Bare numerals, collectivity, and genericity: A new puzzle*

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Abstract

The semantics of number words (or *numerals*), like *three*, has been the subject of intense research, and controversy, over the last few decades. The classic approach to numerical interpretation, pioneered by Horn (1972), contends that the basic, literal meaning of *three* in a sentence like *Three people attended* is 'at least three' (a one-sided meaning), and that the upper bound it typically implies arises by way of scalar implicature. More recent approaches argue that the literal meaning of *three* is 'exactly three' (a two-sided meaning), implemented in one of a variety of ways (e.g. Geurts 2006; Breheny 2008; Kennedy 2015). With a few exceptions, the primary data set has, up to now, consisted of simple existential sentences with distributive predicates, as well as modal sentences. The goal of this article is to reassess these two basic approaches to numerals (the 'at least' approach and a representative family of 'exactly' approaches) in light of a broader range of data that includes existential sentences with collective predicates, such as *Three people lifted the piano together* (cf. Koenig 1991), *Four players formed a team*, and *Five students have the same name*, and sentences with generically interpreted numerical indefinites, such as *Three people can fit in the car* and *Three people can lift the piano* (cf. Link 1987, 1991). I show that a brand new puzzle emerges: on the one hand, (a version of) the classic account gets the collective facts exactly right, but it fails to predict a previously unnoticed asymmetry in the distribution of two-sided readings

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of numerical indefinites embedded under a downward-entailing operator; on the other hand, (a version of) the 'exactly' approach beautifully predicts the embedding facts, but overgenerates in the collective domain. I conclude by offering some speculation as to how the puzzle might be resolved in favor of the classic account.

Keywords: numerals; plurality; distributivity; collectivity; genericity; scalar implicature

1 Introduction

This article concerns the semantics of number words, or *numerals*, such as the word *three*. Despite several decades of research on the topic, there is still no consensus on what the basic, literal meaning of a word like *three* is. One reason is that its interpretation is, in some sense, variable: in some environments, it seems to mean 'exactly three', while in other environments, it seems to mean 'at least three'.[1] For example, an utterance of (1) naturally implies that it is not the case that more than three people attended; hence, (1) is typically construed as expressing the proposition that *exactly* three people attended.

(1) Three people attended.

By contrast, an utterance of (2) does *not* imply that Ann is required not to solve more than three problems, i.e. that in order to pass, she needs to solve exactly three problems; rather, (2) is typically construed as expressing the proposition that Ann is required to solve *at least* three problems.

(2) Ann is required to solve three problems (in order to pass).

There are two broad approaches to this challenge.[2] The classic approach, pioneered by Horn (1972), contends that the basic, literal meaning of *three* in a sentence like (1) is 'at least three' — i.e. a one-sided meaning — and that an upper-bound inference arises by way of scalar implicature, resulting in the reading 'at least three but not at least four', i.e. 'exactly three' — a two-sided meaning. No analogous implicature arises for (2). I will discuss the basic idea behind the classic

1 I use *italics* to refer to object-language expressions (as well as for emphasis and for introducing terminology), sans serif to refer to metalanguage expressions, 'single quotes' to informally describe particular readings or interpretations of an expression, and "double quotes" for direct quotations from the literature.

2 My discussion of the various approaches to numerals draws heavily on Spector 2013. I have simplified Spector's classification of approaches (he divides them into four types) by lumping the 'exactly'-only view and the ambiguity views into one category and by not discussing the underspecification view at all. (See Spector 2013 for arguments against the underspecification view, namely his conclusion that "there is no compelling evidence that a bare numeral could ever receive an 'at most' interpretation, and there is in fact evidence against such a view".)

view, and its merits, in §2 and present a compositional implementation of the account in §3. In §4, I will point out a number of correct predictions it makes regarding the interpretation of numerals in sentences with collective predicates, such as (3), (4), and (5). One contribution of this article is to highlight the classic account's record of success when it comes to collectivity.

(3) Three people lifted the piano together.

(4) Four players formed a team.

(5) Five students have the same name.

Another contribution of this article is to introduce new data involving generically interpreted numerical indefinites (§5). The relevant examples fall into two categories. The first category is exemplified by (6). On its generic reading, (6) means, roughly, that any group of three people can fit in the car. As in the case of (1), an utterance of (6) implies an upper bound, viz. that four people are too many.

(6) Three people can fit in the car.

The second category is exemplified by (7). On its generic reading, (7), just like (6), means, roughly, that any group of three people can lift the piano. However, in this case, a lower bound is implied, viz. that two people are too few.

(7) Three people can lift the piano.

I show that the classic account derives these readings using well-motivated syntactic, semantic, and pragmatic mechanisms, and in a way that is completely parallel to its account of numerical interpretation in the existential domain. However, this uniformity of analysis turns out to be a problem. The classic account, I show, is unable to cope with a previously unnoticed asymmetry in the availability of two-sided readings of numerals embedded under a downward-entailing (DE) operator: briefly, when (1) is embedded under a DE operator, a two-sided reading is still accessible, but when (6) or (7) is, only a one-sided reading is accessible.

In recent years, a number of competing theories have been proposed that contend roughly the opposite of the classic view — namely, that the basic, literal meaning of *three* in (1) is 'exactly three', and that its 'at least three' reading is somehow derived on the basis of the 'exactly three' meaning. I present three such accounts (Geurts 2006; Breheny 2008; Kennedy 2015) (§6) and show that, quite surprisingly, two of them view beautifully predict the aforementioned distribution of two-sided readings of numerals in DE environments (§7). However, I also show that all three accounts make incorrect predictions for numerical sentences like (3), (4), and (5) involving collective predicates (they derive unattested readings) (§8).

A brand new puzzle thus emerges: the collective data swing in favor of (a version of) the classic account, while the generic data swing in favor of (a version of) the 'exactly' account. I conclude (§9) by offering some speculation as to how the puzzle might be resolved in favor of the classic account.[3]

2 The classic, scalar implicature view

In this section, I describe the basic idea and motivation behind the 'at least', or one-sided, view of numerals. In §3, I will present a compositional implementation of it.

2.1 The basic idea

The classic view is summed up best by Horn (1972) himself.

> "Let us assume that these conversational postulates [Grice's maxim of quantity] govern the interpretation of given occurrences of a cardinal number. Numbers, then, or rather sentences containing them, assert lower-boundedness — *at least n* — and given tokens of utterances containing cardinal numbers may, depending on context, implicate upper-boundedness — *at most n* — so that numbers may be interpreted as denoting an exact quantity." (Horn 1972, p. 41)

As we will see, the qualification "or rather sentences containing [numbers]" is important: many sentence types do not imply any upper bound at all.

Consider again (1), repeated below.

(1) Three people attended.

Preliminary evidence that the upper-bound inference associated with (1) is an implicature is that it is cancellable, suspendable, etc., just like the 'not all' implicature of *some* and the 'not both' implicature of *or*.[4]

(8) a. Three people attended, $\left\{ \begin{array}{c} \text{and possibly even} \\ \text{if not} \end{array} \right\} \left\{ \begin{array}{c} \text{more} \\ \text{*fewer} \end{array} \right\}$.

b. Three people attended; in fact, four did. (cf. Horn 1972, p. 38)

3 A quick note on example sentences. The literature on plurality contains many examples like *Three boys together carried a sofa up the stairs* (Koenig 1991), *Three men can lift the piano* (Link 1987, 1991), and so on. In keeping with the LSA's guidelines for inclusive language (`http://www.linguisticsociety.org/content/guidelines-inclusive-language`), I have taken the liberty of changing, without warning, occurrences of *boys* and *men* to just *people*, and of injecting female names into discussions where male names have historically been used.

4 Since all the sentences discussed in this article are syntactically well formed, I use "*" to indicate semantic oddness/infelicity (on a particular reading).

(9) a. Ann read *Hamlet* or *Macbeth*, $\left\{\begin{array}{c}\text{and possibly even}\\ \text{if not}\end{array}\right\}$ both.

b. Beth solved some of the problems; in fact, she solved all of them.

It will be instructive throughout this article to compare the behavior of *three* (*some*, etc.) with that of *only three* (*only some*, etc.), in which the same inferences ('not four', 'not all', etc.) arise, but are known to be semantic entailments. In the case at hand, adding *only* renders the follow-ups infelicitous, thus lending further support to the idea that the inferences without *only* are indeed implicatures, not entailments.

(10) a. *Only three people attended; in fact, four did.

b. *Beth solved only some of the problems; in fact, she solved all of them.

Let us therefore assume that (1) is assigned the meaning 'at least three (i.e. three or more) people attended'. (In §3, I will present several ways of cashing this out compositionally.) Let us also assume that numerals form a so-called *Horn scale* (Horn 1972): ⟨. . . , *two*, *three*, *four*, . . .⟩. Then the upper-bound inference of (1) can be derived as a scalar implicature on the basis of familiar neo-Gricean reasoning, sketched out informally as follows.[5]

(11) a. The speaker uttered (1), hence believes that three or more people attended (*maxim of quality*).

b. If the speaker believed that four (or five, or . . .) people attended, it would have been preferable to utter *Four (five, . . .) people attended*, since that is more informative (*maxim of quantity*).

c. Therefore, it is not the case that the speaker believes that four (or five, or . . .) people attended.

d. Assuming that the speaker is knowledgable about whether four (or five, or . . .) people attended, it follows that the speaker believes that four (or five, or . . .) people did *not* attend, hence that *exactly* three people attended.

5 For a more precise implementation of this reasoning process, see Sauerland 2004. For a game-theoretic approach, see Franke 2011 and Frank and Goodman 2012. Note also that the scalar implicature view of upper-bound inferences need not be cast in neo-Gricean (or rational behavior) terms. One could also assume that scalar implicatures are grammatically derived via a covert exhaustification operator (Chierchia, Fox, and Spector 2012; Fox 2007). In fact, this is precisely what Spector (2013) ultimately proposes for the analysis of numerals. Likewise, we need not rely on Horn scales, but could instead adopt the view that alternatives are structurally defined, in the sense of Katzir 2007.

2.2 Predictions

Since upper-bound inferences are treated as scalar implicatures, the prediction of this account is that numerals should pattern like other scalar items, e.g. *some* and *or*, in terms of inferences. Let us return to (2), repeated below, and recall that *three* is construed as 'at least three' here; that is, we do not infer that it is forbidden for Ann to solve more than three problems, in the way that we do for (12).[6]

(2) Ann is required to solve three problems (in order to pass).

(12) Ann is required to solve only three problems (in order to pass).

A similar pattern holds for *some* and *or*: *some* in (13a) is naturally interpreted as 'some or all' (i.e. we do not infer that it is forbidden for Ann to read all of Shakespeare's plays), and *or* in (13b) is naturally interpreted inclusively ('... or both'; i.e. we do not infer that it is forbidden for Ann to read both *Hamlet* and *Macbeth*).[7]

(13) a. Ann is required to read some of Shakespeare's plays.

b. Ann is required to read *Hamlet* or *Macbeth*.

Moreover, from (2) we infer that Ann is not required to solve more than three problems (i.e. three is sufficient). Similarly, (13a) implies that Ann is not required to read all of Shakespeare's plays, and (13b) implies that Ann is not required to read both *Hamlet* and *Macbeth*.

All of these facts follow straightforwardly from the scalar implicature view. Here is why. The relevant alternatives under consideration are those in (14), and negating these results in inferences of the form 'Ann is not required to ...', rather than 'Ann is required not to ...'.[8]

(14) a. Ann is required to solve four problems (in order to pass).

b. Ann is required to read all of Shakespeare's plays.

c. Ann is required to read *Hamlet* and *Macbeth*.

So far so good. Another prediction concerns downward-entailing (DE) envi-

6 Note that (12) also has a separate, evaluative sort of 'at least' reading (where *only* takes widest scope), which states that three problems is the most that Ann is required to solve in order to pass, and that three counts as small. On this reading, Ann is not forbidden from solving more than three problems.

7 More precisely, these are the natural interpretations under the narrow-scope readings of the scalar items. On their widescope readings, (13a) means that there are some particular Shakespeare plays that Ann is required to read, and (13b) means that either Ann is required to read *Hamlet*, or she is required to read *Macbeth*, and the speaker is unsure which.

8 Of course, the grammatical approach to scalar implicature (cf. fn. 5) would predict both inferences to be possible, depending on the scope of the covert exhaustivity operator relative to the modal.

ronments, where scalar implicatures typically disappear.[9] To illustrate, *or* in the examples in (15) is most naturally interpreted inclusively, not exclusively. For example, (15a) means that Ann read neither *Hamlet* nor *Macbeth* (hence not both, either); it does not have the meaning 'it is not true that Ann read *Hamlet* or *Macbeth* and not both', which is equivalent to 'either Ann read neither *Hamlet* nor *Macbeth*, or she read both of them'. Similar remarks hold for the other examples.[10]

(15) a. Ann did not read *Hamlet* or *Macbeth*.
b. Everyone who read *Hamlet* or *Macbeth* passed/failed.
c. Beth doubts that Ann read *Hamlet* or *Macbeth*.
d. If Ann reads *Hamlet* or *Macbeth*, she will pass/fail.

These facts have a natural explanation: in DE contexts, the relevant alternatives (e.g. *Ann did not read* Hamlet *and* Macbeth) are *weaker* than the original assertion; that is, they are entailed by the assertion and hence cannot be negated, and so no scalar strengthening occurs.

When it comes to numerals, however, the scalar implicature account appears to make incorrect predictions: while they can certainly receive a one-sided, 'at least' interpretation in DE contexts, numerals can also easily receive a two-sided, 'exactly' interpretation. For example, (16a) can easily be construed as meaning that Ann did not read *exactly* three plays, as evidenced by the fact that it can be followed up with *... she read six!* Such a follow-up would be contradictory on an 'at least' construal of *three*.

