlds make up \( \text{Dox}(e_a \sqcup e_b \sqcup e_k) \), \( e_a \sqcup e_b \sqcup e_k \) successfully serves as a witness to the event-quantification in (18), meaning that we rightly predict (8) to be true.

In short, then, we see that adopting Lewis-Kratzer premise negotiation as a way of relating the belief worlds of individuals and pluralities generates the right results for the examples discussed in the previous section. Next we will look at an alternative analysis with a somewhat simpler ontology, as well as a somewhat more complex view of the semantics of plurals.

### 3.3 Analysis II: Beliefs under cover

Suppose we adopt a simpler principle of belief summing: \( \text{Dox}(e_a \sqcup e_b) \) is the intersection of \( \text{Dox}(e_a) \) and \( \text{Dox}(e_b) \) if this intersection is non-empty, and is otherwise undefined. This is stated for the more general case in (19):

\[(19) \quad \text{Belief summing, Analysis II: } \]

\[\text{Dox}(e) \text{ is defined iff } \bigcap \{\text{Dox}(e') \mid e' \in \text{Atm}(e)\} \neq \emptyset.\]

Where defined, \( \text{Dox}(e) = \bigcap \{\text{Dox}(e') \mid e' \in \text{Atm}(e)\}. \]
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For the scenarios in which the beliefs of the individual experiencers were all mutually compatible (namely, (4) and (6)), (19) generates the same results as before, since there the set of ideal worlds was the intersection of the individual experiencers’ belief worlds. Moreover, as discussed in the previous section (7) is true on a distributive reading, meaning that this example is also unproblematic for the simpler ontological principle in (19).

This just leaves the three-cousin case of (8). If we assume a denotation for this sentence along the lines of (18) in conjunction with the principle in (19), we do not predict truth. Since Beatrice and Kate’s beliefs are mutually incompatible, Dox(e_a) and Dox(e_b) have an empty intersection, which in turn entails that Dox(e_a), Dox(e_b), and Dox(e_k) have an empty intersection. But this means that Dox(e_a ∪ e_b ∪ e_k) is undefined, so (18) will not return true. Since (8) is also false on a distributive reading—that is, after all, what made the example interesting to begin with—we are left without an explanation for the truth of (8).

However, we can regain an account of (8) if we adopt a more nuanced view of the semantics of plural definites. More specifically, Gillon (1987, 1990) and Schwarzschild (1996) have shown that distributive readings and “true” cumulative/collective readings constitute two ends of a whole spectrum of possible interpretations. Consider (20), due to Lasersohn (1989):

(20) The TAs were paid exactly $14,000 last year.

Suppose that each of several courses was assigned two TAs, and that precisely $14,000 in combined TA salary was allotted for each course. (20) can be true here, in spite of being false on both a distributive reading (the TAs did not get $14,000 apiece) and a cumulative reading (the TAs got more than $14,000 total).

We can account for the truth of (20) by generalizing the notion of distributivity: rather than always restricting ourselves to quantifying over the atomic individuals that make up a plurality, as in the case of true distributivity, we can also quantify over plural individuals whose sum is the plurality. That is, if x is the large plurality in question (e.g., the sum of all of the TAs), we can quantify over the elements of any contextually determined mereological cover of x, as defined in (21):

(21) A is a mereological cover of x iff ∪A = x.

If Covc(x) is the contextually determined cover of x, then on this view the truth conditions for (20) would be as semi-formally represented in (22):

1 Lasersohn actually uses (20), in a slightly different context, to argue against the sort of analysis espoused by Gillon and Schwarzschild. However, Gillon (1990) offers a convincing rebuttal to Lasersohn, and with our revised context (20) in fact becomes a convenient illustration of the theory.

2 Both Gillon and Schwarzschild adhere to a set-theoretic view of plurality in which pluralities are sets of atomic individuals, in contrast to the Linkian lattice-theoretic framework adopted here (Link 1983). This distinction is immaterial for the analysis at hand.
On the (false) distributive reading of (20), \(\text{Cov}^e(\text{-TAs})\) will be the set containing all and only the individual TAs, so that the sentence will be true iff each of these TAs was paid \$14,000. On the (again false) cumulative reading, \(\text{Cov}^e(\text{-TAs})\) will be a singleton set containing the mereological sum of all of the TAs; (20) will then be true if this plurality earned exactly \$14,000. Finally, on our intermediate reading, \(\text{Cov}^e(\text{-TAs})\) will be a set of pluralities, where each plurality is a pair of TAs who worked together on some course. The result is that (20) is true on this reading, since each of the pairs in this set was paid exactly \$14,000.

