Some remarks on inference patterns involving epistemic modality

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Abstract

This paper identifies and resolves a previously unnoticed puzzle emerging from recent literature on quantifiers and epistemic modality. Specifically, I look at inference patterns involving superlative quantifiers ("at least *m*", "at most *m*"), which give rise to epistemic implications, and comparative quantifiers ("more than *n*", "fewer than *n*"), which do not. I identify an asymmetry in the inference patterns involving superlative quantifiers that I argue is not readily explained by either of two competing theories of superlative quantifiers. The solution I propose is that subjects in inferential tasks make certain default (but defeasible) assumptions: anything that is not known to be false is assumed to possibly be true. This proposal makes a number of predictions regarding inference patterns with overt epistemic modals, which I show are broadly correct. The end result suggests a new point of divergence between the notion of semantic entailment and that of an intuitively valid inference.

1 Introduction

Generalized quantifier theory (GQT; Barwise and Cooper 1981) standardly takes comparative quantifiers (CQs: "more than *n*", "fewer than *n*") and superlative quantifiers (SQs: "at least *m*", "at most *m*") to be semantically equivalent, modulo the numeral.[1] Thus, (1) is taken to be synonymous

[1] This assumption stems in part from the fact that the operators $>$ and $<$, which traditionally provide the semantics for CQs, are mathematically interdefinable with the operators $\geq$ and $\leq$, which traditionally provide the semantics for SQs. For any two numbers m and n, we have $n > m$ iff $n \geq m + \epsilon$ for some $\epsilon > 0$. For example, on a scale of units, $n > 2$ iff $n \geq 3$. Throughout this paper, I assume that we're dealing with a scale of units, e.g., the number of children Floyd has.

with (2), i.e., each entails the other, and judgments seem to agree: from (1) we readily infer (2), and from (2) we readily infer (1).[2]

(1) Floyd has more than two children.

(2) Floyd has at least three children.

Under GQT, then, CQs and SQs share the same entailment patterns and are thus expected to participate in the same inference patterns, too.

Recently, however, several inference patterns have been identified that are intuitively valid with CQs, but intuitively invalid with their SQ counterparts. For example, in Geurts and Nouwen's (2007) words, "it isn't nearly as evident" that (2) follows from (3) as it is that (1) does.

(3) Floyd has exactly three children.

Intuitively, the reason we don't readily infer (2) from (3) is that (2) implies that the speaker considers it possible that, or is uncertain whether, Floyd has more than three children, which contradicts (3), which is normally interpreted to mean that the speaker is certain that Floyd has exactly three children. (1), however, has no such epistemic implication, hence the intuitive validity of inference from (3) to (1).

Geurts and Nouwen 2007 take this contrast in inference patterns as evidence that SQs are semantically different from CQs, i.e., that epistemic implications (EIs) are part of the truth-conditional content of SQs. Büring 2008 and Cummins and Katsos 2010, however, propose that the contrast is pragmatic, i.e., that EIs are implicatures arising from the use of SQs but not of CQs.

In this paper I make the observation that, in judging whether a conclusion q follows from a premise set P, EIs arising from q that are not entailed by P can be of two types: *contradictory* (they contradict some premise $p \in P$) or *noncontradictory* (they do not contradict any premise $p \in P$). Crucially, judgments indicate that contradictory EIs invalidate

[2]In this paper I use "infer" (and its derivatives) and "to intuitively follow from" very generally, with no committment as to whether such inferences are entailments or otherwise; the judgments are simply pre-theoretic intuitions reported in the literature. On the other hand, by "entail" I mean semantic, or logical, entailment as normally defined: P entails q (in symbols, $P \vDash q$) iff, if each $p \in P$ is true, then q must also be true. In addition, if $p_1(, p_2, \ldots, p_n)$ is (are) premises, I'll often write "$p_1(, p_2, \ldots, p_n)$ entail(s) q" instead of "the set containing $p_1(, p_2, \ldots, p_n)$ entails q".

otherwise semantically valid inferences, whereas noncontradictory EIs, even when unentailed, do not. For example, the EI arising from (2) that Floyd possibly has more than three children contradicts (3), rendering the inference from (3) to (2) intuitively invalid. Conversely, that same EI neither contradicts nor is entailed by (1)—as I show more explicitly later on, it does not necessarily follow from (1) that Floyd possibly has more than three children (or that the speaker considers this possible)—and yet the inference from (1) to (2) is intuitively valid.

I provide several arguments for why this asymmetry in inferences between contradictory and noncontradictory unentailed EIs is unexpected under both the semantic account of Geurts and Nouwen 2007 and the pragmatic account of Büring 2008: both accounts predict that, if contradictory EIs invalidate otherwise valid inferences, then so should noncontradictory EIs. The intuition that (2) follows from (1) thus remains a puzzle.

To explain this puzzle, I propose that subjects in inferential tasks assume the EIs of a conclusion to be true by default, whenever possible, and that this reasoning process stems from a more general reasoning process whereby anything that is not known by the subject to be false, i.e., anything not explicitly given as a premise, is assumed by the subject to possibly be true. I provide a precise characterization of this reasoning and show that it makes several clear predictions, including predictions regarding inference patterns with overt epistemic modals. These predictions are indeed borne out, broadly speaking; however, there are some subtle intuitions that indicate that the basic, strong proposal must be suitably constrained in ways that I sketch out. The end result suggests a new point of divergence between the notion of semantic entailment and that of an intuitively valid inference.

2 Some preliminary data

In this section I lay out the data to be accounted for. These data are divided into two parts: (i) uncontroversial, intuitively valid inferences, and (ii) intuitively invalid inferences, as reported in recent literature (followed by a table summarizing all the data).

To start with, two sentences that differ only in that one has a CQ, and the other its SQ counterpart, are judged to be synonymous. Thus, for the so-called lower-bound quantifiers, (2) intuitively follows from (1), and vice versa, as we've already seen.

(1) Floyd has more than two children.

(2) Floyd has at least three children.

Similarly, for the upper-bound quantifiers, (5) intuitively follows from (4), and vice versa.

(4) Floyd has fewer than five children.

(5) Floyd has at most four children.

In addition, "at most *n*" licenses negative polarity items (NPIs), e.g., "give a damn" in (6), and so under the classic account of NPI licensing (Ladusaw 1979) is expected to be downward entailing. That is, inferences from sets to subsets should be intuitively valid.

(6) At most three students give a damn about Pavarotti.

(7) At most three students smoke.

(8) At most three students smoke cigars.

Indeed, that (8) follows from (7) seems uncontroversial.[3] Note that the same observations regarding NPI licensing and downward entailingness hold for "fewer than *n*", as well.

Finally, consider (3), repeated from earlier, and (9).

(3) Floyd has exactly three children.

(9) Floyd has exactly four children.

Both (1) and (4) intuitively follow from (3) as well as from (9).