(16) a. Ann did not read three plays.
b. Everyone who read three plays passed/failed.
c. Beth doubts that Ann read three plays.
d. If Ann reads three plays, she will pass/fail.

Horn, who later renounced his support of the classic view in light of such observations (Horn 1992, 2006), also notes the contrast in (17). Whereas the follow-

9 The following examples involve *or*. I set aside *some* due to the complications it introduces in being a positive polarity item (i.e. in DE contexts *some* usually becomes *any*).

10 The reason for including both *passed* and *failed* is to control for contextual factors. For instance, it would perhaps be odd if everyone who read either *Hamlet* or *Macbeth* but not both passed, while not everyone who read both passed. (Typically, the more you read, the better your chances of passing.) Thus, it is arguable that *or* can indeed be construed exclusively here with *passed*, and that the contextual entailment that everyone who read both passed is responsible for the perception that *or* is interpreted inclusively. However, if we switch to *failed*, no analogous contextual entailment arises: if everyone who read either *Hamlet* or *Macbeth* but not both failed, it hardly follows that everyone who read both also failed. The fact that *or* nevertheless cannot be interpreted exclusively with *failed* suggests that it only has an inclusive interpretation in the restrictor of *everyone*. (Similar remarks hold for the antecedent of a conditional.)

up in (17a) intuitively contradicts the first sentence (because adoring entails liking), the follow-up in (17b) does not intuitively contradict the first sentence (because having four kids apparently does not necessarily entail having three kids).

(17) a. *Neither of us liked the movie — she adored it, and I hated it.

b. Neither of us has three kids — she has two, and I have four.

As Spector (2013) points out, there is a potential response to this shortcoming. Note first of all that even standard scalar items like *some* and *or* can receive their strengthened interpretation in DE contexts if they are focused, i.e. are pronounced with special intonation (indicated by capitalization below).[11]

(18) a. *Ann did NOT read some of Shakespeare's plays — she read ALL of them!

b. Ann didn't read SOME of Shakespeare's plays — she read ALL of them!

(19) a. *Ann did NOT read *Hamlet* or *Macbeth* — she read BOTH!

b. Ann didn't read *Hamlet* OR *Macbeth* — she read BOTH!

Importantly, numerals do not require special intonation for their two-sided, 'exactly' construal (though they can have it, too).

(20) a. Ann did NOT read three plays — she read SIX!

b. Ann didn't read THREE plays — she read SIX!

The response, then, is that numerals "are intrinsically focused, in the sense that they automatically activate their alternatives (i.e. other numerals)" (Spector 2013). This view is also in line with the assumption of Krifka (1999) that numerals "can introduce alternatives without the help of focus".[12, 13]

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11 I use the term "strengthened" here to mean locally strengthened, within the DE context. Obviously, strengthening a scalar item locally, within a DE context, typically leads to global weakening. Nevertheless, I trust that no confusion will arise by using "strengthen" in this way. For numerals, I will stick to "two-sided", so that this issue does not arise at all.

12 While this idea may seem rather stipulative, there is reason to believe that the numerical scale is indeed privileged relative to other scales. For example, children typically fail to strengthen quantifiers like *some*, but have no trouble interpreting numerals in an exact way (Papafragou and Musolino 2003). One explanation is that children simply lack the ability to compute scalar inferences and hence that numerals have an 'exactly' semantics, rather than an 'at least' semantics (cf. §6). But another explanation is that children can and do compute scalar inferences, provided they have access to the relevant scales, and that they have access to the numerical scale but not to scales like ⟨*some*, *all*⟩ (Barner and Bachrach 2010). After all, children are explicitly taught numerals from an early age, but not other scales, and indeed, even adults would be hard-pressed to recite non-numerical scales if asked to do so.

13 There is a missing piece here: How exactly does inherent focus, or automatic activation of alternatives, help to derive two-sided readings in DE environments? The intuition here is that whatever mechanism derives strengthened meanings of focused scalar items like *some* and *or* in DE environments should likewise derive two-sided meanings of (inherently focused) numerals in DE environments, but this

So, at this point, the scalar implicature view does quite well: it predicts an 'exactly' reading of numerals in the basic cases, it does not predict an 'exactly' reading of numerals in the scope of universal modals (and it predicts just the right inference, viz. 'not required . . . more than'), and it predicts an 'at least' reading of numerals in DE contexts (which is accessible). In addition, its main drawback, viz. that it does not also predict the availability of an 'exactly' reading in DE contexts, can plausibly be explained by assuming that numerals are intrinsically focused, or that they can evoke alternatives without focus, and thus that they can, like overtly focused scalar items, be strengthened in DE environments.

3 Compositional implementation of the classic view

Recall that, on the classic, scalar implicature approach to the interpretation of numerals, a sentence like (1), repeated below, is assigned a meaning that can be paraphrased as 'at least three (three or more) people attended'. The 'exactly' reading arises due to an upper-bound scalar implicature, viz. that it is not the case that four or more people attended. The question we now address is how exactly the basic, 'at least' meaning arises compositionally.

(1) Three people attended.

3.1 Predicative uses of numerals

In deciding on the basic, lexical meaning of a numeral like *three*, we need to decide whether we want to assign *three* an 'at least' meaning in *all* sentences in which it occurs, or just in certain types. It turns out that in some types of sentences (or environments), *three* never means 'at least three'. A case in point is when *three* occurs in predicate position. Consider (21). As Landman (2004), Geurts (2006), and Rothstein (2013) all argue, *three* really seems to mean 'exactly three' here, and a theory that derives the 'exactly' meaning via scalar implicature is not a favorable theory.

(21) We are three people.

Geurts (2006) reasons as follows: while (1) and (21) appear, on first glance, to both have 'exactly' interpretations, only (1) licenses inferences to lower numerals. As he puts it (and switching now to his actual examples), "while there is no way of construing the number words so as to make [(22b)] come out valid, it is at least

proposal remains to be spelled out. For an explicit attempt to explain embedded strengthening for non-numerical scalar expressions like *some* and *or*, see Fox and Spector 2017. It is not clear to me that their proposal extends to the case of numerals, but I leave that question for future work.

arguable that [(22a)] is valid in some sense".[14]

(22) a. Five cows mooed. So: Four cows mooed.

b. These are five cows. So: These are four cows.

Landman (2004, pp. 22f), citing Partee 1987, observes furthermore that the upper bound associated with a numeral in predicate position, (23b), is not cancellable, suspendable, etc. in the way that it is for a numeral in argument position, (23a).[15]

(23) a. Three girls came in; in fact, four girls came in.

b. *The guests are three girls; in fact, they are four girls.

In sum, an empirically adequate implementation of the scalar implicature account of numerals needs to assign an 'at least' meaning when numerals occur in argument position, as in (1), (22a), and (23a), but not when they occur in predicate position, as in (21), (22b), and (23b).

3.2 Syntactic-semantic implementation

Keeping in mind the observations above, let us now turn to the question of how to compositionally assign the proper interpretation to various sentences containing numerals. Like most contemporary work on quantification, we will assume that our domain of individuals contains not only ordinary, singular (or *atomic*) individuals, but also *sums* (or pluralities, or groups) of individuals (Link 1983). I will use the symbol "#" to denote the function that maps a sum x to the number of atoms that are part of x, so that "$\#x = 3$" means that x has (exactly) three atomic parts. (In such a case, I will often say that x has *cardinality* 3, to borrow a term from set theory.)

Given the preceding discussion, we want to assign an 'exactly' interpretation to sentences involving predicative *three*. In other words, the meaning of (21) should be represented as in (24a), not as in (24b).

(24) a. $\#(\mathsf{we}) = 3 \wedge \mathsf{people}(\mathsf{we})$

b. $\#(\mathsf{we}) \geq 3 \wedge \mathsf{people}(\mathsf{we})$

As for *three* in argument position, we want to assign an 'at least' interpretation. For (1), the representation in (25a), with the obvious relation "≥", is perfectly suitable: it means that a group of three or more people attended, which seems to

14 As we will see in §6, Geurts (2006) argues that (22a) is ambiguous between an 'at least' reading and an 'exactly' one, while (22b) has only an 'exactly' reading. His point here is that, even though the preferred reading of (22a) is the 'exactly' one, it *can* be interpreted in the 'at least' way, thus validating inferences to lower numerals; by contrast, in his system, (22b) can never be interpreted in an 'at least' way, hence never validates inferences to lower numerals.

15 Landman (2004) attributes this observation "to Barbara Partee, or Nirit Kadmon, or both". See also Rothstein 2013.

be just what we want. However, it turns out that a representation of the meaning of (1) involving "=" instead of "≥", as in (25b), is also an 'at least' meaning, due crucially to the distributivity properties of the predicates in the sentence. Here is why. What (25b) says is that a group of *exactly* three people attended. However, even if more than three people attended, then it is still true that a group of *exactly* three people attended: just take the total group of people who attended, pick any 3-membered subgroup, and that is a group of exactly three people who attended. In other words, (25a) and (25b) are truth-conditionally equivalent.

(25) a. $\exists x[\#x \geq 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

b. $\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

To arrive at the representations in (24a) and (25a)/(25b) compositionally, let us momentarily assume, for the sake of exposition, that *three* has two lexical entries, depending on whether it occurs in predicate or in argument position, and let us remain agnostic about whether we want "≥" or "=" in argument position. Then the following two lexical entries would seem to do the trick.[16]

(26) a. $[\![\text{three}_{\text{pred}}]\!] = \lambda x_e \,.\, \#x = 3$

b. $[\![\text{three}_{\text{quant}_{\geq}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\#x \geq 3 \wedge P(x) \wedge Q(x)]$

c. $[\![\text{three}_{\text{quant}_{=}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\#x = 3 \wedge P(x) \wedge Q(x)]$

Using these entries, our running examples are assigned the right meanings on the basis of the following logical forms (LFs). Note that the type clash that occurs when $three_{pred}$ (type *et*) combines with *people* (type *et*) is resolved by a rule of semantic interpretation that allows two expressions of type *et* to combine intersectively (cf. the rule of *predicate modification* in Heim and Kratzer 1998).[17]

(27) a. we [are [$\text{three}_{\text{pred}}$ people]]

b. [$\text{three}_{\text{quant}_{\geq}}$ people] attended

c. [$\text{three}_{\text{quant}_{=}}$ people] attended

16 Rather than having the simple condition that $\#x = 3$ in the meaning of $three_{pred}$, we could have a more sophisticated meaning along the lines of Ionin and Matushansky 2006, involving a partition of x into three subparts of which some property P (e.g. the nominal complement of *three*) holds, which would allow us to compositionally analyze *three hundred* on the basis of the meanings of *three* and *hundred*.

17 Alternatively, we could assume that $[\![\text{three}_{\text{pred}}]\!] = \lambda P_{et} \,.\, \lambda x_e \,.\, \#x = 3 \wedge P(x)$, so that $three_{pred}$ takes a predicate like *people* as its argument. A potential piece of evidence in favor of this higher predicative type, at least for English, is the following: while a sentence like *We are three* is somewhat commonplace (e.g. uttered to the host at a restaurant), structurally identical sentences are quite bad in English, e.g. *?? The books on the table are three* (cf. *The number of books on the table is three*). This suggests that *three* must typically combine with a common noun in order to be used predicatively, as in *The books on the table are three dictionaries*, which is perfectly natural. (Similar remarks do not hold for languages like Italian and German, in which the equivalent of *three* can be productively used in predicate position without first combining with a noun.)

Now, it would be preferable to assign just one meaning to *three* in a way that still manages to assign to each sentence the right overall meaning. We can do this by assuming the predicative meaning to be the basic one, and, following the vast body of literature on existential indefinites, posit a separate mechanism that contributes existential quantification. There are many ways that existential quantification could be introduced, but for concreteness, let us avail ourselves of the typeshifting operator A (Partee 1987), which takes two sets and says that they have an element in common.[18]

(28) $\mathsf{A} = \lambda P_{\alpha t} \,.\, \lambda Q_{\alpha t} \,.\, \exists x_\alpha [P(x) \wedge Q(x)]$

The idea is that *three$_{pred}$* combines intersectively with a noun phrase (on the basis of its predicative meaning), and then A connects everything up.

(29) a. [three$_{\text{pred}}$ people] attended

b. $\mathsf{A}(\mathsf{three}(\mathsf{people}))(\mathsf{attend})$
$\equiv \exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

On this account, the representation we end up with for *three* in argument position involves "=", not "≥". This fact will become important when we move from distributive to collective predicates in §4.

At this point, we have what I would consider to be a suitable, contemporary version of the classic view of numerals, often referred to as the *adjectival* theory of numerals because *three*, like any run-of-the-mill adjective, is taken to be a property of individuals, albeit plural individuals (see especially Landman 2004 and the references therein).[19] However, let me make one small adjustment (which has been argued for in various places in the literature), mainly for presentational purposes (to maximize similarity across theories), because Kennedy 2015 contains (something like) it. The adjustment is to reduce the denotation of *three* even further by identifying it with the number 3 (type *d*), (30), which arguably accounts for its singular-term use in sentences like *The number of children Homer has is three* and *One and two make three*.[20]

(30) $[\![\text{three}]\!] = 3$

18 Another possibility is to assume a rule of global existential closure (Heim 1982). Yet another move would be to assume the presence of a null existential determiner (Link 1987; Krifka 1999).

19 "Contemporary" is maybe a stretch: the adjectival view goes back to at least Verkuyl (1981), Hoeksema (1983), and Link (1987). But it is contemporary in the sense that (i) it is quite different from the original conception of Horn (1972), and from the view of Generalized Quantifier Theory (Barwise and Cooper 1981), and (ii) most neo-Griceans still subscribe to the same general view, modulo the different technical choices mentioned here in footnotes.