Let’s adopt this cover-based analysis, so that belief ascriptions will have a denotation templatically represented in (23):

\[
[\alpha \text{ believe(s) that } p]^c = 1 \text{ iff } \forall x \in \text{Cov}^e(\alpha) \left[ \exists e [\text{Exp}(e) = x \land \forall w \in \text{Dox}(e)[p(w)]] \right]
\]

In those cases where the belief ascription is truly cumulative/collective—(4) and (6)—\(\text{Cov}^e(\alpha)\) will be the singleton set \(\{\alpha\}\), i.e., \(\{k_{1-6}\}\) and \(\{a \sqcup b\}\), respectively. Universally quantifying over this singleton set generates precisely the same truth conditions as we had before covers were introduced, meaning that for these sentences, which were unproblematic for the cover-free analysis, we continue to correctly predict truth. For (7), which is true on a distributive reading, \(\text{Cov}^e(a \sqcup b)\) will be \(\{a,b\}\), meaning that we predict truth iff Arnie and Beatrice each believe the weak proposition denoted by the embedded clause; since this is the case, (7) is true.

Finally, there is (8). Since the sum of the cousins in the three-cousin scenario is \(a \sqcup b \sqcup k\), we need to find out what \(\text{Cov}^e(a \sqcup b \sqcup k)\) is. Suppose that it’s \(\{a \sqcup b, a \sqcup k\}\). Note that it is not a problem that \(a\) appears twice here: \((a \sqcup b) \sqcup (a \sqcup k) = a \sqcup b \sqcup k\), so this set fits the conditions for a mereological cover as defined in (21). Given our new coverful analysis of belief ascriptions, we now predict (8) to be true iff for each (plural) individual in the set \(\{a \sqcup b, a \sqcup k\}\), there is a state of that individual believing that Paul’s husband is a rich man from either New York or Iowa.

For \(a \sqcup b\), the belief state in question will be \(e_a \sqcup e_b\). Since \(\text{Dox}(e_a)\) and \(\text{Dox}(e_b)\) are mutually compatible, by (19) \(\text{Dox}(e_a \sqcup e_b)\) will be their intersection. Because all \(\text{Dox}(e_a \sqcup e_b)\) worlds are worlds in which Paul marries a rich New Yorker, it will also be the case that all \(\text{Dox}(e_k)\) worlds will be worlds in which Paul marries a rich man from either New York or Iowa. We get the same thing for \(a \sqcup k\). Since \(\text{Dox}(e_a)\) and \(\text{Dox}(e_k)\) are mutually compatible, \(\text{Dox}(e_a \sqcup e_k) = \text{Dox}(e_a) \cap \text{Dox}(e_k)\), meaning that all \(\text{Dox}(e_a \sqcup e_k)\) worlds are rich Iowan worlds, in turn entailing that all \(\text{Dox}(e_a \sqcup e_k)\) worlds are rich New Yorker or rich Iowan worlds. Thus each member of the cover has a corresponding state of believing that Paul married a rich man from New York or Iowa, meaning that we rightly predict (8) to be true.
3.4 Summary

In this section, we have seen two ways of accounting for the observation that arose in Section 2. The two theories were in agreement that cases where conjunctive inferences arose were due to intersection of the sets of belief worlds of the individual experiencers. Where the two theories differed was in their analyses of those cases in which disagreements led to the licensing only of weaker, disjunctive inferences: for the first theory, this was a result of the mechanisms of a Lewis-Kratzer premise semantics, while for the second theory it was the result of a cover-based semantics for plural definites. I will not argue for one of these theories over the other in this paper. Instead, I will next turn to a problem faced by both of these theories as currently formulated, namely, the problem of irrelevant disagreement.

4 Irrelevant disagreement and aboutness

4.1 The problem

Consider again the sentence in (6), this time in the revised context below:

_As before, Arnie believes that Paul married a rich man, and Beatrice believes that he married a New Yorker, with no other relevant beliefs. In addition, Arnie mistakenly believes that Mozart was born in 1755, while Beatrice correctly believes him to have been born in 1756._

(6) Paul’s cousins think he married a rich New Yorker.

It seems that (6) remains true in this new scenario, but we currently fail to predict this. The scenario dictates that all worlds in Dox(e_a) are worlds in which Mozart was born in 1755, and those in Dox(e_b) worlds in which he was born in 1756. Assuming nobody thinks Mozart could have been born twice, this means that Dox(e_a) ∩ Dox(e_b) = ∅. Thus, on both of the analyses in the previous section, we predict (6) to be false. Instead, the result should essentially be the same as in the rich Marylander/poor New Yorker scenario in (7), in that the strongest claim we can make should be the weaker (24):

(24) Paul’s cousins think he married someone who is rich or a New Yorker.