In addition to being intuitively valid inferential relations, the above sentence pairs are also all semantically valid entailment relations under GQT, in which CQs compare set cardinalities with the operators $<$ and $>$, and SQs do so with $\leq$ and $\geq$. That is, the above intuitions correspond precisely with the predictions that GQT makes, under the basic assump-

[3]These three examples, as well as the judgment, are all taken from Chierchia and McConnell-Ginet 2000. Presumably, "at least *n*" should likewise be analyzed as upward entailing, and indeed inferences from sets to supersets seem to be intuitively valid.

tion that a valid semantic entailment translates into an intuitively valid inference.

However, when we switch out CQs with their SQ counterparts, intuitions indicate that some of the inferences are no longer valid. First, while (1) intuitively follows both from (3) and from (9), (2) does not intuitively follow from either. Second, while (4) intuitively follows both from (3) and from (9), (5) does not intuitively follow from either.[4,5]

These judgments are all summarized in Table 1, where "$A \vDash^* B$" stands for "B intuitively follows from A", or "the inference from A to B is intuitively valid"; and "$A \nvDash^* B$" is the negation of "$A \vDash^* B$". The table is drawn in such a way that column A includes standard, uncontroversial inferences, and column B is derived from column A by substituting quantifier types (CQs for SQs and SQs for CQs). Under the GQT view that CQs and SQs are semantically equivalent, inferences—viewed as judgments on entailment relations—in the same row are expected to have the same status (valid or invalid). However, two asymmetries emerge: first, rows 4–7 reveal that there are some inference patterns that CQs and SQs do not share; and second, column B as a whole reveals that judgments with SQs involve an asymmetry that judgments with CQs do not share.

Based on the first asymmetry (more specifically, rows 4–5 across both columns), Geurts and Nouwen 2007 conclude that CQs and SQs must have different meanings. They propose that the contrast is due to a speaker-oriented, epistemically modal component of SQs, which CQs lack. Specifically, (2), but not (1), implies that the speaker considers it possible that Floyd has more than three children. This epistemic implication contradicts both (3) and (9), hence the intuitive invalidity of the inference from (3) to (2) or from (9) to (2).

[4]Most of these judgments were first reported in Geurts and Nouwen 2007 and tested experimentally in Geurts et al. 2010 and Cummins and Katsos 2010. The exception (my own observation) is the intuition that neither (2) nor (5) follows from (9). I show in section 3.1 that this missing data point leads to an accidental, unwanted asymmetry in Geurts and Nouwen's (2007) semantics for SQs. I therefore take it to be a simple oversight on their part and revise their account accordingly, which turns out to bring it exactly in line with Büring's (2008) pragmatic account in terms of what EIs it derives and what inference patterns it predicts.

[5]Geurts and Nouwen 2007 actually use a bare numeral (without "exactly") in their premise. Given that bare numerals can have weak interpretations (equivalent to "at least" quantifiers), an inference with "exactly" in the premise (and "at least" in the conclusion) is expected to be even less plausible.

Table 1: Summary of inference patterns involving CQs and SQs

	Column A			Column B		
1	"at least 3 P Q"	(2) $\vDash^*$ (1)	"more than 2 P Q"	"more than 2 P Q"	(1) $\vDash^*$ (2)	"at least 3 P Q"
2	"at most 4 P Q"	(5) $\vDash^*$ (4)	"fewer than 5 P Q"	"fewer than 5 P Q"	(4) $\vDash^*$ (5)	"at most 4 P Q"
3	"fewer than 4 P Q"	$\vDash^*$	"fewer than 4 P $R_{\subseteq Q}$"	"at most 3 P Q"	(7) $\vDash^*$ (8)	"at most 3 P $R_{\subseteq Q}$"
4	"exactly 3 P Q"	(3) $\vDash^*$ (1)	"more than 2 P Q"	"exactly 3 P Q"	(3) $\nvDash^*$ (2)	"at least 3 P Q"
5	"exactly 3 P Q"	(3) $\vDash^*$ (4)	"fewer than 5 P Q"	"exactly 3 P Q"	(3) $\nvDash^*$ (5)	"at most 4 P Q"
6	"exactly 4 P Q"	(9) $\vDash^*$ (1)	"more than 2 P Q"	"exactly 4 P Q"	(9) $\nvDash^*$ (2)	"at least 3 P Q"
7	"exactly 4 P Q"	(9) $\vDash^*$ (4)	"fewer than 5 P Q"	"exactly 4 P Q"	(9) $\nvDash^*$ (5)	"at most 4 P Q"

Curiously, however, under this account, the rest of column B (rows 1–3) ought to be judged invalid as well—the conclusions contain epistemic implications which, although they don't contradict their premises, are nevertheless unentailed by them—and so the second asymmetry remains a mystery. The aim of this paper is primarily to explain this second asymmetry, i.e., why it is that contradictory epistemic implications invalidate otherwise valid inferences, whereas noncontradictory but nevertheless unentailed ones do not.

3 Epistemic implications

In this section I discuss two accounts (one semantic, one pragmatic) of how epistemic implications (EIs) arise from the use of SQs. I demonstrate explicitly how neither account predicts the particular asymmetry in intuitions reported in section 2 regarding contradictory and noncontradictory EIs.

3.1 A semantic account

Geurts and Nouwen 2007 offer the following semantic denotations to sentences with SQs like (2) and (5), repeated below. The denotation of each sentence is simply the conjunction of the two modal statements. Note that "$\Box$" denotes a speaker-oriented necessity operator, which I gloss as "the speaker is certain that", and "$\Diamond$" denotes a speaker-oriented possibility operator, which I gloss as "the speaker considers it possible that".[6] Furthermore, "x" denotes a plural individual, and "$\#x$" the number of atomic parts of x.

(2) Floyd has at least three children.

a. $\Box\exists x[\#x = 3 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

b. $\Diamond\exists x[\#x > 3 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

(5) Floyd has at most four children.

a. $\neg \Diamond \exists x[\#x > 4 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

b. $\Diamond\exists x[\#x = 4 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

Under this account, (2) conveys that the speaker is certain that Floyd has three (or more) children, and considers it possible that he has more than three children. Similarly, (5) conveys that the speaker considers it possible that Floyd has four children (or more), and does not consider it possible that Floyd has more than four children.[7]

In contrast, the CQ counterparts of (2) and (5), as well as the versions containing "exactly", all repeated below, contain no such epistemic statements in their denotations.[8]

(1) Floyd has more than two children.

a. $\exists x[\#x > 2 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

(4) Floyd has fewer than five children.

a. $\neg\exists x[\#x = 5 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

(3) Floyd has exactly three children.

a. $\exists! x[\#x = 3 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

(9) Floyd has exactly four children.

a. $\exists! x[\#x = 4 \wedge \text{children}(x) \wedge \text{has}(f, x)]$

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[6]These operators can also be thought of as quantifiers over possible worlds, in which case "$\Box$" can be glossed as "in all possible worlds consistent with the speaker's evidence", and "$\Diamond$" as "there is a possible world consistent with the speaker's evidence".