20 Frege (1884, §57) argued that a numeral denotes a "proper name" (*Eigenname*), on the basis of singular-term uses like in his famous example *Die Zahl der Jupitersmonde ist vier* ('The number of Jupiter's moons is four'), in which *die Zahl der Jupitersmonde* "denotes the same object as" *vier*.

The predicative meaning of *three* then arises by combining it with a silent ⟨*many*⟩, (31), which shifts the number-denoting meaning of a numeral to a predicative meaning.[21, 22]

(31) $[\![\langle \text{many} \rangle]\!] = \lambda n_d \, . \, \lambda x_e \, . \, \#x = n$

4 Evidence from collectivity: A triumph for the classic account

As first noted (I believe) by Koenig (1991), only sentences in which the numerical subject combines with a distributive predicate, such as *attend*, license inferences to lower numerals. When the numerical subject combines with a collective predicate like *carry a sofa up the stairs together*, as in (32), downward inferences do not hold.

(32) Three people together carried a sofa up the stairs.

(33) Three people together carried a sofa up the stairs.
$\not\Rightarrow$ Two people together carried a sofa up the stairs.

As Koenig points out further, this difference between distributive and collective predicates extends to judgments on upward compatibility, as shown by the contrast in (34). The felicitous follow-up *in fact four* in (34a) cancels the implicature of the first part that the group in question does not consist of more than three people. By contrast, this very same follow-up is infelicitous in (34b), which suggests that the first part does *not* (merely) implicate that the group in question does not consist of more than three people — it entails as much.

(34) a. Three people came, in fact four.
b. *Three people together carried a sofa up the stairs, in fact four.

In other words, a sentence like (32) seems to mean that (a group of) *exactly* three people carried a sofa up the stairs, not that (a group of) *at least* three people did

21 Rothstein (2013) proposes that the predicative (type *et*) meaning is basic and that the singular-term (type *d*) meaning is the "individual property correlate" of (the set denoted by) the predicative meaning (cf. Chierchia 1985). In other words, the type *d* meaning is derived from the type *et* meaning, not the other way around, which Rothstein argues is "a direct instantiation of Frege's insight that a property has 'two modes of presentation', one unsaturated in which it applies to an argument to form a sentence, and one saturated, in which it can itself be the subject of a predication".

22 Hackl (2000) proposes that numerals combine with an existential-determiner-like version of ⟨*many*⟩, given below. Clearly, $[\![$[three ⟨many⟩] NP$]\!]$ for Hackl is equivalent to A($[\![$[three ⟨many⟩] NP$]\!]$) on the current account, so for existential sentences, the choice of ⟨*many*⟩ does not matter. However, for generic sentences (§5), it is important (for the classic account) that existential quantification not be built into the numerical indefinite, so that a generic operator can apply instead.

(i) $[\![\langle \text{many} \rangle]\!] = \lambda n_d \, . \, \lambda P_{et} \, . \, \lambda Q_{et} \, . \, \exists x[\#x = n \wedge P(x) \wedge Q(x)]$

so. To be sure, an utterance of (32) may implicate that only one group of people carried a sofa up the stairs, in which case it follows that no group of more than three people carried a sofa up the stairs. However, the sentence is still compatible with, say, a group of six people carrying a sofa up the stairs, just as long as (at least) one group of three people did so, too.[23]

The same remarks hold for (3) and (4), repeated below. For example, if four players formed a team, it certainly does not follow that three players formed a team. This fact is due in part to the fact that if a group x consisting of four players formed a team, then it does not follow that any subgroup y of x also formed a team.

(3) Three people lifted the piano together.

(4) Four players formed a team.

As Koenig observes, the adjectival account of numerals straightforwardly explains these facts. The derived meanings involve existential quantification and "=" (not "≥"), just like for (1) with *attend*. However, because *lift the piano together* and *form a team* do not license inferences from groups to subgroups (unlike *attend*, which does), the derived meanings are not 'at least' meanings.

(35) a. [[three ⟨many⟩] people] [lifted the piano together]

b. $\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{lift}(x)]$

(36) a. [[four ⟨many⟩] players] [formed a team]

b. $\exists x[\#x = 4 \wedge \mathsf{players}(x) \wedge \mathsf{form.a.team}(x)]$

Notably, if we had continued to assume that *three* is polysemous between $\textit{three}_{pred}$ and $\textit{three}_{quant_{\geq}}$, with "≥" (see (26b)), then we would have incorrectly derived 'at least' readings here as well, which would have left the above judgments unexplained.

Nevertheless, there do appear to be collective predicates that license inferences from groups to subgroups (not discussed by Koenig). A case in point is *have the same name*. If a group x consisting of six students has the same name, then it follows that every subgroup of y of x (such that $\#y \geq 2$) has the same name, hence that a group of five students has the same name. On the adjectival theory, the meaning of a sentence like (5), repeated below, is still represented with "=", but because *have the same name* licenses downward inferences, the resulting meaning is an 'at least' meaning, for precisely the same reason that the derived meaning for (1), with *attend*, is an 'at least' meaning.

23 For instance, the following sentence is perfectly felicitous: *Three people together carried a sofa up the stairs; in fact, so did four (other) people.* Here, the follow-up is not canceling the implication that first group mentioned consists of no more than three people (which, as we saw, would be infelicitous); rather, it cancels the implication that only one group of people (the group of three) carried a sofa up the stairs.

(5) Five students have the same name.

(37) a. [[five ⟨many⟩] students] [have the same name]

b. $\exists x[\#x = 5 \wedge \mathsf{students}(x) \wedge \mathsf{same.name}(x)]$

As a result, (5), like (1), and unlike (3) and (4), is predicted to trigger an upper-bound implicature, viz. that not more than five students have the same name, which seems exactly right. For instance, (38) is judged to be much more coherent than the analogous example with *lift the piano together* or *form a team*.

(38) Five students have the same name, in fact six.

Note also the contrasts in (39). Switching from *have the same name* to *lift the piano together* or *form a team* removes any contrast.

(39) a. Five students have the same name; in fact, {more / *fewer} do.

b. Five students have the same name, if not {more / *fewer}.

In sum, the classic, adjectival account of numerals uniformly assigns a semantic representation to existential sentences like (1), (3), (4), and (5) involving existential quantification over groups and a cardinality check with "=". Whether or not inferences to lower numerals are licensed, and, in turn, whether or not an upper-bound implicature is triggered, depends on the logical (inferential) properties of the predicates in the sentence. In the case of *attend* and *have the same name*, an upper-bound implicature is triggered because these predicates license inferences from groups to subgroups, while in the case of *lift the piano together* and *form a team*, no upper-bound implicature is triggered because these predicates do not license inferences from groups to subgroups. In other words, the presence or absence of an upper-bound inference is tightly connected to the logical properties of the predicates involved, which seems exactly right.

5 Evidence from genericity: A problem for the classic account

We now turn from the interpretation of existential numerical indefinites to the interpretation of generic ones. I start by presenting a very simplistic analysis of genericity, just enough to give us some theoretical scaffolding, before proceeding to the discussion of bare numerals in generic sentences.

5.1 Basic generic sentences

The example in (40) is a run-of-the-mill *generic* (or generalizing, or characterizing) sentence (Carlson 1978; Schubert and Pelletier 1987; Krifka et al. 1995): it states a generalization of some kind (in this case, about cats).

(40) Cats meow.

In particular, (40) states, roughly, that in general every cat meows (possibly with exceptions), or that every typical cat meows, hence the intuitive validity of the following inference.

(41) Cats meow.
Snowball is a (typical) cat.
⇒ Snowball meows.

We can represent this reading as follows, where "$\forall_{\text{Gen}}$" is a quasi-universal (i.e. restricted universal) quantifier that quantifies over all 'typical' individuals (of some sort or other).[24]

(42) $\forall_{\text{Gen}} x[\mathsf{cats}(x) \rightarrow \mathsf{meow}(x)]$

Assuming that the extension of plural expressions like *cats* may contain both atomic and properly plural individuals (see, e.g., Krifka 1999, 2003; Sauerland, Anderssen, and Yatsushiro 2005; Spector 2007), the derived reading, viz. 'every (typical) group of cats has the property of meowing', entails that every (typical) cat meows, as desired.

A common way to derive this reading compositionally is to assume a silent generic operator, *Gen*, at LF, whose semantics involves "$\forall_{\text{Gen}}$", as shown in (43).[25]

(43) $[\![\text{Gen}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[P(x) \rightarrow Q(x)]$

(44) a. [Gen cats] meow
b. $\forall_{\text{Gen}} x[\mathsf{cats}(x) \rightarrow \mathsf{meow}(x)]$

5.2 Bare numerals in generic sentences

Generic sentences with bare numerals appear to work similarly. Consider (6) and (7), repeated from §1. (6) means, roughly, that any group of three people can fit in

24 I will refrain from providing a model theory for "$\forall_{\text{Gen}}$", and in particular leave open the question of what it means for an individual to be 'typical'. See Krifka et al. 1995 for a survey of a number of proposals, any one of which could be employed here. The exact treatment of "$\forall_{\text{Gen}}$", in particular how exceptions are allowed for, is an important issue in the semantics of genericity, but is, as far as I can tell, not very important for what I will have to say about generic numerical indefinites.

25 I have assumed here that *Gen* is a dyadic operator and that it combines first with the nominal predicate and then with the verbal predicate. It may be more appropriate to assume that *Gen* is located in the verbal projection, hence combines first with the verbal predicate, or that *Gen* is a monadic (sentential) operator. As far as I can tell, none of these distinctions matter much to the discussions in this article, and so for simplicity I analyze *Gen* essentially as a silent determiner.

the car, and (7), adapted from Link 1987, 1991, means (as Link also observes) that any group of three people can lift the piano.[26]

(6) Three people can fit in the car.

(7) Three people can lift the piano. (Link 1987, 1991)

One way to see that these readings are quasi-universal is to note the robustness of the intuitive validity of the following inferences.

(45) Three people can fit in the car.
Ann, Bill, and Carol are three (typical) people.
⇒ Ann, Bill, and Carol can fit in the car.

(46) Three people can lift the piano.
Ann, Bill, and Carol are three (typical) people.
⇒ Ann, Bill, and Carol can lift the piano.

Assuming (uncontroversially) that *three* has, or can have, a predicative meaning, as it does on the classic, adjectival theory developed in §3, then these readings fall out naturally.

(47) a. [Gen [[three ⟨many⟩] people]] [can fit in the car]

b. $\forall_{\text{Gen}} x[[\#x = 3 \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.fit}(x)]$

(48) a. [Gen [[three ⟨many⟩] people]] [can lift the piano]

b. $\forall_{\text{Gen}} x[[\#x = 3 \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.lift}(x)]$

These observations are not new. The earliest relevant observation that I have managed to find is by Hoeksema (1983), who writes, "I do believe that numeral-noun combinations may have a generic reading, just like ordinary bare plurals (Carlson 1978)", and gives the following examples.

(49) a. Ten people can't form a soccer team.

b. Three cars are more expensive than two.

Hoeksema furthermore observes that "the familiar equation of *n* and *at least n* leads to wrong results here [. . .] [W]ith generic readings, we need *exactly n*". He is also

26 I write *can lift the piano* instead of *can lift the piano together* because *together* is now redundant: on its generic reading, (7) necessarily states a generalization about groups of people being able to do a collective action. This is why **Three people can speak French* is completely anomalous on a generic reading. One explanation for this generalization is the following: the generic reading with a distributive predicate like *can speak French* would state that any group of three people is such that each of them can speak French, but this is tantamount to just saying that every single person can speak French. In other words, the numeral does no semantic work. Such readings would thus be ruled out by a constraint like the one that Buccola and Spector (2016) propose for independent reasons to explain the range of readings of certain modified numerals.

careful to point out that "when the bare plural is read existentially, not generically, this amounts to the same results as in the Barwise and Cooper 1981 definition", i.e. an 'at least' interpretation, for precisely the reasons mentioned in §3.2.

Similarly, Link (1987, 1991), arguing against a Generalized Quantifier (GQ) theoretical approach to numerals, had this to say.[27]

> "Numerals play a double role within the prenominal domain. First there is a clear indication that they occur as adjectival modifiers with an intersective meaning. Consider the pair *three people lifted the piano* vs. *(any) three people can lift the piano*; if the numeral *three* is treated like a quantifier with an in-built existential force appropriate for the first sentence, then the universal (or generic) force of the second sentence cannot be accounted for." (Link 1991)
>
> "The first sentence [*three people lifted the piano*] expresses a particular 'historical' fact (in a certain context), the second [*three people can lift the piano*] can be used as a generic statement about the piano in question. It is really these different uses here that introduce the existential and universal force, respectively. GQ theory cannot say that, as we saw, since it considers the (existential) quantificational force as inherent in the numeral." (Link 1987)

As we will see in §6, however, this criticism does not really extend to proposals like that of Geurts (2006) or Breheny (2008), even though they, too, build existential force into the numeral — because they also have a way to shift the numeral to an adjectival type. In addition, Link (and Hoeksema) did not consider the bound inferences that generic sentences with numerals can trigger, which is the topic of the next subsection. The analysis of such inferences, and the comparison of the competing proposals discussed in this article, can therefore be seen as a natural extension of Link's investigation into the "double role" that numerals play.

5.3 Bound inferences in generic sentences

Despite their structural and even semantic similarity, the two generic sentences in (6) and (7) nevertheless pattern differently in terms of inferences, viz. whether they license inferences to lower vs. higher numerals, and whether they imply an upper vs. a lower bound.

For instance, (6), with *can fit*, licenses inferences to lower numerals: if three people can fit in the car, then certainly so can two. This inference is a consequence of the more general fact that *can fit in the car* licenses inferences from groups to

27 Link (1987, 1991) only considers the predicate *can lift the piano*, not *can fit in the car*. As we will see in §5.3, these two predicates have quite different properties.

subgroubs: if a group x can fit in the car, then it follows that every subgroup of x can, too. As a result, if every group of three people can fit in the car, then so can every subgroup of every group of three people, which of course includes every group of two people.