The conundrum we face, and the one that I will address in this section, is as follows. As currently formulated, the highly relevant disagreements in (7) and (8) and the completely irrelevant disagreement above have the same status: both entail a non-overlap of belief worlds. But the two have different outcomes. We thus need a way to “filter out” irrelevant disagreements between individual experiencers, while retaining their relevant disagreements.
4.2 Beliefs are about situations

The way in which I will filter out irrelevant disagreements between experiencers will be by positing that beliefs are about situations, where situations are partial possible worlds and possible worlds are maximal situations (cf. Barwise & Perry 1983, Kratzer 2002). On the semantic end of things, the denotation of believe will now include an additional conjunct stating that the belief state is about a contextually determined situation $s^c$, as in (25):

$[\text{believe}]^c_{\lambda e. \text{about}(e) = s^c \land \forall w \in \text{Dox}(e)[p(w)]}$

Importantly, if about($e$) = $s$, then Dox($e$) is the set of worlds compatible with what the experiencer of $e$ believes specifically about $s$. For example, my beliefs about the situation containing my apartment might entail that I left the stove on, but would not entail that my friends are at the Purple Pub. Meanwhile, my beliefs about the situation containing the Purple Pub might entail that my friends are there, but not that I left the stove on. This notion of “aboutness” of beliefs will serve to filter out those beliefs that are contextually determined to be irrelevant, i.e., those that are not about the situation under discussion.

Before showing how such aboutness can be used to account for (ir)relevance of disagreement, one relatively harmless but nonetheless noteworthy stipulation must be put in place. We have assumed—and will continue to assume—that $\text{Exp}(e_1 \sqcup e_2)$ is $\text{Exp}(e_1) \sqcup \text{Exp}(e_2)$. Furthermore, in the previous section we saw two possibilities for the relationship between Dox($e_1$), Dox($e_2$), and Dox($e_1 \sqcup e_2$). But now that there is an additional conjunct in the definition of believe, we must ask what the relationship is between about($e_1$), about($e_2$), and about($e_1 \sqcup e_2$). Once again, as far as the mechanics of the proposal is concerned, this choice does not seem to matter. However, I will assume that about($e_1 \sqcup e_2$) = about($e_1$) $\sqcup_s$ about($e_2$), where $s_1 \sqcup_s s_2$ is the sum of $s_1$ and $s_2$ (i.e., the minimal situation containing both $s_1$ and $s_2$).

Now let us see how aboutness can save us from the Mozart problem. The predicted denotation for Arnie thinks that Mozart was born in 1755 can be seen in (26). The denotation for Beatrice thinks that Mozart was born in 1756 is the result of swapping out $a$ for $b$, and 1755 for 1756.

$[\text{Arnie thinks that Mozart was born in 1755}]^c = 1$ iff

$\exists e[\text{Exp}(e) = a \land \text{about}(e) = s^c \land \forall w \in \text{Dox}(e)[\text{born-in-1755}(w)]]$

For our purposes, it does not matter what the situation is about which Arnie believes that Mozart was born in 1755. All that matters is that such a situation exists and can be picked out by context. In this case (26) is, of course, true.

3 One possibility is to say that for beliefs like this, which seem to be more general and less pinned to particular situations, the about-situation is the whole world of utterance. But again, the theory at hand does not require that we pinpoint a particular situation that this belief state is about.
As for (6), suppose that \textit{s\textsubscript{wed}} is the situation containing, say, Paul’s wedding and reception. While Arnie in general believes that Mozart was born in 1755, his beliefs \textit{specifically about s\textsubscript{wed}} do not entail this. Thus, if \textit{e\textsubscript{a\textsubscript{wed}}} is Arnie’s belief state about \textit{s\textsubscript{wed}}, \text{Dox}(e\textsubscript{a\textsubscript{wed}}) will contain some worlds in which Mozart was born in 1755, some in which he was born in 1756, and some in which he was born in other years. However, \text{Dox}(e\textsubscript{a\textsubscript{wed}}) \textit{will} entail that Paul’s husband is a rich man, as this is a belief Paul has about \textit{s\textsubscript{wed}}. Similar facts hold for \textit{e\textsubscript{b\textsubscript{wed}}}, Beatrice’s belief state about \textit{s\textsubscript{wed}}: \text{Dox}(e\textsubscript{b\textsubscript{wed}}) will entail that Paul married a New Yorker, but not that Mozart was born in 1756. \text{Dox}(e\textsubscript{a\textsubscript{wed}}) and \text{Dox}(e\textsubscript{b\textsubscript{wed}}) therefore have a non-empty intersection, meaning that on either of the analyses in the previous section \text{Dox}(e\textsubscript{a\textsubscript{wed}} \cup e\textsubscript{b\textsubscript{wed}}) is this intersection. As a result, if the contextually determined about-situation is \textit{s\textsubscript{wed}}, then there is a belief state that is about that situation, that has \textit{a} \cup \textit{b} as its experiencer, and that is such that in all doxastic worlds, Paul marries a rich New Yorker: namely, \textit{e\textsubscript{a\textsubscript{wed}}} \cup \textit{e\textsubscript{b\textsubscript{wed}}}. In other words, we again predict (6) to be true.