[7]It might seem confusing that the second conjunct of the meaning of (5) has an "or more" meaning, since intuitively "at most" expresses an upper bound; however, under this denotation, the upper bound comes about from the modal statement in (5)a., which effectively removes the "or more" possibility of (5)b.

[8]Note that "$\exists! x$" means there is a *unique* x.

Although these denotations contain no epistemic operators, we can assume that, because speakers generally only say what they believe to be true, utterances of this type are interpreted as being in the scope of a universal epistemic operator. That is, we can imagine the above denotations as having a "$\Box$" in front, as if there were an implicit "I am certain that" in the syntax.[9]

At this point we can ask whether this semantic account captures the intuitively invalid inference patterns (rows 4–7 of column B). It turns out that it captures only some of them. Given a premise like (3), the account correctly predicts that neither (2) nor (5) intuitively follows, since each of these contains, as part of its semantic content, a modal statement not entailed by (and in fact, contradictory with) (3).

Specifically, (2) asserts that the speaker considers it possible that Floyd has more than three children, and (5) asserts that the speaker considers it possible that Floyd has exactly four children. Both of these assertions contradict (3), which is interpreted as meaning that the speaker is certain that Floyd has exactly three children. And indeed, as noted, neither (2) nor (5) intuitively follows from (3); thus, the right predictions are made here.

Recall now that neither (2) nor (5) is likewise judged to follow from (9). Under this account, however, (9) actually entails both (2) and (5): it is interpreted as conveying that the speaker is certain that Floyd has exactly four children; and from this it follows (i) that the speaker is certain that Floyd has three (or more) children and that the speaker considers it possible that Floyd has more than three children, and (ii) that the speaker considers it possible that Floyd has four children (or more) and that the speaker does not consider it possible that Floyd has more than four children.[10] Since (i) and (ii) are the full meanings conveyed by (2) and (5), respectively, these sentences are entailed by (9), under this account; thus, the inferences from (9) to (2) and from (9) to (5) are predicted to be judged

[9]See Fox 2007 for a general discussion of this assumption. For the remainder of this paper, I assume that all (non-modal) sentences are in fact interpreted universally. This assumption is crucial to determining both the relevant entailment relations and the relevant inference patterns later on.

[10]Crucially, I take it that to be certain of p entails considering p possible. In possible-worlds talk, if in all possible worlds consistent with the speaker's evidence, p is true, then (assuming this set of possible worlds is nonempty) there is clearly at least one possible world consistent with the speaker's evidence in which p is true.

valid, contra fact.

The latter examples actually expose an unwanted asymmetry in how lower-bound and upper-bound SQs are treated under this account: "exactly n P Q" entails "at most n P Q" but not "at least n P Q", and "exactly n P Q" entails "at least $n-1$ P Q" but not "at most $n+1$ P Q".

This asymmetry can easily be fixed by strengthening the meanings of (2) and (5) in the following way: (2) asserts not only that the speaker considers it possible that Floyd has more than three children, but also that she considers it possible that Floyd has exactly three children; and (5) asserts not only that the speaker considers it possible that Floyd has exactly four children, but also that she considers it possible that Floyd has fewer than four children (and no more). Thus, we make it so that both the lower- and upper-bound SQs convey both a comparative and an "exactly" possibility. The revised denotations are given below. Notice that (2)a. and (5)a. correspond to the epistemically interpreted meanings of (1) and (4), respectively. In other words, the additional epistemic meaning of SQs, i.e., (2)b.-c. and (5)b.-c., are what we've been calling the epistemic implications (EIs), and I will continue referring to them as such.[11]

(2) Floyd has at least three children.

a. $\Box\exists x[\#x = 3 \land \text{children}(x) \land \text{has}(f, x)]$
b. $\Diamond\exists x[\#x > 3 \land \text{children}(x) \land \text{has}(f, x)]$
c. $\Diamond\exists! x[\#x = 3 \land \text{children}(x) \land \text{has}(f, x)]$

(5) Floyd has at most four children.

a. $\neg\Diamond\exists x[\#x > 4 \land \text{children}(x) \land \text{has}(f, x)]$
b. $\Diamond\exists x[\#x = 4 \land \text{children}(x) \land \text{has}(f, x)]$
c. $\Diamond\exists! x[\#x < 4 \land \text{children}(x) \land \text{has}(f, x)]$

As the reader can check, these extended denotations make it so that "exactly n P Q" no longer entails "at most n P Q" or "at least $n-1$ P Q": each SQ conclusion now contains an EI that contradicts the (epistemically interpreted) "exactly" premise. Moreover, since these denotations are strictly stronger than Geurts and Nouwen's original ones, it still holds that "exactly n P Q" entails neither "at least n P Q" nor "at most $n+1$ P Q", as the reader can also check. Hence, neither (2) nor (5) is entailed by

[11] As we'll see, the EIs of our revised version of Geurts and Nouwen's account are precisely the same as those that Büring 2008 derives, too, except as implicatures.

(3) or (9)—there is always a contradictory EI—which coincides precisely with the inference patterns in rows 4–7, column B of Table 1.

This revised version of Geurts and Nouwen's account thus makes the right predictions for the intuitively *invalid* inferences (rows 4–7), but what about the intuitively *valid* ones (rows 1–3)? Consider first whether (1) entails (2). (1) is interpreted to mean that that speaker is certain that Floyd has more than two children. This does *not* entail that the speaker considers it possible that Floyd has exactly three children, or that the speaker considers it possible that Floyd has more than three children, both of which are part of the truth conditions of (our revised version of) (2). To see why, note that (1) could be felitously and truthfully uttered in a context where the speaker is certain that Floyd has, say, exactly three children, or exactly six children: in either case, one of the EIs of (2) is false.[12] Thus, (1) does not entail (2), meaning that (2) is not predicted to intuitively follow from (1). However, as reported in row 1, column B of Table 1, this inference is intuitively valid.

Similarly, (4) does not entail (5): (4) is interpreted to mean that the speaker is certain that Floyd has fewer than five children (and no more than that), which does *not* entail that the speaker considers it possible that Floyd has exactly four children, or that the speaker considers it possible that Floyd has fewer than four children (and no more than that). Since these are part of the truth conditions of (our revised version of) (5), it follows that (4) does not entail (5), meaning that (5) is not predicted to intuitively follow from (4). However, as reported in row 2, column B of Table 1, this inference is intuitively valid.

Finally, we can show that "at most *n*" is not downward entailing under this account, e.g., that (7) does not entail (8), repeated below.

(7) At most three students smoke.

(8) At most three students smoke cigars.