(50) Three people can fit in the car. $\rightsquigarrow$ Two people can fit in the car.

By contrast, (7), with *can lift*, licenses inferences to higher numerals: if three people can lift the piano, then certainly so can four. This inference is a consequence of the more general fact that *can lift the piano* licenses inferences from groups to supergroups: if a group x can lift the piano, then it follows that every supergroup of x can, too (the larger the group, the easier the piano lifting). As a result, if every group of three people can lift the piano, then so can every supergroup of every group of three people, which of course includes every group of four people.[28]

(51) Three people can lift the piano. $\rightsquigarrow$ Four people can lift the piano.

In addition, (6) and (7) each license bound inferences, but in different directions. In particular, (6) implies an upper bound, while (7) implies a lower bound.

(52) a. Three people can fit in the car. $\rightsquigarrow$ Four people cannot fit in the car.

b. Three people can lift the piano. $\dashrightarrow$ Two people cannot lift the piano.

Admittedly, the lower-bound inference for (7) is not as strong as the upper-bound inference for (6), as indicated visually by the use of different implication symbols above. (In the next subsection, I will give one plausible reason why this is the case.) Nevertheless, both bound inferences appear to be real, and furthermore they appear to be scalar implicatures (recall the hallmark traits reviewed in §1): they are cancellable, suspendable, etc., which is only felicitous if the inferences exist in the first place.

(53) a. Three people can fit in the car; in fact, $\left\{\begin{array}{c}\text{more}\\ \text{*fewer}\end{array}\right\}$ can.

b. Three people can fit in the car, if not $\left\{\begin{array}{c}\text{more}\\ \text{*fewer}\end{array}\right\}$.

28 This example is reminiscent of an example discussed in Beck and Rullmann 1999, namely *Four eggs are sufficient (to bake this cake)*, which likewise licenses inferences to higher numerals (if four eggs are sufficient, then so are five, six, and so on). For the specific purpose of investigating the meaning of numerical indefinites, I prefer to avoid such examples because it is unclear that *four eggs* should be analyzed as quantifying over sums of individuals (i.e. groups of eggs); for example, Rett (2014) argues that determiner phrases in such sentences denote degrees. Instead, I stick to examples like (6) and (7) and work under the assumption that the relevant expression (*three people*) ranges over sums of individuals, in the spirit of Hoeksema 1983 and Link 1987, 1991.

(54) a. Three people can lift the piano; in fact, $\left\{\begin{array}{c}\text{*more}\\ \text{fewer}\end{array}\right\}$ can.

b. Three people can lift the piano, if not $\left\{\begin{array}{c}\text{*more}\\ \text{fewer}\end{array}\right\}$.

In addition, the bound inferences do not persist when the numerical indefinite occurs in the scope of a universal modal. For example, (55) means that three *or more* people must be able to fit in the car; that is, it is compatible with more than three people being able to fit. It does not require that three and *no more than three* people be able to fit in the car.

(55) It is required that three people be able to fit in the car.

Similarly, (56) means that three *or fewer* people must be able to lift the piano; that is, it is compatible with fewer than three people being able to lift it. It does not require that three and *no fewer than three* be able to lift the piano.

(56) It is required that three people be able to lift the piano.

Finally, it is easy to construct examples where the bound inference disappears when the numerical indefinite occurs in a DE environment. For instance, the most natural reading of (57) entails that Ann doubts that more than three people can fit in the car, i.e. *three* is construed as 'three or more'.

(57) Ann doubts that three people can fit in the car.

Similarly, the most natural reading of (58) entails that Ann doubts that fewer than three people can lift the piano, i.e. *three* in construed as 'three or fewer'.

(58) Ann doubts that three people can lift the piano.

In sum, both (6) and (7) license inferences to other numerals as well as bound inferences; however, the inferences they license are the reverse of one another: (6) licenses inferences to lower numerals as well as an upper-bound inference, while (7) licenses inferences to higher numerals as well as a lower-bound inference. In addition, the bound inferences in both cases exhibit the hallmark traits of scalar implicature.

5.4 The classic view revisited

Assuming that the inferences from higher (respectively, lower) to lower (respectively, higher) numerals reported in §5.3 are valid for (6) and (7), then clearly the concomitant upper-bound (respectively, lower-bound) inferences that those sentences trigger follow from standard scalar reasoning, hence can be explained in terms of scalar implicature, in exactly the same way as the upper-bound implicature for (1) arises.

More precisely, let us assume that the following statements are true.

(59) a. *n people can fit in the car* is more informative than *m people can fit in the car* just in case $n > m$.

b. *n people can lift the piano* is more informative than *m people can lift the piano* just in case $n < m$.

Then the upper-bound inference of (6) can be derived as a scalar implicature on the basis of familiar neo-Gricean reasoning, sketched informally as follows (cf. the derivation of the upper-bound inference for (1) in §2).

(60) a. The speaker uttered (6), hence believes that three people can fit in the car (*maxim of quality*).

b. If the speaker believed that four (or five, or ...) people could fit in the car, it would have been preferable to utter *Four (five, ...) people can fit in the car* since that is more informative (*maxim of quantity*).

c. Therefore, it is not the case that the speaker believes four (or five, or ...) people can fit in the car.

d. Assuming that the speaker is knowledgable about whether four (or five, or ...) people can fit in the car, it follows that the speaker believes that it is *not* the case that four (or five, or ...) people can fit in the car.

The lower-bound inference for (7) can be derived in a completely analogous way by replacing each occurrence of *can fit in the car* with *can lift the piano* and by replacing each occurrence of *four (or five, or ...)* with *three (or two, or ...)*.

Recall now the difference in intuitive strength between the upper bound implied by (6) and the lower bound implied by (7). If these bound inferences are all scalar implicatures, then this variation in strength is perhaps to be expected, for the following reason: the downward inferences responsible for the upper bound of (6) are arguably logical, due in part to the distributivity (an arguably logical property) of *fit*, whereas the upward inferences responsible for the lower bound of (7) depend on encyclopedic knowledge of the world. That is, one can imagine possible worlds in which three people can lift the piano, but four cannot (e.g. because they cannot all fit around the piano), but one cannot imagine any possible world in which three people can fit in the car, but two people cannot.

5.5 Downward-entailing environments revisited

Although the classic, adjectival theory appears to explain everything discussed so far, there is a puzzle lurking beneath the surface. Recall from §2 that, while numerals can certainly have their one-sided readings in DE environments, they can also easily have their two-sided readings (unlike standard scalar items like *some* and *or*) — a fact that is inconsistent with a naive, scalar implicature explanation of the upper bound. We therefore assumed that numerals are special: either they "are intrinsically focused, in the sense that they automatically activate their alternatives" (Spector 2013), or they "can introduce alternatives without the help of focus" (Krifka 1999). In either case, generic numerical indefinites embedded under a DE operator should, it seems, likewise be able to have their two-sided readings.[29]

This prediction, however, is not borne out: both upper- and lower-bound inferences in generic sentences are are systematically inaccessible when the generic numerical indefinite occurs in a DE environment.[30]

For instance, I observed in §5.3 that the most natural reading of (57), repeated below, is one-sided: *three* is construed as 'three or more'. However, a stronger claim seems to hold: it does not seem possible to also interpret (57) as asserting that Ann doubts (the proposition) that three and no more than three can fit, i.e. to access a two-sided reading of *three*.

(57) Ann doubts that three people can fit in the car.

To see this more clearly, note the contrast in acceptability between the follow-up sentence in (61a), which is perfectly fine, and the follow-up sentence in (61b), which intuitively contradicts the first sentence.

(61) a. Ann doubts that only three people can fit in the car — she thinks four can.
b. *Ann doubts that three people can fit in the car — she thinks four can.

Similarly, I observed that the most natural reading of (58), repeated below, is one-sided: *three* is construed as 'three or fewer'. Once again, a stronger claim seems to hold: it does not seem possible to also interpret (58) as asserting that Ann doubts (the proposition) that three and no fewer than three can lift it, i.e. to access a two-sided reading of *three*.

(58) Ann doubts that three people can lift the piano.

Note, again, the contrast in acceptability between the follow-up sentence in

29 In §9, I will briefly explore a more nuanced position which, instead of assuming automatic activation of alternatives, takes the notion of "intrinsic focus" a bit more seriously.

30 Spector (2015) has independently made a similar point with regard to sentences like *It is sufficient to solve three problems to pass* and *On this highway, it is forbidden to drive 60 mph*, which pattern essentially like our running generic example (7) (*Three people can lift the piano*): they license inferences to higher numerals and also imply a lower bound, viz. that it is not sufficient to solve two problems and that it is not forbidden to drive 59 mph, but when these sentences are embedded under a DE operator, these inferences disappear.

(62a), which is perfectly fine, and the follow-up sentence in (62b), which intuitively contradicts the first sentence.[31]

(62) a. Ann doubts that three, but no fewer than three, people can lift the piano — she thinks two can.

b. *Ann doubts that three people can lift the piano — she thinks two can.

Similar remarks hold for the antecedent of a conditional (another DE environment). Take the contrast in (63): (63b) cannot be interpreted in the same way as (63a) to mean that if three, but no more than three, people can fit in the car, then we are too many.

(63) a. If only three people can fit in the car, then we are too many.

b. *If three people can fit in the car, then we are too many.

Similarly, (64b) cannot be interpreted in the same way as (64a).

(64) a. If three, but no fewer than three, people can lift the piano, then we are too few.

b. *If three people can lift the piano, then we are too few.

Note, finally, the following contrast, modeled on the example from Horn 2006 in (17) from §2.

(65) a. Neither lecture was attended by three people — two people attended the first one, and four people attended the second one.

b. *Neither car fits three people — this one fits two people, and that one fits four.

c. *Neither piano can be lifted by three people — two people can lift this one, and four people can lift that one.

The moral of the story is that there is a systematic asymmetry in the availability of two-sided readings of numerical indefinites occurring in the scope of a DE operator: they are available for existential indefinites, but not for generic indefinites. The classic, adjectival account that was motivated and developed in §2 and §3

31 For some reason, *Ann doubts that only three people can lift the piano* also fails to have the reading that Ann doubts that the lower bound is three, which is why I use *three, and no fewer than three*. This fact is surely connected to the fact that **Only three people can lift the piano* is very odd as a generic sentence: it cannot seem to be used to express the generalization that three people can lift the piano, but no fewer than three (cf. *It takes three people to lift the piano*). I have no explanation for why *only* is restricted in this way, but my hunch is that for some reason *only* seems to care about the natural ordering of numbers, and hence only ever negates numerical alternatives containing a numeral that is greater than the numeral in the prejacent. I leave a more detailed investigation of this puzzle for future research.

is unable to cope with this observation. Either two-sided readings of numerical indefinites embedded under a DE operator should never be available (the standard view), or, if numerals automatically evoke their alternatives, then their two-sided readings should always be available.

5.6 Brief excursus: Against a widescope modal analysis

A reviewer asks whether sentences like (6) and (7) are really generics, or whether, instead, what is going on has more to do with the modality being expressed. To be more concrete, perhaps the quasi-universal reading should be represented by an LF where the modal *can* takes wide scope and the propositional argument of the modal is a simple existential statement, as shown below. (I will use just the piano-lifting example for illustration.)

(66) a. can [[[three ⟨many⟩] people] [lift the piano]]

b. $\Diamond \exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{lift}(x)]$

This interpretation can be paraphrased as 'there is an accessible world w such that, in w, a group of three people lift the piano'.

First, it seems clear that numerical indefinites can be interpreted generically even in the absence of any modal, as in *Three cars are more expensive than two* (Hoeksema 1983), *Two's company, three is a crowd* (Link 1987), *Five people make a good team*, and so on.[32] In fact, even (6) need not contain the modal *can*: *Three people fit in the car* (not on its episodic reading, of course) means the very same thing as (6). The main reason for focusing on examples that feature the ability modal *can* is that these examples give rise to clear lower- and upper-bound inferences, which in turn reveal a new challenge for the classic, scalar implicature theory of numerical interpretation.

Nevertheless, just because numerical indefinites *can* have generic readings (presumably involving *Gen*) does not, in and of itself, mean that they do in (6) and (7). Perhaps in those cases, a widescope modal analysis like (66) is indeed correct. I have three arguments against such an analysis.

32 The same reviewer, citing Declerck 1988, points out that numerical indefinites seem to resist generic interpretations, on the basis of contrasts like the following.

(i) a. Lions eat antelopes when they live in prides.

b. *Three lions eat antelopes when they live in prides.

However, this example crucially features the distributive predicate *eat antelopes*. As I observed in fn. 26, it is actually distributivity that causes numerical indefinites in such cases to resist generic interpretations, because the numeral ends up being vacuous. In this case, (ib) states that any group of three lions is such that (when they live in prides) each of them eats antelopes, which is tantamount to just saying that every single lion (when it lives in a pride) eats antelopes, which is precisely what (ia) asserts.

Argument #1. The first, and perhaps most important, argument is that the interpretation derived on this analysis is far too weak to represent the quasi-universal reading of (7) that we are interested in. Recall that (7) entails that if Ann, Bill, and Carol are three (typical) people, then they can lift the piano. Without some kind of further strengthening, the widescope modal interpretation is too weak to capture this inference: it just says that in *some* accessible world, there is *some* group of three people (perhaps three extraordinarily strong people, i.e. not necessarily Ann, Bill, and Carol) who lift the piano. More formally, the following argument is invalid.

(67) $\Diamond\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{lift}(x)]$
$\mathsf{people}(a \sqcup b \sqcup c)$
$\not\Rightarrow$ $\Diamond\mathsf{lift}(a \sqcup b \sqcup c)$

Argument #2. Second, observe that weak negative polarity items (NPIs) are licensed inside of generically interpreted numerical indefinites.