4.3 Context-sensitivity

The proposal outlined above pins the distinction between relevant and irrelevant disagreement to a contextually determined about-situation. As a result, whether or not experiencers qualify as being in agreement is predicted to be a matter partially determined by the context of utterance. In fact, this turns out to be a good thing. For example, consider the following revision of the original context for (4) (the house-building scenario):

\begin{quote}
Sam owns a construction company and has six clients, none of whom know of the others’ existence. She has convinced each client that she would build a house for him and him alone. In reality, she is a con artist and built no houses at all.
\end{quote}

In this revised context, (27) is clearly true:

\begin{equation}
(27) \quad \text{Each client believes that Sam built a house for him and him alone.}
\end{equation}

In order for (27) to be true, Sam’s six clients must have non-intersecting belief worlds: all of Client 1’s belief worlds are worlds in which he gets a house, and no one else (including Clients 2–6) does. Since there are no worlds in which six different clients are the only client to get a house from Sam, there are no worlds shared by all six clients’ belief states.

However, suppose that the six clients later find each other and catch on to Sam’s scam. They then file a joint lawsuit against her. In this context, while justifying the proposed amount of combined damages, it seems that the clients’ attorney can truthfully say (28):

\begin{equation}
(28) \quad \text{In reality, she is a con artist and built no houses at all.}
\end{equation}
Each of the contracts was for a massive house worth over $5 million. So the reason the combined damages are over $50 million is because Sam’s clients thought she had built six very expensive houses for them.

While the lawyer is adding a bit of contextual backdrop, the underlined sentence in (28) is essentially the same as that in (4).

So here is the issue. In order for (27) to be true, the six clients must have mutually contradictory beliefs, since each client’s beliefs exclude the possibility of any other clients getting a house. But in order for (28) to come out as true, the clients’ beliefs must not count as mutually contradictory, in order for their six house-beliefs to be conjoined into one six-house belief. In other words, the clients’ belief of exclusivity qualifies as relevant for (27), but not for (28), since the exclusivity clause is presumably irrelevant in the tabulation of combined damages.

To see how aboutness can account for this, imagine for simplicity’s sake that each client $k_i$’s contract with Sam is as in Figure 4. Situation $s_i$ contains all and

![Figure 4](image_url)

**Figure 4** About-situations for house-building, with and without exclusivity

only those portions of the contract that state that Sam will build a house for $k_i$. Meanwhile, $s_i'$, which properly includes $s_i$, also includes those portions of the contract guaranteeing that Sam will not build a house for anyone else. So much like in the original house-building scenario for (4), $k_i$’s beliefs about $s_i$ entail that $k_i$ is getting a house, but not necessarily that $k_j$ isn’t getting a house. But $k_i$’s beliefs about $s_i'$ entail not only that $k_i$ is getting a house, but also that $k_i$ is the only one getting a house.

With our scenario now clarified, we can move on to (27) and (28). In order to actually get (27) right, a slight tweak to the semantics is in order. To see why, consider (29), which is what we currently predict the denotation for (27) to be:

\[\forall x : \text{client}(x)[\exists e [\text{Exp}(e) = x \land \text{about}(e) = s^c \land \forall w \in \text{Dox}(e)[\text{only-house}(x, w)]]]]\]

The reason (29) is problematic is because it requires that all of the clients’ beliefs be about the same situation $s^c$. That is, there must be some situation (determined by
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context) such that each client \(k_i\)’s beliefs about that situation entail that \(k_i\) and only \(k_i\) is getting a house. But that is not what we have here. Rather, \(k_i\)’s beliefs are about \(s_i\), \(k_j\)’s beliefs are about \(s_j\), etc. Something must be changed in order to allow the about-situation to vary across quantified-over experiencers.