[12] In possible-worlds talk, if in all possible worlds consistent with the speaker's evidence, Floyd has more than two children, it's not necessarily the case that one of those worlds is a world in which Floyd has exactly three children (they could all be ones in which he has more than three children), or that one of those worlds is a world in which Floyd has more than three children (they could all be worlds in which Floyd has exactly three children).

The relevant thing to consider here is this: (8) conveys, among other things, that the speaker considers it possible that exactly three students smoke cigars. This statement, however, is *not* entailed by (7), which conveys nothing at all about the number of cigar-smoking students the speaker considers there to be. The speaker could, for example, know for a fact that no students smoke cigars.[13] Thus, (7) does not entail (8), meaning that (8) is not predicted to intuitively follow from (7). However, as reported in row 3, column B of Table 1, this inference is intuitively valid.[14]

Thus, our revised version of Geurts and Nouwen's semantic account makes the right predictions for the intuitively *invalid* inferences reported in rows 4–7: the conclusions contain EIs that contradict the premise. However, this account does not capture the intuitively *valid* inferences in rows 1–3: although the EIs of the conclusions do not contradict the premises, they are nonetheless unentailed by the premises; thus, the entailment relation does not hold, so the inference is predicted to be intuitively invalid.

More generally, it should be clear from this discussion that, under this semantic account, unentailed EIs—being part of the semantic content of sentences with SQs—are enough to invalidate an entailment relation, regardless of whether they're contradictory or not. Hence, inferences involving noncontradictory unentailed EIs in a conclusion are expected to be judged invalid, contra fact.

In the next subsection, I review the pragmatic account of Büring 2008 and Cummins and Katsos 2010 and show that it doesn't fare much better in accounting for all the judgments in column B.

3.2 A pragmatic account

Büring 2008 and Cummins and Katsos 2010 attempt to derive EIs from general, neo-Gricean pragmatic principles. They posit that SQs are inter-

[13]In possible-worlds talk, if it's the case that in all possible worlds consistent with the speaker's evidence, at most three students smoke, it doesn't necessarily follow that one of those worlds is a world in which exactly three students smoke cigars: they could all be worlds in which no students smoke cigars.

[14]It should be clear from this discussion that this semantic account makes "at most *n*" nonmonotone. Thus, if NPI licensors necessarily create a semantically downward monotone environment, this account cannot be right. I return to this issue briefly in section 6.

preted disjunctively, which gives rise to a quantity implicature similar to the familiar clausal implicature associated with disjunction.[15] For example, an utterance like (10) implies that the speaker is uncertain whether Lenny called and is uncertain whether Carl called (she is certain only that one of them called).

(10) Lenny or Carl called.

a. asserts: Lenny called OR Carl called.
b. implicates: (i) the speaker is uncertain whether Lenny called, and (ii) the speaker is uncertain whether Carl called.

Following Sauerland 2004 and Fox 2007, these implicatures can be derived as follows: "Lenny called" and "Carl called" are stronger alternatives to (10) (each asymmetrically entails what (10) asserts); assuming the speaker is cooperative, she must have some reason for not using one of these alternatives; the reason, presumably, is that she lacks adequate evidence for either alternative; this, together with the assertion, entails that the speaker is uncertain of each alternative.

The meaning that Büring 2008 offers for "at least" is given syncategorematically in (11), and the full meaning of (2) is given thereafter.[16]

(11) "at least n P Q" := "exactly n OR more than n"

[15] Geurts and Nouwen (2007) actually consider, but in the end discard, the possibility that EIs are conversational implicatures, noting that under the assumption that CQs and SQs are semantically equivalent, they "fail to see how any conversational implicature associated with one expression could fail to be associated with the other."

Büring (2008), recognizing the problem, notes that his proposal "is committed to a view of implicatures that allows a given implicature to be triggered by a class of expressions (or some more abstract property characteristic of that class), rather than particular expressions," but gives no further indication of what that might mean for the present theory of implicatures. Cummins and Katsos's (2010)'s solution is to propose that CQs involve the strict comparative operators $<$ and $>$, whereas SQs involve the non-strict comparative operators $\leq$ and $\geq$; and although strict and non-strict operators are mathematically interdefinable (making CQs and SQs semantically equivalent), non-strict operators are "regarded as disjunctions, at some non-linguistic level of representation" (making CQs and SQs *psychologically* different). For an alternative solution, see also Schwarz 2011, which spells out the implicatures of SQs in terms of Horn sets: "at least" and "at most" belong to a Horn set that also includes "exactly", but not "more than" or "fewer than".

[16] The symbol ":=" here is Büring's and means something like "is interpreted as". For discussion, see footnote 15.

(2) Floyd has at least three children.

a. asserts: Floyd has exactly three children OR Floyd has more than three children.

b. implicates: (i) the speaker is uncertain whether Floyd has exactly three children, and (ii) the speaker is uncertain whether Floyd has more than three children.

The EIs of (2) are derived as implicatures along the same lines as before: "Floyd has exactly three children" and "Floyd has more than three children" are stronger alternatives to (2) (they each asymmetrically entail what (2) asserts); assuming the speaker is cooperative, she must have some reason for not using one of these alternatives; the reason, presumably, is that she lacks adequate evidence for either alternative; this, together with the assertion, entails that the speaker is uncertain of each alternative. Moreover, being uncertain of each alternative, together with the assertion, amounts to considering each alternative possible, which in turn derives the same epistemic implications as our revised version of Geurts and Nouwen's (2007) theory, albeit as implicatures and not as semantic content.[17]

Although Büring 2008 discusses only "at least", one can imagine a similar take on "at most".

[17]In possible-worlds talk, if it's not the case that in all worlds consistent with the speaker's evidence is p true, nor in all possible worlds consistent with the speaker's evidence is q true, but in all such worlds p or q is true, then in at least one of those worlds, p is true (e.g., the one in which q is false), and in at least one of those worlds, q is true (e.g., the one in which p is false). But these are precisely the epistemic implications of our revised version of Geurts and Nouwen's (2007) account.

Moreover, a translation can be done in the opposite direction, too, i.e., the epistemic implications of our revised version of Geurts and Nouwen's (2007) account can derive the same ones that this pragmatic account does. If there is a possible world consistent with the speaker's evidence in which p is true, and a possible world consistent with the speaker's evidence in which q is true, and if p and q are mutually inconsistent (cannot both be true in the same possible world), it follows that (i) it's not the case that in all possible worlds consistent with the speaker's evidence is p true, and (ii) it's not the case that in all possible worlds consistent with the speaker's evidence is q true. Since "exactly n P Q" and "more than (fewer than) n P Q" are always mutually inconsistent, (i) and (ii) can always be derived. But these correspond precisely to the (glosses of the) epistemic implications derived by Büring (2008). Thus, the epistemic implications that each account derives are, for all intents and purposes, equivalent.

(5) Floyd has at most four children.
  a. asserts: Floyd has exactly four children OR Floyd has fewer than four children (and no more than that).
  b. implicates: (i) the speaker is uncertain whether Floyd has exactly four children, and (ii) the speaker is uncertain whether Floyd has fewer than four children (and no more than that).