(68) Three people with any experience in the moving business can lift the piano.

This observation is fully expected on the generic analysis in (48), because there *three* occurs in a DE environment, viz. the restrictor of *Gen*, just like the restrictor of *every* (Ladusaw 1979). However, it is not expected on the analysis in (66), because there *three* occurs in an upward-entailing environment.

Argument #3. Finally, note that (69) is perfectly acceptable and interpretable but, crucially, is intuitively weaker than (7) (on its quasi-universal reading).

(69) It is possible for three people to lift the piano.

For instance, the following argument is intuitively invalid.

(70) It is possible for three people to lift the piano.
Ann, Bill, and Carol are three (typical) people.
$\not\Rightarrow$ It is possible for Ann, Bill, and Carol to lift the piano.

Moreover, (69) fails to license weak NPIs.

(71) It is possible for three people with $\left\{ \begin{array}{c} \text{*any} \\ \text{some} \end{array} \right\}$ experience in the moving business to lift the piano.

That (7) and (69) differ intuitively both in meaning and in NPI licensing is strong evidence for different representations. Specifically, (69) appears to be best represented with a widescope modal, as in (66), while (7) is best represented with a generic operator, as in (48).

In sum, while (66) may represent one reading of (7), it does not represent what I, and Link (1987, 1991), take to be the quasi-universal reading of (7), which I henceforth assume involves the generic operator, *Gen*, as in (48).

6 Alternative views

In this section, I present three recent approaches to the semantics of numerals, which differ from the classic approach in that they all take the basic, literal meaning of a numeral like *three* to be one that gives rise to a two-sided, 'exactly' interpretation in a simple existential sentence like (1), discussed in §2. In other words, the 'exactly' construal of *three* in (1) is not a scalar implicature, but rather a semantic entailment arising from the literal meaning of *three*. One advantage of such an approach is that it immediately and correctly predicts the ability of numerals, observed in §2.2, to have a two-sided, 'exactly' reading even in DE environments: it is simply their normal meaning. In §7, I will show that (a version of) this approach also correctly predicts the *inability* of generic numerical indefinites to acquire a two-sided reading in DE environments, thus explaining the puzzling asymmetry between existential and generic contexts that was revealed in §5.5. Despite this achievement, however, in §8, I will show that the 'exactly' approach leads to problems of overgeneration when it comes to existential sentences with collective predicates, like (3), (4), and (5), discussed in §4.

6.1 Unique groups (Geurts 2006)

Recall the logic of why the classic, adjectival account of numerals developed in §3 derives a one-sided, 'at least' reading for (1) on the basis of the representation in (25b), repeated below: even if more than three people attended, then it is still true that a group of exactly three people attended. For example, if the attendees were Ann, Bill, Carol, and Dan (four in total), then Ann, Bill, and Carol make a group of exactly three people who attended, and so do Bill, Carol, and Dan (and so do the remaining two combinations of three of them).

(25b) $\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

In general, if a total of n people attended (for $n > 1$), then for any non-zero $m < n$, there is more than one group of m people who attended. Thus, if we wish to turn the one-sided representation in (71) into a two-sided representation, one way that readily comes to mind is to require that there be a *unique* group of three people who attended. (The notation "$\exists x![\ldots]$" is to be read as 'there is a unique (one and only one) x such that …'.)

(72) $\exists! x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

Geurts (2006) proposes exactly this sort of meaning for a sentence like (1). To get there compositionally, he proposes the lexical entry in (73) for the basic meaning of *three*. This entry is identical to the quantifier entry for *three* that we briefly entertained in (26c), repeated below, except for one crucial difference: the presence of "!". Because this account involves a uniqueness condition on existential quantification over groups, I will refer to it as UniqGrp.

(73) $[\![\text{three}_{\text{UniqGrp}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists! x[\#x = 3 \wedge P(x) \wedge Q(x)]$

(26c) $[\![\text{three}_{\text{quant}_=}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\#x = 3 \wedge P(x) \wedge Q(x)]$

Under UniqGrp, (1) is analyzed as in (74b). To reiterate, the effect of the uniqueness condition is to derive an 'exactly' reading of *three* here, because if more than three people attended, then there is necessarily more than one group of three people who attended (hence, no unique such group), and so the sentence is false. As a result, the sentence means that three, and no more than three, people attended.

(74) a. [three$_{\text{UniqGrp}}$ people] attended
b. $\exists! x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

In the specific case of distributive existential sentences, UniqGrp amounts to maximizing over groups, albeit somewhat indirectly: it is the precise combination of existential quantification, distributivity, and uniqueness that leads to maximization. In essence, the uniqueness condition disrupts the inferential pattern that would otherwise lead to a one-sided, 'at least' reading, which we saw manifested in the classic, adjectival account.

6.2 Maximal groups (Breheny 2008)

Rather than impose a uniqueness condition, we could simply maximize over groups directly, by collecting together all the people who attended and requiring that the resulting group have cardinality 3, as described by the formula in (75). It states that the maximal group that can be formed from individuals who are people and who attended has cardinality 3.

(75) $\#(\bigsqcup\{x : \mathsf{people}(x) \wedge \mathsf{attend}(x)\}) = 3$

Breheny (2008) proposes this sort of meaning for a sentence like (1). The lexical meaning of *three* is represented as in (76).[33] What *Three P Q* means is the following:

33 Actually, for the meaning of a numerically quantified noun phrase like *three P*, Breheny proposes the representation in (ia). Translated to our current formalism, in which pluralities are modeled as sums, not sets, (ia) becomes (ib). Abstracting over *P* yields (76) as the meaning of *three*.

the maximal group that can be formed by summing up individuals (or groups) that are both P and Q has cardinality 3. For this reason, I will refer to this account as MaxGrp.

(76) $[\![\text{three}_{\text{MaxGrp}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = 3$

Under MaxGrp, (1) is analyzed as in (77), which, to repeat, is a two-sided, 'exactly' meaning.

(77) a. [$\text{three}_{\text{MaxGrp}}$ people] attended

b. $\#(\bigsqcup\{x : \mathsf{people}(x) \wedge \mathsf{attend}(x)\}) = 3$

Due to the "$\bigsqcup$" operator, MaxGrp forms (hence, asserts the existence of) a maximal group, relative to two properties (and then assigns that group a cardinality). What UniqGrp and MaxGrp have in common, then, is that they build existential quantification into the meaning of *three* and, in the specific case of (1), they maximize over groups, albeit in different ways.[34] This feature (hardwiring existential quantification) is what turns out to allow these two accounts to correctly predict the distribution of two-sided readings of numerals in DE contexts, as I will show in §7. However, as we will see in §8, the two approaches diverge in interesting ways when it comes to existential sentences with collective predicates, and both make incorrect predictions.

6.3 Maximal cardinalities (Kennedy 2015)

Instead of quantifying (and maximizing) over groups, Kennedy (2013, 2015) takes a quite different approach. Building on successful, degree-theoretic analyses of modified numerals (e.g. *more than two*, *fewer than five*), he analyzes *three* as a generalized quantifier over degrees (type $(dt)t$), which encodes a maximality operator, as shown in (78): *three P* asserts that the maximum degree in the extension of P is 3.[35] In the case where P involves quantification over groups (the only case

(i) a. $\lambda Q \,.\, |\bigcup\{X : P(X) \wedge Q(X)\}| = 3$

b. $\lambda Q_{et} \,.\, \#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = 3$

34 To see more clearly that MaxGrp hardwires existential quantification into its lexical entry for *three*, note the equivalence between the formulation in (76) and $\lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists y[\#y = 3 \wedge y = \bigsqcup\{x : P(x) \wedge Q(x)\}]$.

35 Kennedy describes this semantics for numerals as "de-Fregean": for Frege (1884), *three* is a second-order property of individuals, which is true of a property of individuals just in case that property is true of exactly three individuals; and similarly, for Kennedy, *three* is a second-order property of degrees, which is true of a property of degrees just in case that property has a maximum of 3. Put differently, on Frege's view, *three* denotes the equivalence class of sets of individuals whose cardinality is 3, while on Kennedy's view, *three* denotes the equivalence class of sets of degrees whose maximum is 3. An arguably more direct instantiation of a "Fregean" semantics for numerals

considered in this article), this approach amounts to maximizing over cardinalities of groups with certain properties. For this reason, I will refer to this account as MaxCard.

(78) $[\![\text{three}_{\text{MaxCard}}]\!] = \lambda P_{dt} \,.\, \max(P) = 3$

Under MaxCard, the higher-order version of *three* in (78) cannot combine directly with a noun phrase like *people* (or with ⟨*many*⟩), so it must move (QR), leaving behind a trace of type *d* that can combine with ⟨*many*⟩, and thus creating a degree predicate (type *dt*) in its scope, which serves as its argument, as illustrated in (79). The meaning derived for (1) can be paraphrased as 'the maximal cardinality *n* such that a group of *n* people attended is 3', which of course is equivalent to saying that *exactly* three people attended.

(79) a. $\text{three}_{\text{MaxCard}}$ [1 [[[t_1 ⟨many⟩] people] attended]]

b. $\max(\lambda n_d \,.\, \exists x[\#x = n \wedge \textsf{people}(x) \wedge \textsf{attend}(x)]) = n$

Similar to the uniqueness condition under UniqGrp, the presence of a maximality operator under MaxCard disrupts the derivation of an 'at least' reading, which would otherwise obtain, as we saw for the classic account. Unlike UniqGrp and MaxGrp, however, MaxCard does not build existential quantification into the meaning of *three* (the degree predicate may involve *Gen*, for instance) — a fact that will become important when we revisit genericity in §7.

6.4 Deriving 'at least' readings via typeshifting

All three accounts presented above — UniqGrp, MaxGrp, and MaxCard — posit a meaning for *three* that derives an 'exactly' reading in a basic existential sentence like (1). However, we also saw in §2 that *three* can be used predicatively and that it sometimes receives an 'at least' interpretation. All three accounts can derive these uses by appealing to some combination of typeshifting operators (Partee 1987).

The first typeshifter that we need is BE, given in (80). When BE applies to the quantifier meaning of *three*$_{UniqGrp}$ *people* or *three*$_{MaxGrp}$ *people*, it returns the same predicative (adjectival), i.e. type *et*, meaning as *three*$_{pred}$ *people*.[36] (The lexical entry

is provided by Rothstein (2013), who assumes that *three* is born at type *et* and denotes the class of all sums of individuals consisting of exactly three atomic parts — which of course is a version of the classic, adjectival approach to numerals. (See also fn. 21.)

36 Breheny (2008) (whose account was presented as MaxGrp) does not actually assume any typeshifting operator. Instead, he assumes that nouns can be implicitly restricted, so that *Three people attended* means 'exactly three people who are *P* attended', for some arbitrary property *P*. If *P* cannot be determined by context, then the sentence winds up meaning, essentially, 'there is a *P* such that exactly three people who are *P* attended', which is equivalent to 'three or more people attended', since, if more than three people attended, we can always find a *P* such that exactly three of the attendees are *P*.

for *three*$_{pred}$ was introduced in (26a) in §3.2, in the context of the classic, adjectival theory of numerals.)

(80) $\mathsf{BE} = \lambda \mathcal{P}_{(\alpha t)t} \,.\, \lambda x_\alpha \,.\, \mathcal{P}(\lambda y_\alpha \,.\, y = x)$

(81) $\mathsf{BE}(\llbracket \text{three}_{\text{UniqGrp}} \text{ people} \rrbracket)$
$\equiv \mathsf{BE}(\llbracket \text{three}_{\text{MaxGrp}} \text{ people} \rrbracket)$
$\equiv \lambda x_e \,.\, \#x = 3 \wedge \mathsf{people}(x)$
$\equiv \llbracket \text{three}_{\text{pred}} \text{ people} \rrbracket$

When BE applies the degree quantifier meaning of *three*$_{MaxCard}$, it returns the property of being equal to 3. Only 3 itself has this property, and so a pure degree meaning (the same one we eventually posited for *three* on the classic account) can be recovered with our second typeshifter, iota, given in (82). By applying BE and then iota to the meaning of *three*$_{MaxCard}$, we derive the number 3, which can then combine with (the meaning of) ⟨*many*⟩, just like on the classic account, to return a predicative meaning.

(82) $\mathsf{iota} = \lambda P_{\alpha t} \,.\, \iota x_\alpha [P(x)]$

(83) $\mathsf{iota}(\mathsf{BE}(\llbracket \text{three}_{\text{MaxCard}} \rrbracket))$
$\equiv \mathsf{iota}(\lambda m_d \,.\, m = 3)$
$\equiv 3$

(84) $\llbracket \langle \text{many} \rangle \rrbracket (\mathsf{iota}(\mathsf{BE}(\llbracket \text{three}_{\text{MaxCard}} \rrbracket)))$
$\equiv \lambda x_e \,.\, \#x = 3$
$\equiv \llbracket \text{three}_{\text{pred}} \rrbracket$

The BE operator therefore allows UniqGrp, MaxGrp, and MaxCard (along with iota) to analyze predicative uses of *three*, as in (21) (*We are three people*), discussed in §3.1.

The third, and final, typeshifting operator is A, which was introduced in §3.2 for the classic, adjectival theory and is repeated below. Unsurprisingly, all three accounts are able derive an 'at least' reading for (1) by first applying BE to (the meaning of) *three* (and then applying iota and ⟦⟨many⟩⟧, in the case of MaxCard), and then applying A to the resulting predicative meaning, just as we did in §3 for the adjectival approach.