To fix this we will modify the semantics for \textit{believe} one more time. Rather than requiring that the belief state be about some contextually determined situation \(s^c\), our new definition will require that the belief state be about some element in a contextually determined set \(S^c\) of situations:

\[
(30) \quad \text{believe}^c_{\text{take } 3} = \lambda p \lambda e. \text{about}(e) \in S^c \land \forall w \in \text{Dox}(e)[p(w)]
\]

Naturally, the modified denotation for (27) will be as in (31):

\[
(31) \quad \forall x : \text{client}(x)[\exists e[\text{Exp}(e) = x \land \text{about}(e) \in S^c \land \\
\forall w \in \text{Dox}(e)[\text{only-house}(x, w)]]]
\]

If \(S^c = \{s_i^c \mid 1 \leq i \leq 6\}\), we rightly predict (27) to be true: for each client \(k_i\), there is some situation in \(S^c\)—namely, \(s_i^c\)—about which \(k_i\) believes that he is the only one to receive a house.

As for the underlined sentence in (28), the predicted truth conditions are as in (32):

\[
(32) \quad \exists e[\text{Exp}(e) = k_{1-6} \land \text{about}(e) \in S^c \land \forall w \in \text{Dox}(e)[\text{six-houses}(w)]]
\]

Now suppose that for each client \(k_i\) and situation \(s_i\), \(e_i\) is \(k_i\)’s belief state about \(s_i\). Since \(s_i\) contains only the portion of \(k_i\)’s contract guaranteeing him a house (and not the part guaranteeing him exclusivity), for each other client \(k_j\), \(\text{Dox}(e_j)\) is agnostic about \(k_j\) getting a house. So just like in the original house-building scenario for (4), \(\{\text{Dox}(e_i) \mid 1 \leq i \leq 6\}\) has a non-empty intersection, and in all worlds in that intersection Sam builds six houses for her six clients. Thus, all \(\text{Dox}(e_{1-6})\) worlds are six-house worlds. Plus, as stipulated above, \(\text{about}(e_{1-6})\) is \(s_{1-6}\), the sum of \(s_1\) through \(s_6\). We therefore predict (28) to be true if \(S^c\) contains \(s_{1-6}\), since there is a state \(e_{1-6}\) that has experimenter \(k_{1-6}\) and about-situation \(s_{1-6}\), and that is such that in all doxastic worlds Sam builds six houses for the six clients.

5 Conclusion

We have seen a variety of evidence suggesting that (I) it is possible for a plurality to believe \(p\) without any of its atomic parts believing \(p\), and (II) the beliefs of a plurality are to a significant extent predictable from the beliefs of its atomic parts. I have offered two analyses for the facts discussed in Section 2: one in which all

\[\text{This has no effect on the Mozart example discussed earlier in this section, since all that is now required is that } s^{\text{wed}} \in S^c.\]
of the work was accomplished by a robust principle of belief summing, and one that utilized a more slimmed-down ontology together with a more robust semantics for plurals. I then discussed a problem faced by both of these analyses, namely the problem of irrelevant agreement, and offered a solution in terms of the “aboutness” of beliefs. I now conclude by detailing two areas for possible future research.

First, while I have stuck to believe and think in this paper, other attitudes like want allow for non-distributive readings as well:

Sam has six clients, who do not know each other. Each client has asked Sam to build a house for him.

(33) Sam’s six clients want her to build six houses for them.

Extending our current analysis of believe to want poses two new challenges. The first is that the semantics for want has generally been taken to be more complex than the semantics for believe; more specifically, broadly Hintikkan analyses for want generally utilize an ordering relation over possible worlds in terms of their preferability to the experiencer (see, e.g., Heim 1992; von Fintel 1999; Büring 2003; Villalta 2008; Anand & Hacquard 2008, 2013; Rubinstein 2012, 2017; Phillips-Brown 2016). Thus, new principles need to be devised elucidating the relationship between the world-orderings of atomic desire states and those of their sums. In addition, Pasternak (2018, accepted) provides evidence suggesting that states of desire have a robust mereology all on their own, completely independent of the issue of plurality. This adds another layer of complexity to any account of non-distributive desire ascriptions.5

Second, over the course of discussion of this material it was brought to my attention that Schmitt (2017) has similarly argued for the existence of cumulative (and thus non-distributive) belief ascriptions. However, much of the data she discusses are quite different from what was covered in this paper, with the result being that her analysis and my own differ sharply. It is up to future work to determine what the relationship is between Schmitt’s facts and analysis and my own, and perhaps how they might be combined in order to account for the sum of the available evidence.

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5 Pasternak (2018) also provides evidence that such non-distributive readings arise for verba dicendi like say and claim, a fact that poses additional challenges.
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