Under the view that CQs and SQs are semantically equivalent, but that SQs (and not CQs) trigger EIs,[18] how are inferences with SQs in the conclusion predicted to pattern? The answer is not so clear-cut. Recall my observation that the asymmetry in intuitions reported in column B of Table 1 suggests a distinction between contradictory and noncontradictory EIs. Bearing in mind this distinction, there are four logically possible predictions that this pragmatic account might make.

1. Both contradictory and noncontradictory unentailed EIs invalidate inferences.

2. Neither contradictory nor noncontradictory unentailed EIs invalidate inferences.

3. Only noncontradictory, but not contradictory, unentailed EIs invalidate inferences.

4. Only contradictory, but not noncontradictory, unentailed EIs invalidate inferences.

It's unclear which, if any, of these is "the" prediction that this account makes. Option 1 seems particularly reasonable. Magri 2009 and Singh 2010, among others, have argued that ignorance inferences, including those arising from overt disjunctions, are mandatory and non-defeasible, much like asserted content. If the EIs arising from SQs are of the same nature, then, assuming that an unentailed, non-defeasible implicature invalidates an inference (in the same way that unentailed asserted content does), we'd expect both contradictory and noncontradictory unentailed EIs to invalidate inferences.

[18]For convenience I'll continue to just write "EI", which in the context of this pragmatic account can be read as "epistemic implicature", i.e., an epistemic implication derived as an implicature.

If, however, EIs are defeasible (or not mandatory), and assuming there is no pragmatic distinction, in terms of inferential reasoning, between contradictory and noncontradictory EIs, then option 2 seems quite reasonable: neither type of EI is expected to invalidate an inference; rather, they are defeated (or not calculated) when unentailed.

Or, if EIs are defeasible (not mandatory), option 3 also seems reasonable: if only one of the two types of epistemic implicature were able to invalidate an inference, we'd more reasonably expect the noncontradictory type to do so, rather than only the contradictory type. Implicatures are defeated precisely when they're inconsistent with some context or utterance; thus, we'd expect an EI that's inconsistent with a premise set to be defeated, while a noncontradictory EI might remain to invalidate the inference.

That leaves option 4, which seems to be the dark horse: although it's a logical possibility, it has little rationale behind it, especially given the discussion of option 3. That is, either no EIs should ever be expected to invalidate inferences (option 2), or, if EIs were able to invalidate inferences, we'd expect noncontradictory ones do so before (or in addition to) contradictory ones (options 1 and 3). Option 4 seems untenable.

Curiously, option 4 actually gets the data exactly right: contradictory EIs do invalidate inferences, while noncontradictory EIs do not. Conversely, option 3 gets the data exactly backwards, and options 1 and 2 each only get part of the data right.

Notice, too, that option 1 actually makes the same predictions as our revised version of Geurts and Nouwen's (2007) semantic account: each makes the right predictions for rows 4–7 of column B, in which contradictory unentailed EIs result in intuitively invalid inferences, but not for rows 1–3, in which noncontradictory unentailed EIs result in intuitively valid inferences. In other words, option 1 puts the two accounts on a par.[19]

Rather than try to rationalize option 4, and given the independent evidence for the non-defeasibility (mandatoriness) of epistemic implicatures, I assume that option 1 is the only real tenable option. In this next section, I offer a proposal that indeed salvages both option 1 of this pragmatic

[19]The exception is that our revised semantic account makes "at most *n*" nonmonotone, whereas this pragmatic account makes it downward-monotone, just like "fewer than *n*". I return to this issue in section 6.

account as well as our revised version of Geurts and Nouwen's (2007) semantic account.

4 The proposal

To recap, the puzzle we wish to solve is the following: Why do non-contradictory, but nevertheless unentailed EIs—viewed either as semantic content or as non-defeasible (mandatory) implicatures—*not* invalidate otherwise valid inferences, given that contradictory EIs *do*? For example, why does (2) intuitively follow from (1), despite the fact that the EIs of (2) are unentailed by (1)? In the previous section, I showed that, under both the (revised) semantic account of Geurts and Nouwen 2007 as well as the pragmatic account of Büring 2008 (assuming option 1 to be the most tenable), noncontradictory unentailed EIs are expected to invalidate inferences; hence, the inference from (1) to (2) is expected to be intuitively invalid.

When put in those words, the puzzle seems to corner us into concluding that subjects in inferential tasks somehow simply *assume* unentailed EIs to be true, despite not following logically from a given premise set. In other words, the reason that unentailed noncontradictory EIs do not invalidate inferences is that they are assumed by default by the subject to be true, and the reason that contradictory EIs do invalidate inferences is that they are neither entailed by the premise set, nor can they be assumed by the subject do be true (since they contradict the premise set).

Thus, we could for example make the following proposal.[20]

[20] By "explicit premise set", I just mean the (possibly singleton) set containing the sentence(s) given to the subject. Later on, I contrast the explicit premise set with the so-called extended premise set that contains the explicit premises together with all assumptions made by the subject.

Also, what I write as "p", "q", etc. should be interpreted not as a logical formulas (or as logical translations of a natural language sentences), but simply as shorthand for any natural sentence whose complete meaning may be a composite of semantic and pragmatic content, e.g., EIs; and "$\Diamond p$" should likewise be interpreted as shorthand for any natural language sentence of the form "It's possible that" or "It may be the case that", whose translation into logical form would involve the epistemic possibility operator, "$\Diamond$". The difficulty here is that I'm trying to make a generic proposal in terms of both the semantic account, whose predictions are based on entailment relations beteween logical forms, and the pragmatic account, whose predictions involve utterances. The sloppiness here is, I believe, harmless.

(12) **Universal possibility assumption (UPA) (weakest version)**
Given a subject S, an explicit premise set P, and a conclusion q that gives rise to a set of EIs E (whose members are of the form $\Diamond p$), with S's task being to determine whether q follows from P, then for each $\Diamond p \in E$ that is consistent with P, S assumes $\Diamond p$ to be true.

This formulation is quite specific, and it's unclear what principle such reasoning follows from. Instead, I propose something much stronger (more general), for which I provide independent evidence later on. The idea is as follows: whatever is not known by the subject to be false is assumed by the subject to *possibly* be true. In other words, *any* sentence of the form $\Diamond p$ (whether an EI or not) is assumed to be true, provided such an assumption is consistent with P. I call this the *universal possibility assumption* (UPA) made by subjects.[21]

(13) **Universal possibility assumption (UPA) (strong version)**
Given a subject S, an explicit premise set P, and a conclusion q, with S's task being to determine whether q follows from P, for any sentence $\Diamond p$ consistent with P, S assumes $\Diamond p$ to be true.