(28) $\mathsf{A} = \lambda P_{\alpha t} \,.\, \lambda Q_{\alpha t} \,.\, \exists x_\alpha [P(x) \wedge Q(x)]$

More generally, typeshifting allows UniqGrp, MaxGrp, and MaxCard to replicate all the same desirable results that the classic account gets right. This includes deriving a basic existential reading for sentences like (3), (4), and (5), with the collective predicates *lift the piano together*, *form a team*, and *have the same name*, as well as generic readings of numerical indefinites in sentences like (6) (with *can fit* *in the car*) and (7) (with *can lift the piano*).

7 Genericity revisited: A triumph for (two of) the 'exactly' accounts

Let us now return to the two generic sentences, (6) (*Three people can fit in the car*) and (7) (*Three people can lift the piano*), discussed in §5, to see what is predicted by the three alternative accounts presented in §6 (UniqGrp, MaxGrp, and MaxCard).

7.1 UniqGrp and MaxGrp

Typeshifting allows all three accounts to replicate the classic account's analysis of generic readings of numerical indefinites in sentences like (6) and (7), which are assigned a quasi-universal semantics, with an upper or lower bound arising by way of scalar implicature.[37] Notably, there is no way for UniqGrp or MaxGrp to derive the bound inferences in generic contexts as semantic entailments. This is because, as already mentioned (see fn. 34), they both hardwire existential quantification into the basic, lexical meaning of *three*. As a result, under UniqGrp and MaxGrp, upper-bound inferences in basic existential cases like (1) are semantic entailments, while both upper- and lower-bound inferences in generic cases like (6) and (7) are scalar implicatures.

Recall now, from §5.5, the puzzle that we uncovered regarding numerical indefinites embedded under a DE operator: it was shown that when the numerical indefinite is existential, a two-sided reading is available (an old observation), but when it is generic, only a one-sided reading is available (a new observation). Remarkably, because UniqGrp and MaxGrp derive two-sided readings as a matter of semantics only in existential contexts, not in generic contexts, they predict exactly this fact. It is the combination of (i) hardwiring existential quantification and maximization into the meaning of *three* and (ii) allowing for *three* to typeshift into an adjectival type that leads to the right result.

Notably, an approach that achieves this result need not be cashed out exactly as UniqGrp or MaxGrp does. Another option, for example, is to maintain that *three* denotes the number 3 and, in addition to the (weak) ⟨*many*⟩ in (31), to posit another, 'strong' version of ⟨*many*⟩, defined below (Nouwen 2010).

37 The facts surrounding generic readings of numerical indefinites appear to require an amendment to Breheny's original theory, which does not involve any typeshifting. Recall (fn. 36) that Breheny's way of deriving 'at least' readings in existential contexts is *not* to typeshift *three NP* into a predicative type and then existentially quantify on top; rather, he leaves *three NP* as an existential quantifier with an 'exactly' meaning but assumes that the noun phrase can be implicitly restricted: 'For some property *P*, exactly three NPs are *P* and VP'. So, to the extent that generic sentences like (6) and (7) do indeed involve a generic operator, it follows that some kind of type-lowering operation is required, which (as long as a mechanism for introducing existential quantification is also available, e.g. A) thereby renders the assumption that noun phrases are implicitly restricted superfluous.

(85) $[\![\langle\text{many}\rangle_{\text{strong}}]\!] =$

a. $\lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists! x[\#x = n \wedge P(x) \wedge Q(x)]$ (à la UniqGrp)

b. $\lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = n$ (à la MaxGrp)

In any case, the take-home message is that the right result is achieved when existential quantification and maximization occur hand-in-hand, such as in the lexical entries for $\textit{three}_{\textit{UniqGrp}}$ and $\textit{three}_{\textit{MaxGrp}}$, or in either of the two lexical entries for $\langle \textit{many} \rangle_{\textit{strong}}$.

7.2 MaxCard

MaxCard, unlike UniqGrp and MaxGrp, predicts the generic sentences (6) and (7) to be ambiguous between the readings described in §5 and another reading, in which $\textit{three}_{\textit{MaxCard}}$ is not typeshifted, but instead instead scopes above *Gen*, resulting in an upper-bound inference as a matter of semantics.

(86) a. $\text{three}_{\text{MaxCard}}$ [1 [[Gen [[t_1 ⟨many⟩] people]] [can fit in the car]]]

b. $\max(\lambda n_d \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathsf{people}(x)] \to \mathsf{can.fit}(x)]) = 3$

For (6), the reading so derived can be paraphrased as ‘the maximum cardinality n such that, in general, any group of n people can fit in the car is 3’, which is precisely the upper-bound generic reading we want to capture, except that the upper bound is a semantic entailment.

In addition, (7) should have an analogous parse, where the numeral scopes above *Gen*. The reading thus derived is paraphrasable as ‘the maximum cardinality n such that, in general, any group of n people can lift the piano is 3’. This reading is clearly not the lower-bound generic reading we wish to derive. Moreover, if we continue to assume that (or restrict ourselves to contexts in which) (7) licenses inferences to higher numerals, then the reading derived in (87) is a (contextual) contradiction: there is no maximum cardinality n such that any group of n people can lift the piano. Presumably, however, such an LF can be ruled out on the basis of its contradictoriness.

(87) a. $\text{three}_{\text{MaxCard}}$ [1 [[Gen [[t_1 ⟨many⟩] people]] [can lift the piano]]]

b. $\max(\lambda n_d \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathsf{people}(x)] \to \mathsf{can.lift}(x)]) = 3$

Under MaxCard, then, upper-bound inferences in both existential and generic cases are semantic entailments, while lower-bound inferences in generic cases are scalar implicatures. This means that MaxCard correctly predicts a lower-bound reading of (7) to be unavailable when it occurs in a DE environment, but it incorrectly predicts an upper-bound reading of (6) to be available when it occurs in a DE environment: simply embed the LF in (86) in a DE environment, as in *Ann doubts that (86)* (cf. (57)). In other words, because MaxCard, unlike UniqGrp and

MaxGrp, sometimes derives two-sided readings of generic numerical indefinites as a matter of semantics, it (like the classic, adjectival account) fails to explain the embedding facts that UniqGrp and MaxGrp get right.

These points rely, of course, on the assumption that *three* is allowed to scope above *Gen*, i.e. that the problematic LF in (86) is generated in the first place.[38] However, one might try and contest this assumption. In particular, the scope-taking ability of a degree phrase (DegP) is known to be constrained in certain ways. For example, Heim (2000) points out that (88a) has the reading represented in (88b), which says that every girl is such that she is less than 4′ tall, but not the one represented in (88c), which says merely that the shortest girl is less than 4′ tall.

(88) a. (John is 4′ tall.) Every girl is less tall than that.

b. [every girl] [1 [[less than 4′] [2 [t_1 is t_2 tall]]]]
$\forall x[\mathsf{girl}(x) \to \max(\lambda d\,.\,\mathsf{tall}(x)(d)) < 4']$

c. [less than 4′] [2 [[every girl] [1 [t_1 is t_2 tall]]]]
$\max(\lambda d\,.\,\forall x[\mathsf{girl}(x) \to \mathsf{tall}(x)(d)]) < 4'$

The observation here is that the DegP *less than 4′* apparently cannot scope above the quantificational DP *every girl*. The more general observation, known as the Heim-Kennedy Generalization (Kennedy 1997; Heim 2000), can be stated as in (89). The LF in (88b) obeys this generalization, since the scope of *every girl* contains both *less than 4′* and its trace (t_2), but the LF in (88c) does not, since the scope of *every girl* contains the trace of *less than 4′* (t_2) but not *less than 4′* itself.

(89) If the scope of a quantificational DP contains the trace of a DegP, then it also contains that DegP itself.

The question now is whether the problematic LF in (86) violates this generalization. If it does, then it is independently ruled out by whatever is responsible for the Heim-Kennedy Generalization. As far as I am aware, no one has examined the interaction of degree phrases and genericity. However, two simple arguments can be put forth that suggest that DegP movement over *Gen* is *not* subject to the Heim-Kennedy Generalization, hence that the problematic LF in (86) would be derivable on a MaxCard, DegP-style analysis of *three*.

First, if we move from bare numerals to comparative constructions, we find that DegPs are apparently happy to move above *Gen*. Let us start with a simple comparative involving existential quantification: (90) says that the number of people who like coffee exceeds the number of people who like tea. This interpretation is straightforwardly captured on any standard analysis of comparatives. For

38 I thank an anonymous reviewer for pushing me to discuss the interaction of numerals (and degree phrases more generally) and genericity, and in particular for raising the question of whether such interaction might be subject to the Heim-Kennedy Generalization.

concreteness, I will adopt the popular assumption that the comparative morpheme *-er* compares two sets of degrees P and Q and says that the maximum of Q exceeds the maximum of P. The first argument is the degree predicate denoted by the *than*-clause, and the second argument is the degree predicate formed by movement of the DegP *-er 2 (than)* t_2 *⟨many⟩ people like tea*. (I make the simplifying assumption that *⟨many⟩ people like* is silent in the *than*-clause but present at LF.)

(90) a. More people like coffee than tea.

b. [-er 2 [(than) t_2 ⟨many⟩ people like tea]] 1 [t_1 ⟨many⟩ people like coffee]

c. $\max(\lambda n \,.\, \exists x[\#x = n \wedge \mathsf{like.coffee}(x)]) > \max(\lambda n \,.\, \exists x[\#x = n \wedge \mathsf{like.tea}(x)])$

Now consider (91), which is a comparative construction that is similar to our generic example (6), except that, instead of saying that the number of people who, in general, can fit in the car is 3, it says that that number exceeds the number of people who, in general, can fit in the truck. As the LF illustrates, this reading can be captured straightforwardly, and in a parallel way to (90), by allowing the DegP *-er 2 (than) Gen* t_2 *⟨many⟩ people can fit in the truck* to scope above *Gen*.[39]

(91) a. More people can fit in the car than (can fit in) the truck.

b. [-er 2 [(than) Gen t_2 ⟨many⟩ people can fit in the truck]]
1 [Gen t_1 ⟨many⟩ people can fit in the car]

c. $\max(\lambda n \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.fit.car}(x)])$
$> \max(\lambda n \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.fit.truck}(x)])$

Second, the choice of the typeshifting operator A (for introducing existential quantification) was made purely for convenience (see fn. 18). It is also possible that there is a covert existential determiner, ∃, in the syntax (Link 1987; Krifka 1999),

39 The reader may object that the comparative construction in (91) does not involve genericity at all, and that a better analysis involves the modal *can* scoping above existential quantification (cf. §5.6), and the relevant DegP scoping above the modal, as in (ib).

(i) a. More people can fit in the car than (can fit in) the truck.

b. [-er 2 [(than) can [t_2 ⟨many⟩ people fit in the truck]]] 1 [can [t_1 ⟨many⟩ people fit in the car]]

c. $\max(\lambda n \,.\, \Diamond \exists x[\#x = n \wedge \mathsf{people}(x) \wedge \mathsf{can.fit.car}(x)])$
$> \max(\lambda n \,.\, \Diamond \exists x[\#x = n \wedge \mathsf{people}(x) \wedge \mathsf{can.fit.truck}(x)])$

But if this is the correct analysis of (91), then one would expect the analysis of (6) in (iib) to be correct/possible.

(ii) a. Three people can fit in the car.

b. $\text{three}_{\text{MaxCard}}$ [1 [can [t_1 ⟨many⟩ people fit in the car]]]

c. $\max(\lambda n \,.\, \Diamond \exists x[\#x = n \wedge \mathsf{people}(x) \wedge \mathsf{fit}(x)]) = 3$

If so, then we would again incorrectly predict the upper-bound reading of *three* in (6) to be accessible even when the clause occurs in a DE environment, e.g. by parsing (57) (*Ann doubts that three people can fit in the car*) as *Ann doubts that (iib)*.

which numerals clearly must be able to scope above on the MaxCard account, in order to derive upper-bounded construals of sentences like (1) as a matter of semantics.

(92) a. Three people attended.

b. three$_{\text{MaxCard}}$ [1 [∃ t_1 ⟨many⟩ people attended]]

c. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \textsf{people}(x) \wedge \textsf{attend}(x)]) = 3$

Assuming that ∃ and *Gen* are similar covert syntactic elements, it would be unexpected (even, or especially, from the perspective of the Heim-Kennedy Generalization) for numerals to be able to scope freely above ∃ but be banned from scoping above *Gen*.

In sum, I see no reason (syntactic or semantic) why, on a MaxCard-style analysis, bare numerals should be banned from scoping above *Gen*, and in fact there is evidence suggesting that they *should* do so.

8 Collectivity revisited: A problem for all three 'exactly' accounts

We have seen (§7) that two of the three 'exactly' approaches to numerals presented in §6 — UniqGrp and MaxGrp, but not MaxCard — remarkably explain the previously puzzling embedding facts observed in §5.5, viz. that two-sided readings of existential numerical indefinites embedded under a DE operator are accessible, while two-sided readings of generic ones are not. They achieve this result by (i) hardwiring existential quantification and maximization into the basic, lexical meaning of numerals, which derives two-sided readings in existential contexts as a matter of semantics, and (ii) allowing numerals to typeshift into an adjectival type, which derives two-sided readings in generic contexts as implicatures. (By the same token, MaxCard fails to make the right predictions because it allows maximization to occur even in generic contexts, which derives two-sided readings of generic numerical indefinites as a matter of semantics.)

Unfortunately, it is feature (i) that leads to incorrect results when we return to existential sentences with collective predicates. Specifically, all three 'exactly' accounts predict an existential sentence with a collective predicate, like (3) (*Three people lifted the piano together*), (4) (*Four players formed a team*), and (5) (*Five students have the same name*), to be ambiguous between a basic existential reading (the same one derived by the classic, adjectival account, here via typeshifting), which is attested, and a stronger reading (slightly different in each case), which I will argue is unattested. We review each account's predictions in turn.

8.1 UniqGrp

In addition to the basic existential reading of (3) derived by typeshifting, UniqGrp also predicts the following reading to be available, where *three*$_{UniqGrp}$ *people* is not typeshifted at all.