The intuition is that a subject assumes P and only P to be the set of all things she knows. If P does not exclude some possibility, then neither can the subject. The set of all possibilities not excluded by P (hence, consistent with P and assumed by the subject) can be thought of as the "closure" of P under the UPA. Clearly, the closure of P is infinite.

Let's run through an example. Suppose we have (1) as a single premise and (2) as a conclusion, repeated below.[22]

(1) ($\Box$) Floyd has more than two children.

Finally, for a sentence p and a set of sentences P to be consistent, I just mean that there is no $r \in P$ such that p and r are contradictory (cannot both be true).

[21]The present formulation is indeed *too* strong. Evidence for why, as well as suitable constraints to fix the proposal, will be explored in section 5.

[22]The parenthetical "$\Box$" is to remind the reader that by assumption these sentences are interpreted as being in the scope of an epistemic necessity operator, as if there were an "It's certain that" in the syntax. We initially attributed this assumption to Gricean maxims (a speaker only says what she believes or knows to be true). Here, it can be taken as an explicit indicator that the premises are what the speaker "knows" to be true, for the purposes of the inferential task.

(2) Floyd has at least three children.

The premise, (1), is consistent with an infinite number of other sentences, including (14), repeated below, and (15).

(14) It's possible that Floyd has exactly three children.

(15) It's possible that Floyd has more than three children.

In determining whether (2) follows from (1), subjects assume, among many other things, that (14) and (15) are each true.

But (14) and (15) are precisely the EIs of (2).[23] Thus, since the non-EI meaning of (2) is entailed by (1), and the EIs of (2) are assumed to be true in this task, (2) is correctly expected to be judged to follow from (1).

Now suppose a subject is tasked with determing whether (2) follows instead from the single premise (3), repeated below.

(3) ($\Box$) Floyd has exactly three children.

Clearly, (14), one of the EIs of (2), is not only consistent with (3), but in fact weaker than (entailed by) it and thus true with no assumption necessary. However, (15), the other EI of (2), is *not* consistent with (3), and so cannot be assumed by the subject to be true, according to the UPA. Since this EI cannot be rendered true either by entailment by the premise set or by assumption, we correctly expect (2) to not be judged to follow from (3).

More generally, according to this formulation of subjects' reasoning, an EI is expected to invalidate an otherwise valid inference if and only if the EI is not rendered true either by assumption or by entailment by the premise set.

Let's do one last example. Suppose we have (7) as a single premise and (8) as a conclusion, repeated below.

(7) ($\Box$) At most three students smoke.

[23] Actually, we said that EIs are speaker-oriented, i.e., of the form "The speaker considers it possible that". In the case at hand, "It's possible that" really means "The *subject* considers it possible (by assumption) that". One way to deal with this mismatch is to say that, in an inferential task, the subject evaluates the inference in terms of whether she herself can infer *and then utter* the conclusion. That is, the EIs arising from a conclusion are subject-oriented, or rather, speaker-oriented in a hypothetical scenario where the subject is the new speaker. See also footnote 22, in which we likewise take premises to be interpreted with respect to subjects' epistemic state.

(8) At most three students smoke cigars.

For this inference from (7) to (8) to be judged valid under this proposal, the EIs of (8), given below, must either be entailed by (7) or assumed by the subject by the UPA.

(16) It's possible that exactly three students smoke cigars.

(17) It's possible that fewer than three students smoke cigars (and no more than that).

Although not entailed by (7) (as shown earlier), these EIs are nonetheless consistent with (7), and so are assumed to be true by the subject. Thus, the full meaning of (8) follows by either entailment or assumption, and the inference from (7) to (8) is correctly predicted to be judged valid.

A slightly more precise, but equivalent, way of thinking about what it means for a subject to "assume $\Diamond p$ to be true" is to say that a subject extends the initial premise set P to include $\Diamond p$.[24] Let's denote this new, extended premise set, containing all sentences of the form $\Diamond p$ assumed to be true by the UPA, as P^*, the closure of P under the UPA. That is, if P is a premise set explicitly given in an inferential task, and P_{upa} is the set of all sentences of the form $\Diamond p$ assumed to be true by the UPA (with respect to P), then $P^* = P \cup P_{upa}$.

Trivially, if $\Diamond p \in P^*$, then $P^* \vDash \Diamond p$, where "$\vDash$" denotes semantic entailment. Recall now my use of "$\vDash^*$" in Table 1 to denote the intuitive validity of an inference. It follows from the discussion above that, according to my proposal, $P \vDash^* q$ iff $P^* \vDash q$ (where, as before, $P^* = P \cup P_{upa}$). In plain English: An inference from P to q will be judged valid just in case (each part of the meaning of) q is either entailed by P or is assumed to be true via the UPA.

This proposal makes some very clear predictions, which I explore in the next section.

[24]The equivalence is due to the fact that if P does not semantically entail $\Diamond p$, then assuming $\Diamond p$ to be true given P necessarily amounts to updating P to be $P \cup \{\Diamond p\}$; and conversely, updating P to be $P \cup \{\Diamond p\}$ always entails $\Diamond p$.

5 Some predictions

5.1 *Inference blocking*

According to my proposal, a conclusion q is judged to follow from a premise set P just in case (the complete meaning of) q follows from P^*, the set of explicit premises together with the set of all epistemically possible sentences consistent with P. The proposal correctly predicts (2) to intuitively follow from the single premise (1). However, because the members of P_{upa} are fully determined by P, then changing P generally changes P_{upa}, and hence P^*. For example, if $\Diamond p$ is consistent with an initial premise set P, then $\Diamond p \subseteq P_{upa} \subseteq P \cup P_{upa} = P^*$; but if P is updated to include some $(\Box)r$, i.e., we let $P' = P \cup \{(\Box)r\}$ be the updated premise set, and $(\Box)r$ is inconsistent with p, then clearly $\Diamond p \notin P'_{upa}$, hence $\Diamond p \notin P'^*$. In other words, if an explicitly given premise set is extended with further explicit premises, then some assumptions, e.g., EIs, can no longer be made by the subject. In that case, it's predicted that intuitively valid inferences like the one from (1) to (2) can be "blocked", so to speak, with extra premises that prevent the relevant EIs from being assumed by contradicting them. In this section, I show that such a blocking effect is indeed attested.

Recall that (2) intuitively follows from (1) and that (5) intuitively follows from (4), all repeated below.

(1) Floyd has more than two children.

(2) Floyd has at least three children.

(4) Floyd has fewer than five children.

(5) Floyd has at most four children.

Let's now add an extra premise to (1) and (4) that contradicts the EIs of (2) and (5), respectively. I'll do so simply by conjoining the original and the added premises with "and/but" and rename the new, conjunctive premise with a prime symbol, "′".

(1)′ Floyd has more than two children, and/but not more than three.

(4)′ Floyd has fewer than five children, and/but not fewer than four children.