(93) $[\![\text{three}_{\text{UniqGrp}}\ \text{people}]\!]([\![\text{lifted the piano together}]\!])$
$\equiv \exists! x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{lift}(x)]$

This reading can be paraphrased as 'there is a unique group of three people who lifted the piano'. On this reading, the sentence is compatible with smaller and/or larger groups of people having lifted the piano, just as long as one and only one group of *three* people did so. However, no such reading appears to be accessible. For example, if Ann, Beth, and Carol lifted the piano together, and Dan, Evan, and Fred lifted it together, then the sentence is simply true — there is no reading on which it is false. To be more precise, to the extent that the sentence feels false at all, this is presumably due to the potential implicature, described in §3 (see fn. 23), that no more than one group of people lifted the piano. In other words, there is no contrast in truth value judgment for sentence (3) between a context in which two groups of three people lifted it (and no one else did) and a context in which one group of three people lifted it and another group of more or less than three people lifted it (and no one else did), but UniqGrp predicts there to be a contrast: the sentence should be judged true in the latter context and false in the former context.[40, 41]

40 One may try and explain the lack of contrast in terms of the notion of *charity*, which hypothesizes that if a sentence is true on one reading and false on another, listeners will simply judge the sentence true (Gualmini et al. 2008). However, if we embed the sentence in a DE environment, then charity is no longer at play (or more precisely, the 'direction' of charity should be reversed). Consider (i). In a scenario where two different groups of three people lift the piano and Ann loses the bet, UniqGrp predicts (i) to be ambiguous between a true reading and a false one. (Moreover, if charity is at play, listeners should simply judge the sentence true.) However, the sentence is intuitively false in such a scenario. In other words, Ann clearly wins the bet if two different groups of three people lift the piano.

(i) If three people lift the piano together, then Ann wins the bet.

41 An anonymous reviewer, commenting on a previous version of this article, suggests that perhaps the semantic effect of *together* is to require that the event participants introduced by the subject noun phrase (e.g. *Ann and Bill* or *three people*) are participants in the same event. In other words, (3) describes a single event, which would make the uniqueness condition redundant and overall deliver the same weak existential meaning that could be derived by typeshifting ('there is an event *e* of piano-lifting such that there is a (unique) group of three people who are the agent of *e*'). First, let me stress that, even if we remove *together* from the sentence and just consider *Three people lifted the piano*, the uniqueness reading predicted by UniqGrp is not intuitively available: it cannot be interpreted as 'there is a unique group of three people who lifted the piano'. Second, the very same problem arises for (4) (with *form a team*) and (5) (with *have the same name*), despite the fact that these examples do

Similar remarks hold for (4) and (5). For instance, (4) does not have the predicted reading 'there is a unique group of four players who formed a team'. There is no contrast in truth value judgment between a context in which two different groups of four players formed a team (it should be false) and a context in which one group of four players and another group of more or less than four players formed a team (it should be true).

(94) $[\![\text{four}_{\text{UniqGrp}}\text{ players}]\!]([\![\text{formed a team}]\!])$
$\equiv \exists! x[\#x = 4 \wedge \mathsf{players}(x) \wedge \mathsf{form.a.team}(x)]$

Likewise, (5) does not have the predicted reading 'there is a unique group of five students who have the same name'. Recall that *have the same name* licenses downward inferences: if a group x has the same name, then every subgroup y of x (for $\#y \geq 2$) has the same name. As a result, the uniqueness condition rules out the existence of any group of more than five students having the same name (the same as the upper-bounding effect of uniqueness in distributive cases) — which is perhaps a good thing, since (5) is indeed judged odd in a context where more than five students have the same name (cf. §4). However, UniqGrp predicts a contrast in truth value judgment between a context in which exactly five students are named Ann and exactly five students are named Bill (it should be false) and another context in which exactly five students are named Ann and exactly four students are named Bill (it should be true), but no such contrast is detectable.

(95) $[\![\text{five}_{\text{UniqGrp}}\text{ students}]\!]([\![\text{have the same name}]\!])$
$\equiv \exists! x[\#x = 5 \wedge \mathsf{students}(x) \wedge \mathsf{same.name}(x)]$

What is going on is that the uniqueness condition that was imposed to derive maximality effects in distributive cases has disastrous consequences in collective cases: *three NP*, it seems, simply does not assert any kind of uniqueness.

8.2 MaxGrp

As Breheny (2008, fn. 9) observes, MaxGrp's representation for *three* in (76) "allows for a reading of *Three people lifted the piano* which is true if John single-handedly lifted the piano and Bill and Sam also did". This is because, under MaxGrp, the meaning of the sentence is basically 'the maximal group of people who single-handedly lifted the piano, or were part of a group who lifted the piano, is 3'. More precisely, in a context where John (j) single-handedly lifted the piano and Bill and Sam ($b \sqcup s$) also did so together (and no one else did any piano lifting), the maximal group of all groups who are people and who lifted the piano ($j \sqcup b \sqcup s$) has cardinality 3.

not feature *together*. In other words, the problem cannot just be pinned on *together*.

While I grant that Breheny's sentence can be judged true in the context described above, our running example, (3) (*Three people lifted the piano together*), crucially features the word *together*, and it is clear that this sentence is *false* in that same context. I therefore take this to be a wrong prediction for MaxGrp.[42]

But then what about the interpretation of Breheny's example, without *together*? Let me explain what I think is going on here, and in particular how the classic account can capture what I will call 'Breheny's reading' of the sentence, which involves two components: (i) it is compatible with no group of three people actually lifting the piano (e.g. if John single-handedly lifted it, and Bill and Sam also did so together, and no one else did any piano lifting), and (ii) it implies an upper bound on the number of people who lifted, or participated in lifting, the piano.

First, let us note that *people* and *lifted the piano* both have cumulative reference (Krifka 1989): If Ann and Bill are people, and if Carol and Dan are people, then it follows that Ann, Bill, Carol, and Dan are people; and similarly, if Ann and Bill lifted the piano, and if Carol and Dan lifted it, then it follows that Ann, Bill, Carol, and Dan lifted the piano (note the absence of *together*!).[43, 44] Now suppose that John single-handedly lifted the piano and that Bill and Sam also did so together (and that no one else did any piano lifting). Then not only are John (the atomic individual j) and Bill and Sam (the plural individual $b \sqcup s$) in the extension of both *people* and *lifted the piano*, but by the cumulativity of *people* and *lifted the piano*, so is their combined sum ($j \sqcup b \sqcup s$). As a result, it follows that there is a group of three people who 'lifted the piano' (in the cumulative sense), and so the sentence *Three people lifted the piano* comes out true in such a scenario.

What about the upper bound implied under Breheny's reading, which is an entailment under MaxGrp? Krifka (1992, 1999) has independently observed that cumulative predicates give rise to scalar inferences: *Three people ate seven apples* implies that the total number of people who ate apples is three. Since *lift the piano*, like *eat (seven) apples*, can be interpreted cumulatively, an upper-bound scalar implicature is expected to arise.[45] I take its status as an implicature, instead of an

42 One could potentially say something special about the semantic role of *together* that might rule out this reading (see fn. 41). However, as we will see shortly, MaxGrp runs into trouble with (4) and (5), as well, which do not feature the word *together*.

43 One way to capture the cumulative reference of *people* and *lifted the piano* is to assume that they are parsed with a pluralizing operator, usually notated as "*", which closes their extension under sum (Link 1983).

44 For readers who reject cumulative inferences with *lifted the piano*, see fn. 46.

45 Nevertheless, the fact that such bound inferences arise with cumulatively interpreted predicates is actually surprising: on their cumulative readings, there is no entailment relation between *m people ate seven apples* and *n people ate seven apples* (for $m \neq n$). For instance, if a group of four people ate a group of seven apples between them, then it certainly does not follow that a group of three people ate a group of seven apples between. Similarly, on their cumulative readings, there is no entailment relation between *m people lifted the piano* and *n people lifted the piano* (for $m \neq n$). To see this clearly,

entailment, to be a positive feature. For example, it explains why it is hard to judge the sentence false in a scenario where Ann, Bill, and Carol lifted the piano, and at least one other person who is *not* Ann, Bill or Carol also lifted it, or was part of a group who lifted it. In addition, this upper-bound inference seems to disappear in DE environments, as is expected if it is a scalar implicature: in a scenario where Ann makes the bet described by (96), if Beth, Carol, and Dan lift the piano, then Ann clearly wins the bet, regardless of whether some other person who is not Beth, Carol, or Dan also lifts it, or is part of a group who lifts it. If the upper bound were an entailment, as it is on the MaxGrp account, then it should be possible to argue that Ann lost the bet, but this seems wrong.

(96) If three people lift the piano, then Ann wins the bet.

In sum, the classic, adjectival account of numerical interpretation fully replicates Breheny's reading (modulo the upper-bound entailment/implicature difference) in a way that builds on independent observations about cumulative predicates.

We can also now diagnose the problem that MaxGrp faces for (3), which features *together*: MaxGrp effectively builds cumulativity of the two arguments into the meaning of *three*, but of course not every predicate has cumulative reference; thus, in cases where *three* combines with a non-cumulative predicate, the account makes a wrong prediction.[46] Two more cases in point are (4) and (5).

For example, *form a team*, unlike *lift the piano*, does not have cumulative reference:

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suppose that a and b lifted the piano together, and so did c and d, and no one else did. Then the extension of *lifted the piano* is $\{a \sqcup b, c \sqcup d\}$; hence, the extension of **lifted the piano* (see fn. 43) is the closure of this set under sum, viz. $\{a \sqcup b, c \sqcup d, a \sqcup b \sqcup c \sqcup d\}$. As we can see, there is a group of four individuals who 'lifted the piano' (in the cumulative sense), but there is no group of three individuals who did so. Krifka's explanation of the scalar implicature is the following:

> "The typical 'purpose of information exchange' in which sentences with cumulative readings and indefinite NPs occur is such that the main interest is in how many entities of each sort participate in the cumulative relation, not in the individual relations between single entities or subgroups of entities." (Krifka 1992)

However, this explanation is not very satisfying: the question 'Why is n interpreted as the (total) number of people that ate apples/lifted the piano?' cannot be adequately answered with 'Because that is the main interest'. Nevertheless, the crucial point for our purposes is that there is independent evidence that cumulative sentences *do* imply upper bounds and that, whatever the explanation for such inferences is, it carries over straightforwardly to Breheny's example. (For a more worked-out, but rather involved theory of maximal readings of bare and modified numerals in cumulative contexts, see Landman 1998; for a recent, dynamic-semantic account focused only on the modified numeral case, e.g. *Exactly three boys watched exactly five movies*, see Brasoveanu 2013.)

46 Some speakers that I have consulted actually reject Breheny's reading of *Three people lifted the piano* (i.e. judge the sentence false in his scenario), but these same speakers also reject the cumulative reference of *lift the piano*, which further supports the analysis developed in the preceding discussion. My hunch is that there is something special about *lift the piano*, which allows some (but not all) speakers to access cumulative (or cover) readings; by contrast, *lift the piano together*, *form a team*, and *have the same name* never allow for such readings.

if Ann and Bill formed a team, and Carol and Dan formed a team, it does not follow that Ann, Bill, Carol, and Dan formed a team. As expected, the reading for (4) predicted by MaxGrp is not attested: (4) cannot mean 'the maximal group of players who formed a team, or were part of a group who formed a team, is 4'. In a context where Ann and Bill formed a team and Carol and Dan formed a team, (4) is judged false.

(97) $[\![\text{four}_{\text{MaxGrp}}\text{ players}]\!]([\![\text{formed a team}]\!])$
$\equiv \#\bigsqcup(\{x : \mathsf{players}(x) \wedge \mathsf{form.a.team}(x)\}) = 4$

Likewise, *have the same name* does not have cumulative reference: if a group x has the same name (Ann, say) and a group y has the same name (Bill, say), it certainly does not follow that the sum of x and y has the same name. As expected, the reading for (5) predicted by MaxGrp is not attested: (5) cannot mean 'the maximal group of students who have the same name, or are part of a group who has the same name, is 5'. In a context where the students consist of two Ann's, three Bill's, and some other students with unique names, (5) is judged false.

(98) $[\![\text{five}_{\text{MaxGrp}}\text{ students}]\!]([\![\text{have the same name}]\!])$
$\equiv \#\bigsqcup(\{x : \mathsf{students}(x) \wedge \mathsf{same.name}(x)\}) = 5$

8.3 MaxCard

Like UniqGrp and MaxGrp, MaxCard predicts (3) to be ambiguous between a basic existential reading (which is attested), and another reading, in which $\textit{three}_{\textit{MaxCard}}$ is not lowered, and hence has an upper-bounding effect. This reading can be paraphrased as 'the maximal cardinality n such that a group of n people lifted the piano is 3'.[47]

(99) a. $\text{three}_{\text{MaxCard}}$ [1 [[[t_1 ⟨many⟩] people] [lifted the piano together]]]

b. $\max(\lambda n_d\,.\,\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{lift}(x)]) = 3$

On this reading, the sentence should be judged false in a scenario where Ann, Bill, and Carol lifted the piano together, and so did Dan, Ellen, Floyd, and Gwen. However, this prediction seems not to be borne out — the sentence is simply true.[48] To be more precise, the prediction is that there is a contrast in truth value judgment between a context in which a group of two and another group of three people lift the piano (it should be true) and a context in which a group of three and another

47 Once again (see fn. 41 and fn. 42), one could potentially say something special about the semantic role of *together* that might rule out this reading. However, even if *together* is omitted from the sentence, the predicted reading of the resulting sentence is still unavailable, and moreover, MaxCard runs into the same trouble with (4), as we will see shortly.