The inference from (1)′ to (2) is now blocked. More precisely, the judgment here is predicted to be just like the judgment on whether (2) follows from (3), repeated below—which we said it does *not*—since (1)′ amounts to precisely the same thing as (3) (they are synonymous).

(3) Floyd has exactly three children.

Indeed, my intuition is that the two inferences are equally bad.

Similarly, the inference from (4)′ to (5) is blocked. More precisely, the judgment here is predicted to be just like the judgment on whether (5) follows from (9), repeated below—which we said it does *not*—since (4)′ amounts to precisely the same thing as (9) (they are synonymous).

(9) Floyd has exactly four children.

Again, my intuition is that the two inferences are equally bad.

Finally, we can play the same game with the downward entailingness of "at most *n*". Recall that (8) is judged to follow from (7), repeated below.

(7) At most three students smoke.

(8) At most three students smoke cigars.

As before, we can add an extra premise to block this inference.

(7)′ At most three students smoke, and/but no student smokes cigars.

My intuition is that it's certainly more difficult to infer (8) from (7)′. The explanation, of course, is that (8) implies that the speaker considers it possible that exactly three students smoke cigars, which directly contradicts the added premise in (7)′.

The fact that the intuitively valid inferences reported in rows 1–3, column B of Table 1 can be blocked suggests that these inferences are not (purely) cases of semantic entailment. The reason is that entailment has the property of being monotonic: if $P \vDash q$, then $P \cup \{p\} \vDash q$ (provided p is consistent with P). That is, a premise set can be freely extended without affecting valid entailments. Thus, these intuitively valid inferences that can be blocked must involve some sort of nonmonotonic, or defeasible, reasoning. The UPA is precisely the kind of reasoning process that predicts such inference patterns.[25]

[25]For discussion of nonmonotonicity in linguistics, see Thomason 1997.

5.2 Overt modals

A second prediction of my proposal is that, because an infinite number of epistemically possible sentences are assumed by a subject to be true (added to the premise set), then given any explicit premise set, any of those very same epistemically possible sentences is predicted to intuitively follow. That is, if P is an explicit premise set, then each sentence of the form $\Diamond p$ that is assumed to be true by the UPA should be judged to follow from P.

This can easily be tested by asking whether sentences of the form "It's possible that" or "It may be the case that" intuitively follow from a premise. In this section, I show that, broadly speaking they do; however, the original formulation of my proposal seems to be too strong (predict too many inferences), and I provide several ways it can be constrained to fit intuitions. Crucially, whatever constraints are ultimately required to fit the intuitions on overt modals should not affect the proposal's ability to capture the intuitions on SQs, as far as I can tell.

Suppose we have as a single premise like the sentence in (18). There are infinitely many sentences consistent with (18), including (19). Under my proposal, then, (19) is assumed by the subject to be true.

(18) ($\Box$) Floyd went to Italy.

(19) Floyd may have gone to Rome.

If so, then the inference from (18) to (19) is predicted to be intuitively valid. Indeed, I claim that it is. It is a particularly reasonable inference especially when compared with the inference from (18) to (20), which is clearly invalid.

(20) Floyd went to Rome.

Intuitively, the reason that (20) is not inferable from (18) is that, if (18) is true, then for all we know, Floyd may have gone to Florence, and not Rome, in which case (18) is true but (20) is false. In other words, it's easy to think of a counterexample. In contrast, (19) is inferable from (18) in the sense that, if (18) is true, then for all we know, it *may* the case that Floyd went to Rome (we have no evidence or reason to rule out Floyd's going to Rome). If we find out later that Floyd went to Florence and not Rome, then (19) becomes false if *re*-evaluated in terms of this new information, but it was still true, as far as we knew, previously, without any extra information.[26]

My proposal can be seen as a way of characterizing this idea of "for all we know", or "for all the subject knows".[27] Obviously, however, a subject applies more knowledge to an inferential task than just the explicitly given premise set. For example, world knowledge, e.g., the knowledge that Rome is a popular tourist destination in Italy, is clearly important for the task above. Thus, we can perhaps anticipate that not every single epistemically possible sentence that is merely consistent with a premise set will be assumed by the subject to be true, particularly not those that, despite not contradicting the premise set, do in some sense contradict or go against the subject's world knowledge.

For example, it may be that (19) intuitively follows from (18) not simply because it's consistent with (18), but also because of the knowledge that Rome is a popular tourist destination in Italy. In other words, (19) is a *reasonable* (or even *likely*) thing to assume, given the premise together with our knowledge of the world. If we switch out Rome with a less visited or less known place, say, Modica, Sicily, do intuitions differ? That is, does (21) intuitively follow from (18)?

(21) Floyd may have gone to Modica, Sicily.

Given that (21) is obviously consistent with Floyd's going to Italy, my proposal predicts this inference to indeed be intuitively valid. My intuition, however, is that it's more difficult an inference to make, though not totally out of the question.

Aside from world knowledge, a perhaps more serious concern is that

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[26] As in the previous section, the intuitively valid inferences with overt modals can be blocked by adding extra premises.

(18)′ Floyd went to Italy, and/but not to Rome.

Clearly, (19) does not follow from (18)′. Thus, these inferences are likewise nonmonotonic, suggesting that a reasoning process like the UPA is at work.

[27] My proposal is not the only way to account for the inference patterns with overt epistemic modals. A potential alternative analysis is that a premise restricts the modal base (in the sense of Kratzer 2012a,b) relative to which the the modal in a conclusion is interpreted, such that the conclusion is in fact weak enough to be semantically entailed by the premise. However, this analysis does not straightforwardly generalize to the inference patterns with SQs, which involve no modal base.

relevance plays absolutely no role in my proposal. There seems to be a close connection between determining the validity of an inference from A to B and determining the validity (or felicity) of an utterance "A. Therefore, B" or "If A, then B". Since utterances linked by "therefore" or "If ... then" intuitively ought to be relevant to one another, we might therefore expect relevance to play a similar role in inferential tasks.

For example, given the single premise (18), it's clear that (22) is consistent with (18), but does (22) intuitively *follow from* (18)?

(22) Floyd may own a dog.

Not so much, according to my intuitions.[28] There seems to be some kind of expectation by the subject that the conclusion should be relevant to (at least some subset of) the explicit premise set.

One way to constrain my proposal along the lines suggested above is to require a sentence $\diamond p$ not only to be consistent with the premise set P, but also to be consistent with world knowledge (or to be sufficiently reasonable or likely) and to be sufficiently relevant to (at least one premise in) the premise set, as judged by the subject.[29]

(23) **Universal possibility assumption (weaker version)**
Given a subject S, an explicit premise set P, and a conclusion q, with S's task being to determine whether q follows from P, for any sentence p consistent with P together with S's world knowledge, and bearing a relevance relation to (at least one p in) P as judged by S, S assumes $\diamond p$ to be true.

One question we can ask now is whether these constraints affect the ability of the proposal to account for the inference patterns with SQs. They certainly don't have any real effect on the predictions about the inference from (1) to (2) or from (4) to (5): the EIs are clearly both con-

[28]This intuition is consistent with the oddity of (i) and (ii): going to Italy has no perceived relation (causal or otherwise) to owning a dog.

(i) #Floyd went to Italy. Therefore, Floyd may own a dog.

(ii) #If Floyd went to Italy, then Floyd may own a dog.

[29]I say the conclusion should be relevant to at least one premise in the premise set, but it's not at all clear whether this is sufficient.

sistent with world knowledge (e.g., that Floyd may have exactly three children is an entirely reasonable real-world possibility), and relevant to the premises (both deal with the number of children Floyd has).

Things are slightly more complicated in the case of the intuitively valid inference from (7) to (8), repeated below.

(7) ($\Box$) At most three students smoke.

(8) At most three students smoke cigars.

In order for this inference to be correctly predicted to be intuitively valid, it must be the case (among other things we ignore here) that (24) is both consistent with world knowledge and relevant to (7).

(24) It may be the case that exactly three students smoke cigars.

First, (24) is entirely reasonable, given (7) together with world knowledge. Second, since the task deals with the number of student smokers, and since the number of student cigar smokers partly determines the number of student smokers, we can safely assume (24) to be sufficiently relevant for the inference at hand. Hence, we still correctly predict the inference from (7) to (8) to be judged valid.

It's quite easy, however, to add something totally irrelevant.

(25) At most three students smoke and own a dog.

My intuition is that (25) intuitively does follow from (7). However, even though we've *added* something irrelevant, the question is whether all of (26) is relevant to (7); and considering they still both concern the number of student smokers, I'd argue that the relevance is sufficiently strong.

(26) It may be the case that exactly three students smoke and own a dog.

What we can also do is fiddle with the world knowledge (or likelihood) constraint. For example, (27) is highly unlikely in the present day. Thus, under the constrained proposal, a subject would potentially not assume (27) to be true.

(27) It may be the case that exactly three students smoke and have been to the moon.

But (27) is one of the EIs of (28). If it cannot be assumed to be true, and since it's clearly not entailed by (7), then (28) is predicted to not intuitively follow from (7).

(28) At most three students smoke and have been to the moon.

Indeed my intuitions here agree.

Obviously, there are many subtleties here to explore, potentially in the proper experimental setting with real subjects. Only with more reliable data can suitable constraints be made to (the strong version of) my proposal. However, whatever these constraints turn out to be, it seems unlikely that they'd prevent the proposal from capturing the original data on SQs.[30]

5.3 Overt disjunction

The following proposal concerns Büring's (2008) disjunctive analysis of SQs more than Geurts and Nouwen's (2007) semantic analysis. If the ignorance inferences arising from overt disjunctions and the epistemic implications arising from SQs are of the same implicature variety, then should we see the same patterns of inference invalidation with unentailed EIs?

Consider again (10), which as we said has as implicatures (i) that the speaker is uncertain whether Lenny called, and (ii) that the speaker is uncertain whether Carl called.

(10) Lenny or Carl called.

Now suppose (29) is a premise.

(29) ($\Box$) Lenny called.

Since the first implicature of (10) contradicts (29) we expect the inference from (29) to (10) to be intuitively invalid.

Although semanticists generally take a sentence like (29) to *entail* (10), it's unclear to me whether such an inference is really judged valid by

[30]Keep in mind also that the added constraints, as I've given them, refer explicitly to the *subject*'s world knowledge (judgment of likelihood) and to the *subject*'s judgment of relevance. Thus, it's predicted that the intuitive validity of inferences will vary from subject to subject (a prediction I believe is correct), making the data all the more complex.

native speakers untrained in semantics or logic. That is, the idea that (29) entails (10) seems uncontroversial given a suitable theoretical notion of entailment, but whether those unfamiliar with the notion of semantic entailment actually judge the inference from (29) to (10) to be valid, I don't know.

Regarding the intuitions of those who do judge the inference valid, one explanation is that there is only one "or" in English, which is routinely used to express both "logical", non-epistemic statements (especially by logicians, computer scientists, and mathematicians), as well as epistemic ones.[31] If SQs are semantically equivalent to CQs, but only the former trigger EIs, then it perhaps makes sense that a speaker wishing to convey a non-epistemic, purely logical statement (a relation between set cardinalities) will systematically use a CQ, and that SQs, at least in declaratives, really are reserved to imply something epistemic.[32]

6 Concluding remarks

What I hope to have shown in this paper is that inference patterns with epistemic modality are quite complex, involving a sort of default, nonmonotonic reasoning that is difficult to fully and precisely characterize. I proposed that subjects in inferential tasks make the assumption that anything that is not known to be false is possibly true. This proposal explains the asymmetry between contradictory and noncontradictory EIs identified in section 2, which I argued in section 3 is a puzzle for the two accounts of SQs presented in this paper. My proposal leads to several clear predictions, many of which are borne out: intuitively valid inferences can be blocked with extra premises that prevent the EIs of a conclusion from being assumed, and similar nonmonotonic inferences involving

[31] Take, for example, free choice vs. epistemic readings of disjunctions under deontic modals. A sentence like (i) is ambiguous between a free choice reading and an epistemic reading (i.e., with ignorance inferences).

(i) You're allowed to have coffee or tea.

Under the free choice reading, you're allowed to have the coffee and you're allowed to have the tea (and the speaker is certain of this); under the epistemic reading, the speaker is uncertain whether it's coffee or tea that you're allowed to have.

[32] The fact that EIs arising from SQs in the scope of deontic modals actually disappear would remain a mystery under this explanation.

overt epistemic modals are indeed attested and can be blocked. However, intuitions indicate that the strong version of my proposal must be suitably constrained in several ways, e.g., by appealing to subjects' world knowledge as well as relevance. At this point it's unclear what exact constraints are necessary; however, ultimately they would appear to pose no problem to the proposal's ability to capture the data on inferences with SQs.

As a final note, recall that Geurts and Nouwen's (2007) analysis (and our revision of it) makes "at most n" semantically nonmonotone. Their analysis thus predicts (i) that "at most n" should not participate in downward entailing inferences, and (ii) that it should not license NPIs. As was shown, both predictions are incorrect. Interestingly, my proposal saves their account from the first wrong prediction: the UPA allows "at most n" to participate in downward inferences. Assuming, however, that the ability of an expression to license NPIs depends on its *semantic* downward monotonicity (and not simply the inferences it participates in), their account cannot be saved from the second wrong prediction. Nevertheless, although their lexical entry cannot be right for "at most", which licenses NPIs, nothing prevents the existence of an expression in *some* language with such a lexical entry. That expression would have the following properties: it would be semantically nonmonotone and thus not license NPIs, but it would, under my account, nonetheless participate in downward inferences. Whether or not such an expression exists must, of course, await further research.

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