48 Spector (2015) has independently pointed out this same problematic prediction.

group of four people lift the piano (it should be false). However, no such contrast is detectable. Most informants that I have consulted judge (3) to be true in both contexts, and the judgments of those who find (3) to be false/odd in both contexts can be explained on the basis that (3) may implicate that no more than one group lifted the piano (see fn. 23).

Similar remarks hold for (4). This sentence does not have the predicted reading 'the maximum cardinality n such that a group of n players formed a team is 4'. There is no contrast in truth value judgment between a context in which a group of four players and another group of more than four players each formed a team (it should be false) and a context in which a group of four players and another group of fewer than four players each formed a team (it should be true).

(100) $[\![\text{four}_{\text{MaxCard}}\ \text{players}]\!]([\![\text{formed a team}]\!])$
$\equiv \mathsf{max}(\lambda n\,.\,\exists x[\#x = n \wedge \mathsf{players}(x) \wedge \mathsf{form.a.team}(x)]) = 4$

As for (5), MaxCard appears to come out better than UniqGrp and MaxGrp do. The predicted reading states that the maximum cardinality n such that a group of n students have the same name is 5. This imposes an upper bound, and indeed (5) appears to trigger an upper-bound inference, which I showed in §4 can be explained as a scalar implicature, just like in basic distributive cases.

(101) $[\![\text{five}_{\text{MaxCard}}\ \text{students}]\!]([\![\text{have the same name}]\!])$
$\equiv \mathsf{max}(\lambda n\,.\,\exists x[\#x = n \wedge \mathsf{players}(x) \wedge \mathsf{same.name}(x)]) = 5$

Whether or not deriving the upper bound for (5) as a semantic entailment is a positive feature depends in part on whether this inference, like in distributive cases, is accessible even in DE environments. Judgments are subtle, but it seems plausible to me that distributive cases like (1) and collective cases with downward inferences like (5) pattern the same, and hence that the reading derived for (5) under MaxCard is not a problem.

Nevertheless, the two upper-bound readings derived for (3) (with or without *together*) and for (4) are a problem. Moreover, recall (§7) that MaxCard is the only one of the three 'exactly' accounts that fails to explain distribution of two-sided readings of existential vs. generic numerical indefinites embedded under a DE operator.

8.4 Interim discussion

Before concluding this section, let us compare the above results with the results obtained under the classic, adjectival theory of numerals (§4). For all existential sentences, the adjectival theory assigns a meaning involving existential quantification and a cardinality check with "=". Whether or not an upper bound arises (as a scalar implicature) depends entirely on the logical (inferential) properties of the predicates involved. For distributive predicates like *attend* and even collective predicates like *have the same name* that license downward inferences, an upper bound is correctly predicted. For collective predicates like *lift the piano together* and *form a team* that do not license downward inferences, no upper bound is predicted, which is also correct. Moreover, for collective predicates that have cumulative reference, like *lift the piano*, an upper bound — and, more specifically, what I have called Breheny's reading — is derivable, which again is correct.

The problem with the 'exactly' accounts is that they hardwire some kind of upper-bounding mechanism into the semantics of numerals, which in turn derives maximality (or uniqueness) effects across the board. In many cases of collective predication, the results are wrong. In a couple special cases, such as MaxGrp's analysis of *Three people lifted the piano* (which derives Breheny's reading) and MaxCard's analysis of *Five students have the same name*, the resulting reading is attested, but only by accident: it is precisely in these cases that the 'exactly' accounts coincide with the classic account.

The moral of the story, from the view of existential sentences, is that the ability of a numeral to be 'strengthened' (e.g. to acquire a two-sided reading) tracks the logical (inferential) properties of the predicates involved — something that the classic, scalar implicature approach does, but not the 'exactly' approaches, all of which indiscriminately force strengthening across the board.

9 Conclusion

9.1 Recap

Let us step back and take stock of all the observations made. In §2 and §3, I motivated and developed an account of numeral interpretation in which the basic meaning of *three* is an adjectival, or predicative, one that characterizes the set of all plural individuals with (exactly) three atomic parts. (Actually, I assumed that *three* denotes the number 3 and that a silent ⟨*many*⟩ maps *three* to such a predicative meaning.) I called this approach a version of the classic view because in basic distributive cases like (1), *three* ends up meaning 'at least three', while the utterance as a whole implicates 'not more than three', just as was first proposed by Horn (1972). In §4, I showed that this account also correctly predicts a basic existential reading, with no bound inference, in cases of collective predication like (3) and (4), which do not license downward inferences, and an upper-bound reading in cases of collective predication that do license downward inferences, such as (5). Finally, I showed in §5 that this view naturally explains the readings of generically interpreted sentences as well, i.e. their truth-conditional content and the bound inferences they license, which again arise as scalar implicatures. However, the account fails to predict a previously unnoticed asymmetry in the distribution of two-sided readings of numerical indefinites occurring in a DE environment: they are available when the indefinite is existential, but not when it is generic (more on this in a moment).

I then presented in §6 three alternative approaches that take the basic, literal meaning of *three* to be a two-sided, 'exactly' meaning, implemented either with a uniqueness condition on existential quantification over groups (UniqGrp; Geurts 2006), a maximization operator that sums up all the groups being existentially quantified over (MaxGrp; Breheny 2008), or a maximality operator that imposes an upper bound on group cardinalities (MaxCard; Kennedy 2015).

(102) a. $[\![\text{three}]\!] = 3$ (classic view)

b. $[\![\text{three}_{\text{UniqGrp}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists! x[\#x = 3 \wedge P(x) \wedge Q(x)]$

c. $[\![\text{three}_{\text{MaxGrp}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = 3$

d. $[\![\text{three}_{\text{MaxCard}}]\!] = \lambda P_{dt} \,.\, \max(P) = 3$

In addition, each of these alternative approaches has a way to shift the meaning of *three* in such a way as to make *three NP* predicative, which in turn means that they can replicate all the desirable results that the classic account gets right. This includes deriving one-sided, 'at least' readings for sentences with distributive predicates, basic existential readings for sentences with collective predicates, and quasi-universal readings for sentences with generic numerical indefinites.

(103) $[\![\text{[three } \langle\text{many}\rangle\text{] people}]\!]$ (classic view)

$\equiv \mathsf{BE}([\![\text{three}_{\text{UniqGrp}} \text{ people}]\!])$

$\equiv \mathsf{BE}([\![\text{three}_{\text{MaxGrp}} \text{ people}]\!])$

$\equiv [\![\langle\text{many}\rangle]\!](\mathsf{iota}(\mathsf{BE}([\![\text{three}_{\text{MaxCard}}]\!])))([\![\text{people}]\!])$

Moreover, I showed in §7 that UniqGrp and MaxGrp (but not MaxCard) derive a remarkable result when it comes to DE contexts: they correctly predict that existential numerical indefinites can have their two-sided readings when embedded under DE operators (because an upper bound is semantically entailed), while generic numerical indefinites cannot have their two-sided readings when embedded under DE operators (because they must be typeshifted, which removes any semantic upper-bounding mechanism).

Despite this success, I showed in §8 that all three accounts makes wrong predictions when it comes to collective constructions. UniqGrp predicts sentences with collective predicates to be ambiguous between a basic existential reading (which is attested) and a kind of 'unique group of cardinality *n*' reading (which is unattested). MaxGrp predicts such sentences to be true even if no group of cardinality *n* participated in the event, and I showed that this prediction is sometimes wrong; rather, it is only the case when the predicates in the sentence can be interpreted cumulatively (a very special case). Finally, MaxCard predicts collective constructions to be ambiguous between a basic existential reading and a kind of 'maximum group cardinality is *n*' reading, which I argued is unavailable, except when the collective predicate happens to license downward inferences (another very special case).

9.2 Towards solving the puzzle

What emerges is a new puzzle: the classic, adjectival account gets the collective facts right but fails to explain the embedding facts, while a UniqGrp/MaxGrp-style account gets the embedding facts right but fails in the collective domain. The obvious question now is whether we can resolve the puzzle in favor of one approach over the other, i.e. whether we can find a tie-breaker. As far as I can see, there is no way to break the tie in favor of UniqGrp/MaxGrp: if existential distributive sentences are ambiguous, then so necessarily are existential collective ones. Instead, I would like to conclude by presenting some observations that may point to an amendment of the classic account that would get all the facts right.

Recall the idea from §2 that the reason that existentially interpreted numerals in DE contexts can receive their two-sided readings is not because they have an intrinsic 'exactly' meaning, but rather because either they "are intrinsically focused, in the sense that they automatically activate their alternatives" (Spector 2013), or they "can introduce alternatives without the help of focus" (Krifka 1999).

This general view, as stated, cannot seem to distinguish existential from generic contexts: if numerals automatically evoke their alternatives, then two-sided readings of both existential and generic numerical indefinites embedded under a DE operator ought to be accessible, which is incorrect.

However, if we take more seriously the notion of "intrinsically focused" (rather than automatic activation of alternatives), then we may have more success. Specifically, let us assume that numerals are indeed intrinsically focused, but assume nothing about alternative activation, except that it tracks activation of alternatives of overtly focused material. This view then predicts that wherever focused scalar expressions can have their strengthened meanings, then so should (unfocused) numerals; and conversely, wherever focused scalar expressions do *not* have their strengthened meanings, then neither should (unfocused) numerals — at least, not necessarily. In other words, the availability of two-sided meanings of unfocused numerals (which is what we are interested in) is predicted to track the availability of strengthened meanings of focused scalar expressions.

So, the question now is, how do overtly focused numerals and other overtly focused scalar expressions behave when they occur in a generic construction embedded in a DE environment? Judgments are admittedly subtle, but they seem to indicate that strengthened readings are, surprisingly, unavailable, or at least tough to get.

For example, there is not much contrast between the two sentences in (104) or the two sentences in (105): they all sound equally bad, especially when compared to the analogous pair of existential sentences, which both sound totally fine. (Similarly, adding overt focus to *three* in all the odd examples in §5.5 does not rescue them.)

(104) a. *Three people can't fit in the car — FOUR can.
b. *THREE people can't fit in the car — FOUR can.

(105) a. *Three people can't lift the piano — TWO can.
b. *THREE people can't lift the piano — TWO can.

(106) a. Three people didn't attend — FOUR did.
b. THREE people didn't attend — FOUR did.

In other words, focus — whether overt or not — does not seem to help numerals to acquire their two-sided reading in these generic contexts.

Moreover, this observation appears to extend to other scalar expressions, too. For instance, (107), on its generic reading, states that any recliner can fit in the room and that any small couch can fit, and it implicates that it is not the case that a recliner *and* a small couch can fit.

(107) A recliner or a small couch can fit in the room.

Unsurprisingly, this exclusivity inference disappears when the sentence is embedded under a DE operator. What *is* surprising, however, is that, unlike in existential constructions (cf. §2), adding focus to *or* does not make the exclusive reading reappear: the following sentences all sound very strange.

(108) a. *If a recliner OR a small couch can fit in the room, then we should sell our recliner and just bring our couch.
b. *Ann doubts that a recliner OR a small couch can fit — she thinks both can.
c. *It's not true that a recliner OR a small couch can fit — both can.

Contrast the above examples with the following counterparts containing an overt *only*, which sound quite good and have the intended meaning.

(109) a. If only a recliner OR a small couch can fit in the room, then we should sell our recliner and just bring our couch.
b. Ann doubts that only a recliner OR a small couch can fit — she thinks both can.
c. It's not true that only a recliner OR a small couch can fit — both can.

When we move to *can lift the piano*, since the direction of entailment is reversed, we need to use *and* instead of *or*.[49] With this in mind, consider (110). This sentence, on its generic reading, states that any pair consisting of an experienced mover and a strong helper can lift the piano, and it implicates that it is not the case that any experienced mover can lift it on her own (or that any strong helper can lift it on his own).

(110) An experienced mover and a strong helper can lift the piano.

Unsurprisingly, this latter implicature disappears when the sentence is embedded under a DE operator, but as before, adding focus to *and* does not make the strong reading reappear: the following sentences all sound very strange.

(111) a. *If an experienced mover AND a strong helper can lift the piano, then we need to find an extra person.
b. *Ann doubts that an experienced mover AND a stronger helper can lift the piano — she thinks an experienced mover can do it on her own.
c. *It's not true that an experienced mover AND a strong helper can lift the piano — an experienced mover can do it on her own.

Contrast the above examples with the following counterparts containing an overt *only*, which, as before, sound quite good and have the intended meaning.

(112) a. If only an experienced mover AND a strong helper can lift the piano, then we need to find an extra person.
b. Ann doubts that only an experienced mover AND a stronger helper can lift the piano — she thinks an experienced mover can do it on her own.
c. It's not true that only an experienced mover AND a strong helper can lift the piano — an experienced mover can do it on her own.

To the extent that these judgments are reliable, the generalization here is that, for some reason, scalar expressions in generic sentences that are embedded under a DE operator never seem to be able to have their strengthened interpretation, regardless of whether the scalar expression is a numeral or *and*/*or*, and regardless of whether it is overtly focused or not. I have no explanation for this fact (if true). But the point is that the inability of numerals in such contexts to have their strong readings may be a symptom of a more general phenomenon, in which case, the classic, adjectival theory of numerals could potentially be salvaged (pending an explanation of this fact). Moreover, while an account along the lines of UniqGrp or

49 What I mean by "the direction of entailment is reversed" is that *n people can lift the piano* entails *m people can lift the piano* (on their generic readings) just in case $n \leq m$ (rather than $m \leq n$), and *x can lift the piano* entails *y can lift the piano*, for any two (possibly plural) individuals x and y, whenever $x \sqsubseteq y$ (rather than $y \sqsubseteq x$).

MaxGrp does already provide an explanation of the asymmetry between numerals in existential vs. generic constructions, two points should be borne in mind: (i) an explanation for the potential generalization just uncovered would still ultimately be necessary, and (ii) such an account still makes wrong predictions for existential sentences with collective predicates.

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